St George Girls High School

Year 11

Common Test 2

June 2008

Mathematics Extension 1

General Instructions

1. Time – 75 minutes
2. Use only black or blue pens.
3. Board-approved calculators may be used.
4. All necessary working must be shown in all questions.
5. Start each question on a new page.
6. All 6 questions may be attempted.

Question 1 – 12 marks – (Start a new page) Marks

a) Solve $5^x = 30$. Give your answer correct to 3 significant figures. 2

b) The first 3 terms of an arithmetic sequence are $3 - x,\ 1 + 2x,\ 12$. Find the value of x. 2

c) A sequence is defined by:

$T_1 = 2 \quad T_2 = 3$ and $T_{n+2} = T_{n+1} + 2T_n \ (n \geq 1)$. 2

Find the value of T_5

d) How many terms of the geometric sequence 3

3, 6, 12, 24, 48, ... are less than 10^7 ?

e) Solve $\frac{4x}{3x-2} \geq 1$ 3

Question 2 – 12 marks – (Start a new page) **Marks**

a) The lines $3x - 2y + 7 = 0$, $y = x + 1$ and $4x + ky - 3 = 0$ are concurrent. Find the value of k. 3

b) (i) Find the centre and radius of the circle $x^2 + y^2 - 2x + 2y - 2 = 0$ 2

(ii) Using (i) determine whether or not the line $3x + 4y - 11 = 0$ is a tangent to the circle. You must give a reason for your decision. 3

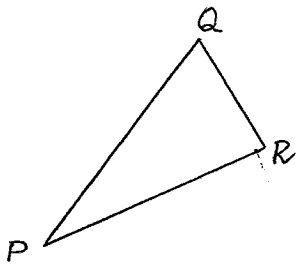
c) (i) Write down, in terms of k, the coordinates of the point A that divides the interval joining $P(3, 2)$ and $Q(-2, 5)$ in the ratio $k:1$ ~~1~~ 2

(ii) Find the ratio in which the line $x + y - 7 = 0$ divides interval PQ. ~~3~~ 2

Given that the point A lies on $x+y-7=0$, find the ratio in which the line $x+y-7=0$ divides the interval PQ

Question 3 – 12 marks – (Start a new page) **Marks**

a) Prove that $\cot\theta + \frac{\sin\theta}{1+\cos\theta} = \operatorname{cosec}\theta$ 3

b) 

The diagram shows 3 points P, Q and R

The bearing from P to Q is $022°T$

The bearing from P to R is $065°T$

The distance PQ and PR are 9km and 12km respectively.

(i) Copy the diagram onto your paper and mark on it the given information. 1

(ii) Find the distance from Q to R. 2

(iii) Find the size of $\angle QRP$ 2

(iv) Find the bearing Q from R. 1

c) Eliminate θ from the pair of equations $x = 3\cos\theta - 2 \quad y = 2\sin\theta + 1$ 3

Question 4 – 12 marks – (Start a new page) **Marks**

a) For an arithmetic sequence $T_1, T_2, T_3, \ldots$ it is given that

$T_3 + T_5 = 37$ and $T_7 + T_{11} = 52$

Find the first term, a, and common difference, d, of this sequence. 3

b) Simplify $\frac{15^x+5^x}{12^x+2^{2x}}$ 2

c) Solve $\log_2(x-1) + \log_2(x+3) = 5$ 4

d) Find the equation of the line that passes through the point of intersection of $3x + 2y + 7 = 0$ and $x + 4y - 1 = 0$ and the point $(1, 2)$ 3

Question 5 – 12 marks – (Start a new page) **Marks**

a) (i) Sketch the graphs of $y = |2x + 1|$ and $y = x + 3$ on the same diagram. 2

(ii) Solve the equation $|2x + 1| = x + 3$ 3

(iii) Using (i) and (ii), or otherwise, write down the solution to the inequation

$|2x + 1| < x + 3$ 1

b) Sketch on separate diagrams the regions defined by

(i) $y \geq \frac{4}{x}$ 2

(ii) $y < \sqrt{16 - x^2}$ 2

c) The n^{th} term of a sequence is given by $T_n = 4 + 5n$. Show that the sequence is arithmetic. 2

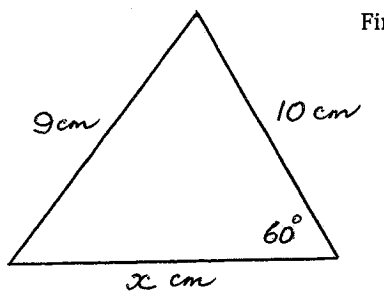
Question 6 – 12 marks – (Start a new page) **Marks**

a) Solve for $0° \leq \theta \leq 360°$

(i) $3\sin 2\theta - 2 = 0$ 2

(ii) $\sin^2\theta - 2\sin\theta\cos\theta - 3\cos^2\theta = 0$ 4

b) Find the value of x. 4



c) Prove that for real numbers a, b 2

$a^2 + b^2 \geq 2ab$

End of Paper

Year 11 Solutions Extension 1 18/6/2008

Question 1

(a) $5^x = 30$

$\log 5^x = \log 30$

$x \log 5 = \log 30$ 1

$x = \frac{\log 30}{\log 5}$ ½

$= 2.11$ ½

(b) $3-x,\ 1+2x,\ 12$

For an AP

$12-(1+2x) = 1+2x-(3-x)$ 1

$12-1-2x = 1+2x-3+x$

$11-2x = -2+3x$ ½

$-5x = -13$

$x = \frac{13}{5}$ ½

(c) $T_1 = 2 \quad T_2 = 3 \quad T_{n+2} = T_{n+1} + 2T_n$

for $n=1$ $T_3 = T_2 + 2T_1$

$= 3+4$

$= 7$ ½

$n=2$ $T_4 = T_3 + 2T_2$

$= 7+6$

$= 13$ ½

$n=3$ $T_5 = T_4 + 2T_3$

$= 13+14$

$= 27$ 1

$\therefore T_5 = 27$

(d) $3, 6, 12, 24, 48 \ldots$

$a=3 \quad r=2$

$ar^{n-1} < 10^7$

$3.2^{n-1} < 10^7$ 1

$2^{n-1} < \frac{10^7}{3}$

$(n-1)\log 2 < \log \frac{10^7}{3}$

$n\log 2 - \log 2 < \log \frac{10^7}{3}$

$n\log 2 < \log \frac{10^7}{3} + \log 2$

$n < \frac{\log \frac{10^7}{3} + \log 2}{\log 2}$ 1

$n < (7.523)$ 22.668...

$\therefore n = (7)\ 22$ 1

$\therefore$ 22 terms are less than 10^7

(e) $\frac{4x}{3x-2} \geq 1$

$4x(3x-2) \geq (3x-2)^2$ 1

$12x^2 - 8x \geq 9x^2 - 12x + 4$

$3x^2 + 4x - 4 \geq 0$

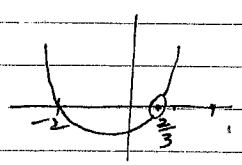
$(3x-2)(x+2) \geq 0$ 1

$\frac{2}{3} \quad -2$

$3x-2 \neq 0$

$3x \neq 2$

$x \neq \frac{2}{3}$



$\therefore x > \frac{2}{3}$ 1

$x \leq -2$

Question 2

(a) $3x - 2y = -7$

$x - y = -1$

$3x - 3y = -3$

$y = -4$

$x + 4 = -1$

$x = -5$

$\therefore (-5, -4)$ also lies on

$4x + ky - 3 = 0$

$4.-5 + k.-4 - 3 = 0$

$-20 - 4k - 3 = 0$

$-4k = 23$

$k = \frac{-23}{4}$

(b)(i) $x^2 + y^2 - 2x + 2y - 2 = 0$

$x^2 - 2x + 1 + y^2 + 2y + 1 = 2 + 2$

$(x-1)^2 + (y+1)^2 = 4$

centre $= (1, -1)$

radius $= 2$

(ii) $d = \frac{|3x_1 + 4y_1 - 11|}{\sqrt{3^2 + 4^2}}$

$= \frac{|3\times 1 + 4\times -1 - 11|}{\sqrt{25}}$

$= \frac{12}{5}$

$= 2.4$

This is greater than radius

$\therefore$ line is not a tangent.

(c)(i) $k:1 \quad (3, 2) \quad (-2, 5)$

$x = \frac{1.x_1 + k.x_2}{k+1} \qquad y = \frac{1.y_1 + k.y_2}{k+1}$

$= \frac{3-2k}{k+1} \qquad = \frac{2+5k}{k+1}$

$\therefore A = \left(\frac{3-2k}{k+1}, \frac{2+5k}{k+1}\right)$

(ii) $\left(\frac{3-2k}{k+1}, \frac{2+5k}{k+1}\right)$ must lie on

$x + y - 7 = 0$

$\therefore \frac{3-2k}{k+1} + \frac{2+5k}{k+1} - 7 = 0$

$3-2k + 2+5k = 7(k+1)$

$5 + 3k = 7k + 7$

$-2 = 4k$

$-\frac{1}{2} = k$

$\therefore$ ratio $-\frac{1}{2} : 1$

$-1 : 2$

Question 3

$$\frac{\cot\theta + \sin\theta}{1+\cos\theta} = \operatorname{cosec}\theta$$

$$LHS = \frac{\cot\theta + \sin\theta}{1+\cos\theta}$$

$$= \cot\theta + \frac{\sin\theta}{1+\cos\theta} \cdot \frac{1-\cos\theta}{1-\cos\theta}$$

$$= \cot\theta + \frac{\sin\theta - \sin\theta\cos\theta}{1-\cos^2\theta}$$

$$= \cot\theta + \frac{\sin\theta}{\sin^2\theta} - \frac{\sin\theta\cos\theta}{\sin^2\theta}$$

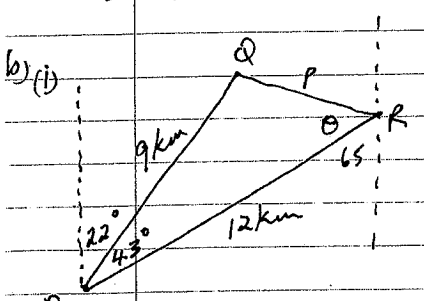
$$= \cot\theta + \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta + \operatorname{cosec}\theta - \cot\theta$$

$$= \operatorname{cosec}\theta$$

$$= RHS.$$

b) (i)



(ii) $p^2 = 9^2 + 12^2 - 2.9.12.\cos 43$

$p^2 = 105.09$

$p = 10.25$

$\therefore QR = 10.25$ km

(iii) $\frac{\sin\theta}{9} = \frac{\sin 43}{10.25}$

$\sin\theta = \frac{9\sin 43}{10.25}$

$\therefore \theta = 36.78^\circ$

$= 36^\circ 47' \approx 37^\circ$

(iv) bearing $= 36^\circ 47' + 65 + 180$

$= 281^\circ 47' \approx 282$

(c) $x = 3\cos\theta - 2 \qquad y = 2\sin\theta +$

$\frac{x+2}{3} = \cos\theta \qquad \frac{y-1}{2} = \sin$

$\left(\frac{x+2}{3}\right)^2 = \cos^2\theta \qquad \left(\frac{y-1}{2}\right)^2 = \sin^2$

$\therefore \left(\frac{x+2}{3}\right)^2 + \left(\frac{y-1}{2}\right)^2 = \cos^2\theta + \sin^2\theta$

$\frac{(x+2)^2}{9} + \frac{(y-1)^2}{4} = 1$

Question 4

(a) $T_3 + T_5 = 37 \Rightarrow a+2d+a+4d=37$

$T_7 + T_{11} = 52 \quad a+6d+a+10d=52$

$\therefore 2a+6d = 37 \quad \cdots ①$

$2a+16d = 52 \quad \cdots ②$

② − ①

$10d = 15$

$d = \frac{15}{10}$

$2a + 6.1\frac{1}{2} = 37$

$2a = 28$

$a = 14$

$\therefore a = 14 \quad d = 1\frac{1}{2}$

b) $\frac{15^x + 5^x}{12^x + 2^{2x}} = \frac{3^x.5^x + 5^x}{3^x.4^x + 4^x}$

$= \frac{5^x(3^x+1)}{4^x(3^x+1)}$

$= \frac{5^x}{4^x}$

$= \left(\frac{5}{4}\right)^x$

(c) $\log_2(x-1) + \log_2(x+3) = 5$

$\log_2(x-1)(x+3) = 5$

$(x-1)(x+3) = 2^5$

$x^2 + 2x - 3 = 32$

$x^2 + 2x - 35 = 0$

$(x+7)(x-5) = 0$

$x = -7, 5$ but $x-1$ must be positive

(d) $3x+2y+7 + k(x+4y-1) =$

$3+4+7+k(1+8-1) = 0$

$14 + 8k = 0$

$8k = -14$

$k = -\frac{7}{4}$

$3x + 2y + 7 - \frac{7}{4}(x+4y-1) =$

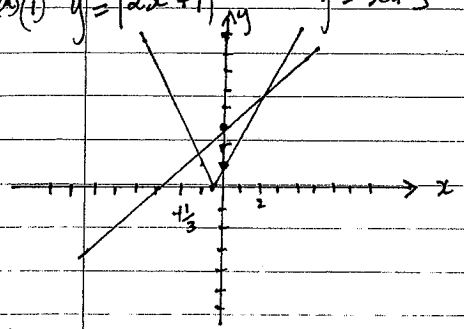
$12x + 8y + 28 - 7x - 28y +$

$5x - 20y + 35 = 0$

$x - 4y + 7 = 0$

Question 5

(a)(i) $y = |2x+1|$ $\quad y = x+3$



)

$|2x+1| = x+3$

$2x+1 = x+3$ $\qquad 2x+1 = -(x+3)$

$x = 2$ $\qquad 2x+1 = -x-3$

$3x = -4$

$x = -\frac{4}{3}$

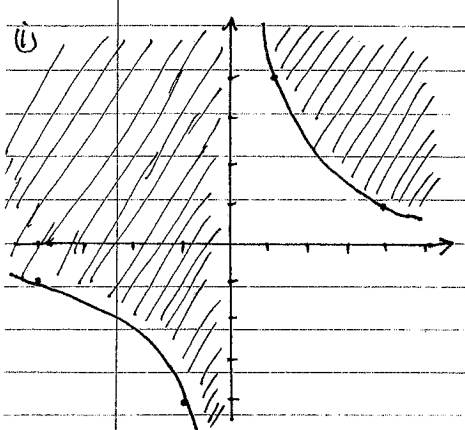
"rom the graph there are two solutions $\therefore x = 2, -\frac{4}{3}$

i) $|2x+1| < x+3$

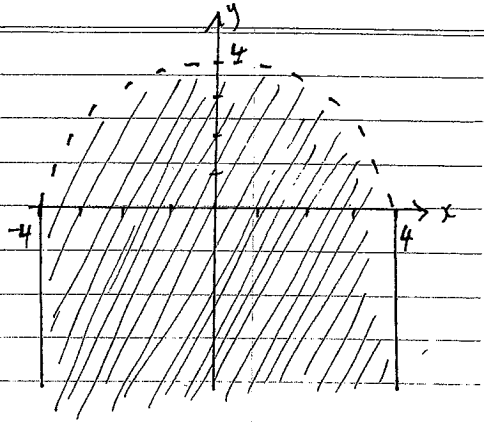
from the graph

$-\frac{4}{3} < x < 2$

ii)



(ii)



(c)

$T_n = 4 + 5n$

$T_1 = 9$

$T_2 = 14$

$T_3 = 19$

$T_4 = 24$

$9, 14, 19, 24, \ldots$

$T_2 - T_1 = 5$

$T_3 - T_2 = 5$

$\therefore$ Sequence is an AP with a common difference $= 5$.

Question 6

(a) (i) $3\sin 2\theta - 2 = 0 \qquad 0 < \theta < 360$

$3\sin 2\theta = 2 \qquad 0 < 2\theta < 720$

$\sin 2\theta = \frac{2}{3}$

$\therefore 2\theta = 41.8, 138.2, 401.8, 498.2$

$\therefore \theta = 20.9^\circ, 69.1^\circ, 200.9^\circ, 249.1^\circ$

(ii) $\frac{\sin^2\theta}{\cos^2\theta} - \frac{2\sin\theta\cos\theta}{\cos^2\theta} - \frac{3\cos^2\theta}{\cos^2\theta} = 0$

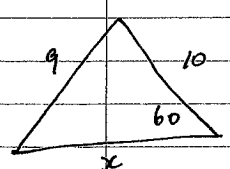
$\tan^2\theta - 2\tan\theta - 3 = 0$

$(\tan\theta + 1)(\tan\theta - 3) = 0$

$\tan\theta = -1, 3$

$\therefore \theta = 135^\circ, 315^\circ, 71^\circ 34', 251^\circ 34'$

(b)



$9^2 = x^2 + 10^2 - 2.x.10\cos 60$

$81 = x^2 + 100 - 20x \cdot \frac{1}{2}$

$81 = x^2 + 100 - 10x$

$0 = x^2 - 10x + 19$

$x^2 - 10x = -19$

$x^2 - 10x + 25 = -19 + 25$

$(x-5)^2 = 6$

$x - 5 = \pm\sqrt{6}$

$x = 5 \pm \sqrt{6}$

(c) $a^2 + b^2 \geq 2ab$

$a^2 + b^2 - 2ab = (a-$

since $(a-b)^2 \geq 0$ for all real numb

$a^2 + b^2 \geq 2ab$