St George Girls High School

Year 11

Common Test – 2

June 2007

Mathematics

Time Allowed: 65 minutes

Instructions

1. All questions should be attempted.
2. Show all working.
3. START EACH QUESTION ON A NEW PAGE.
4. Write on one side of each page only.
5. Marks will be deducted for careless work or poorly presented solutions.
6. Diagrams are not to scale.
7. On the cover sheet of the answer booklet clearly show:
   a) your name
   b) your mathematics class and teacher

Question 1 (10 marks) – **Start a New Page** **Marks**

a) Solve the following inequations:

(i) $11x+4 \le 8x-8$ 2

(ii) $-1 \le 3-2x < 4$ 2

(iii) $x^2-3x-10 \le 0$ 3

b) Find θ to the nearest minute if θ is an acute angle and

(i) $\tan\theta = 0.25$ 1

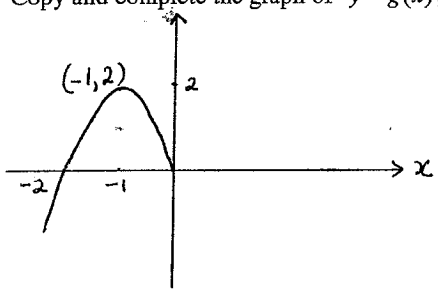
(ii) $\sec\theta = 1.5$ 2

Question 2 (10 marks) – **Start a New Page** **Marks**

a) (i) Write down the algebraic condition used to show that $f(x)$ is an odd function 1

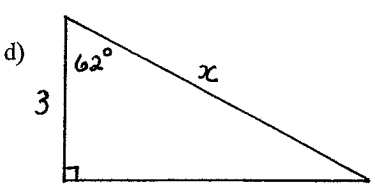
(ii) Show that $f(x)=\dfrac{x^3}{x^2+5}$ is an odd function 2

b) Copy and complete the graph of $y=g(x)$, given that it is an even function. 1

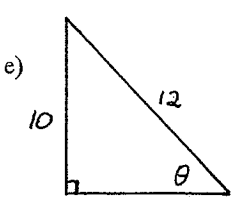


c) Find the exact value of:

(i) $\cos 315^\circ$ (ii) $\sin 600^\circ$ 1 each

d) 

Find the value of x correct to 1 decimal place 2

e) 

Find the value of θ correct to the nearest minute 2

Question 3 (10 marks) – **Start a New Page** **Marks**

a) Solve for x

(i) $|x-7|=3$ 2

(ii) $|x+2|\leq 7$ 2

(iii) $|3x-1|>4$ 3

b) If $\tan\theta=\dfrac{7}{5}$ and $\cos\theta<0$ find the exact value of $\sin\theta$ 3

Question 4 (10 marks) – **Start a New Page** **Marks**

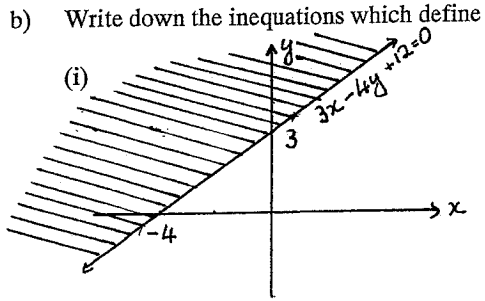
a) On separate diagrams sketch the following regions.

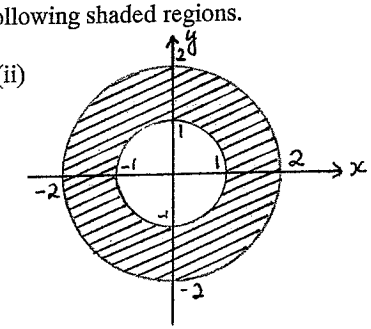
(i) $x+y\leq 4$ 2

(ii) $x^2+y^2<9$ 2

(iii) the intersection of $y>x$ and $y\leq 6-x^2$ 3

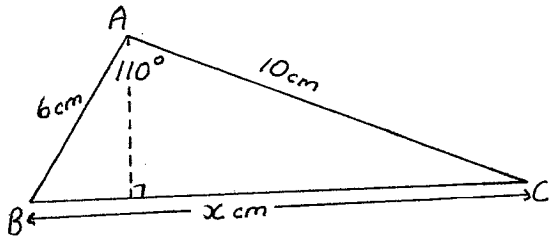
b) Write down the inequations which define the following shaded regions. 1 2

(i) 

(ii) 

Question 5 (10 marks) – **Start a New Page** **Marks**

a)



(i) Find the value of x, correct to 1 decimal place. 2

(ii) Find the area of ΔABC 2

(iii) Hence, or otherwise, find the length of the perpendicular from A to BC. 1

b) Solve for $\theta : 0^\circ \le \theta \le 360^\circ$. Give your answer correct to the nearest degree.

(i) $3\sin\theta - 1 = 0$ 2

(ii) $\tan(\theta + 50^\circ) = -2$ 3

Question 6 (10 marks) – **Start a New Page** **Marks**

a) (i) On the same set of axes sketch the graphs of

$$y = |2x-3| \quad \text{and} \quad y = x+1$$ 3

(ii) Solve the equation $|2x-3| = x+1$ using an algebraic method. 3

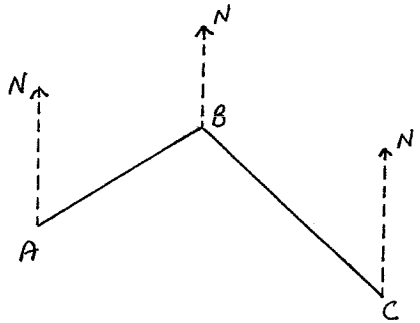
(iii) Using (i) and (ii) or otherwise solve $|2x-3| \le x+1$ 1

b) Solve $0^\circ \le \theta \le 360^\circ$ $\sin 2\theta = \frac{\sqrt{3}}{2}$ 3

Question 7 (10 marks) – **Start a New Page** **Marks**

a) A ship sails from a port A on a bearing of 050° for 150km to a port B. It then sails on a bearing of 127° to an island C which is 295km from B.

(i) Copy the diagram and mark on it all the information given above. 1



(ii) Explain why $A\hat{B}C = 103^\circ$ 1

(iii) Calculate the distance from A to C. 2

(iv) What is the bearing of C from A? (correct to the nearest degree) 3

b) Prove that $\text{cosec}^2\theta + \sec^2\theta = \text{cosec}^2\theta.\sec^2\theta$ 3

End of Paper

Solutions Year 11 Mathematics MM 5
Task 2.

Question 1

(a) (i) $11x + 4 \leq 8x - 8$

$3x \leq -12$

$x \leq -4$

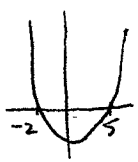
(ii) $-1 \leq 3 - 2x < 4$

$-4 \leq -2x < 1$

$2 \geqslant x > -\frac{1}{2}$

$\therefore -\frac{1}{2} < x \leq 2$

(iii) $x^2 - 3x - 10 \leq 0$



$(x-5)(x+2) \leq 0$

$-2 \leq x \leq 5$

(b) (i) $\tan\theta = 0.25$

$\theta = 14^\circ 2'$

(ii) $\sec\theta = 1.5$

$\frac{1}{\cos\theta} = 1.5$

$\frac{1}{1.5} = \cos\theta$

$\cos\theta = 0.6666$

$\theta = 48^\circ 11'$

Question 2

(a) (i) $f(-x) = -f(x)$

(ii) $f(x) = \frac{x^3}{x^2+5}$

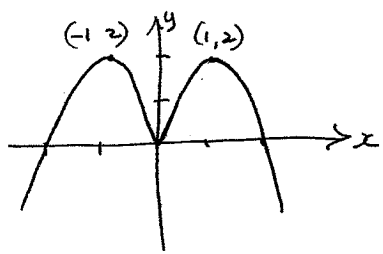
$f(a) = \frac{a^3}{a^2+5}$ $\quad f(-a) = \frac{(-a)^3}{(-a)^2+5}$

$= \frac{-a^3}{a^2+5}$

$= -f(a)$

$\therefore f(x)$ is odd.

(b)



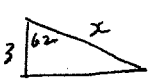
(c) (i) $\cos 315 = \cos 45$

$= \frac{1}{\sqrt{2}}$

(ii) $\sin 600 = -\sin 60$

$= -\frac{\sqrt{3}}{2}$

(d)



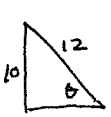
$\cos 62 = \frac{3}{x}$

$x = \frac{3}{\cos 62}$

$= 6.39$

$= 6.4$

(e)



$\sin\theta = \frac{10}{12}$

$\theta = 61^\circ 27'$

Question 3.

(i) $|x-7| = 3$

$x - 7 = 3 \quad x - 7 = -3$

$x = 10 \quad x = 4$

check

$x = 10$ $|10-7| = |3|$ $\quad x = 4$ $|4-7| = |-3|$

$= 3$ $\quad = 3$

$\therefore x = 10, \; x = 4$

(ii) $|x+2| \leq 7$

$-7 \leq x + 2 \leq 7$ *

$-9 \leq x \leq 5$

(ii) $|3x-1| > 4$

$3x - 1 > 4 \quad 3x - 1 < -4$

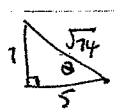
$3x > 5 \quad 3x < -3$

$x > \frac{5}{3} \quad x < -1$

$\tan\theta = \frac{7}{5}$ $\quad \cos\theta$ negative

$\therefore$ Q3

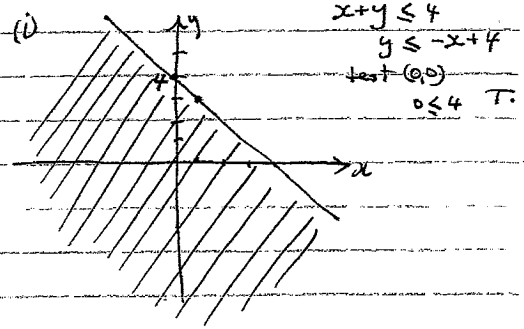
$\therefore \sin$ negative



$\sin\theta = -\frac{7}{\sqrt{74}}$

Question 4.

(a) (i)



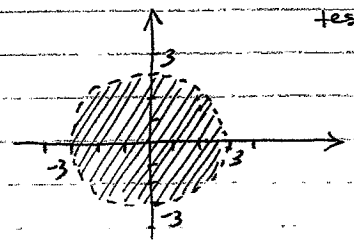
$x + y \leq 4$

$y \leq -x + 4$

test (0,0)

$0 \leq 4$ T.

(i)

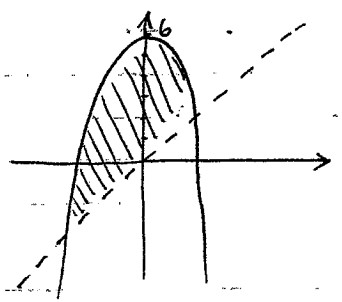


test (0,0)

$x^2 + y^2 < 9$

$0 < 9$ T

(iii)



test (0,3)

$3 > 0$ T

test (0,0)

$0 \leq 6$ T

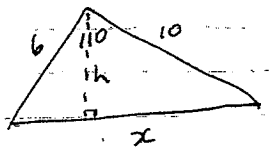
(b)

(i) $3x - 4y + 12 \leqslant 0$

(ii) $x^2 + y^2 \leqslant 4$ and

$x^2 + y^2 \geqslant 1$

Question 5



$x^2 = 10^2 + 6^2 - 2.6.10.\cos 110$

$= 177.04$

$x = \sqrt{177.04}$

$= 13.3$ cm

Area $= \frac{1}{2} \cdot 6.10 \cdot \sin 110$

$= 28.2$ cm^2

Area $= \frac{bh}{2}$

$28.2 = \frac{13.3\,h}{2}$

$h = \frac{2 \times 28.2}{13.3}$

$h = 4.24$ cm.

i) $3\sin\theta + 1 = 0$ $\quad 0 \le \theta \le 360$

$3\sin\theta = -1$ $\quad \therefore \theta$ lies in $Q_1 + Q_2$

$\sin\theta = -\frac{1}{3}$

$\theta =$

$= \quad 180 - 19$

$19^\circ \quad 161^\circ$

i) $\tan(\theta + 50) = -2$ $\quad 50^\circ \le \theta + 50^\circ \le 410$ Q_2, Q_4

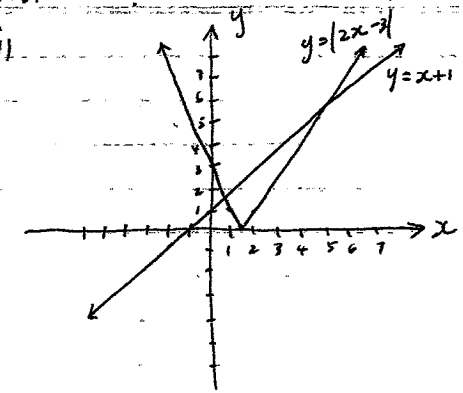
$\theta + 50 = 180 - 63,\ 360 - 63$

$\theta + 50 = 117,\ 297$

$\theta = 67^\circ,\ 247^\circ$

Question 6

(a) (i)



(ii) $|2x - 3| = x + 1$

$2x - 3 = x + 1$ $\quad 2x - 3 = -(x + 1)$

$x = 4$ $\quad 2x - 3 = -x - 1$

$3x = 2$

$x = \frac{2}{3}$

test $x = 4$

$|2x - 3| = |8 - 3|$

$= |5|$

$= 5$

$x + 1 = 4 + 1$

$= 5$

$\therefore x = 4$

test $x = \frac{2}{3}$

$|2x - 3| = |1\frac{1}{3} - 3|$

$= |-1\frac{2}{3}|$

$= 1\frac{2}{3}$

$x + 1 = \frac{2}{3} + 1$

$= 1\frac{2}{3}$

$\therefore x = \frac{2}{3}$

$\therefore$ solution

$x = \frac{2}{3},\ 4$

(iii) $|2x - 3| \le x + 1$

$\frac{2}{3} \le x \le 4$

(b) $\sin 2\theta = \frac{\sqrt{3}}{2}$ $\quad 0 \le \theta \le 36$ $Q_1 + Q_2$

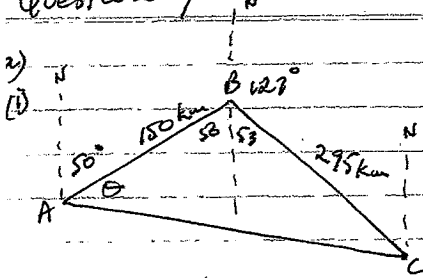
$2\theta = 60^\circ, 120^\circ, 420, 480$

$\theta = 30^\circ, 60^\circ, 210^\circ, 240^\circ$

Question 7

a)

(i)



ii) $A\hat{B}C = 50 + 53$ (alternate angles on $\parallel$ lines and angles on a straight line)

$= 103^\circ$

iii) $AC^2 = 150^2 + 295^2 - 2\,150.295.\cos 103$

$AC^2 = 129433.168$

$AC = 359.768$

≈ 360 km.

iv) $\frac{\sin\theta}{295} = \frac{\sin 103}{359.768}$

$\sin\theta = \frac{\sin 103}{359.768} \times 295$

$\theta = \sin^{-1}\left(\frac{\sin 103 \times 295}{359.76}\right)$

$= \sin^{-1}(0.7980)$

$\theta = 53^\circ$

Bearing of $= 50^\circ + 53^\circ$

$= 103^\circ$

b) $\text{cosec}^2\theta + \sec^2\theta = \text{cosec}^2\theta \cdot \sec^2\theta$

LHS $= \text{cosec}^2 + \sec^2\theta$

$= \frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta \cos^2\theta}$

$= \frac{1}{\sin^2\theta} \cdot \frac{1}{\cos^2\theta}$