St George Girls High School

Year 11

Common Test – 2

June 2008

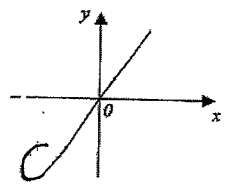
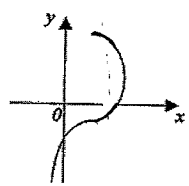
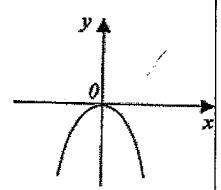
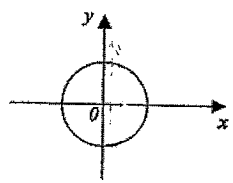
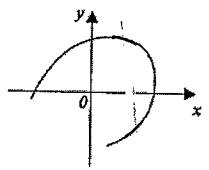
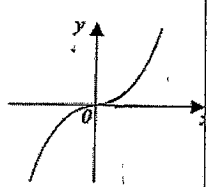
Mathematics

Time Allowed:65 minutes

Instructions

1. All questions should be attempted.
2. Show all working.
3. START EACH QUESTION ON A NEW PAGE.
4. Write on one side of each page only.
4. Marks will be deducted for careless work or poorly presented solutions.
5. Diagrams are not to scale.
5. On the cover sheet of the answer booklet clearly show:

a) your name
b) your mathematics class and teacher

Question 1 – Start a new page	Marks (9)
a) Which of the following graphs represent functions? A)  B)  C)  D)  E)  F) 	1
b) If $f(x) = x^2 - 2x$ (i) Evaluate $f(-3)$ (ii) Find a if $f(a) = 0$ (iii) Express $f(k+1)$ in its simplest form	1 2 2
c) Find the natural domain of each function (i) $f(x) = 7x - 3$ (ii) $f(x) = \frac{2}{5-x}$	1 2

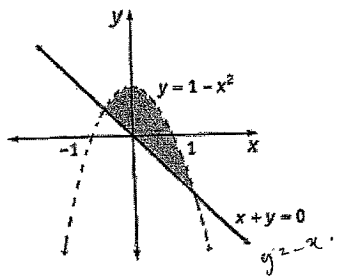
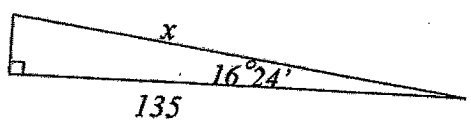
Question 2 – Start a new page	Marks (9)
a) If $f(x) = \begin{cases} 2x-1, & for\ x \geq 0 \\ x^2, & for\ x < 0 \end{cases}$ Create a table of values for $-3 \leq x \leq 3$ and sketch the graph.	3
b) Sketch the parabola $y = x^2 - 2x - 3$, showing the y - intercept, the x – intercepts and the vertex.	3
c) (i) Write down the equation of the function $y = 3^x$ after it has been translated down 2 units. (ii) Sketch the graph of the new function showing the y –intercept and the asymptote.	1 2

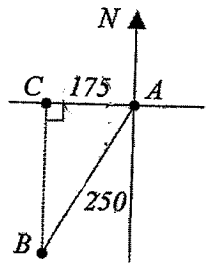
Question 3 – Start a new page	Marks (8)
a) Prove that $\tan^2\beta \cos\beta + \cos\beta = \sec\beta$	3
b) For $y = (x-1)(x+2)^2$ (i) Copy and complete the table of values (ii) Sketch the graph, marking all x-intercepts and y-intercepts.	4
c) Write down the algebraic condition for the function $f(x)$ to be even.	1

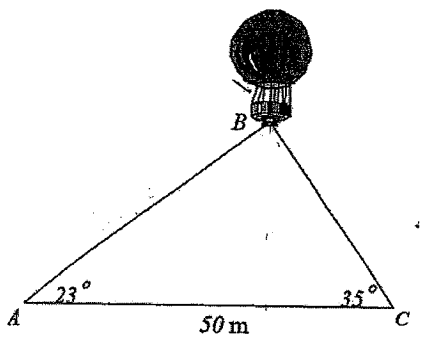
x	-3	-2	-1	0	1	2
y						
sign						

Question 4– Start a new page	Marks (9)
a) Solve for x: (i) $-4 < 1 - \frac{1}{3}x \leq 3$ (ii) $x^2 - 5x + 4 \geq 0$	3 3
b) Solve for x : $\|3x - 5\| \leq 4$	3

Question 5– Start a new page	Marks (8)
a) On separate diagrams sketch the following regions (i) $y > x - 2$ (ii) $x^2 + y^2 \leq 4$ (iii) The intersection of $y > x - 2$ and $x^2 + y^2 \leq 4$	2 2 2
b) Show that $f(x) = \frac{x}{x^2-4}$ is an odd function.	2

Question 6– Start a new page	Marks (8)
a) For the following diagram write down the pair of inequations that determine the shaded region 	3
b) Find θ to the nearest minute if θ is an acute angle and (i) $\sin\theta = \frac{3}{4}$ (ii) $\sec\theta = 1.2230$	1 2
c) Find x correct to one decimal place. 	2

Question 7– Start a new page	Marks (9)
a) Find the bearing of B from A to the nearest degree 	2
b) Find the exact value of (i) $\cos 225°$ (ii) $\tan 330°$ (iii) $\cos 45° \sin 30°$	1 1 2
c) If $\operatorname{cosec}\alpha = \frac{7}{3}$ and $\cos\alpha < 0$, find the $\tan\alpha$	3

Question 8– Start a new page	Marks (8)
a) A hot air balloon B is secured at two points A and C, 50 metres apart on the ground. The line from A to the balloon makes an angle of 23° with the ground, whilst the line from C to the balloon makes an angle of 35°. Calculate the lengths of the line AB and the height of the balloon.  	4
b) Solve for $0 \leq \theta \leq 360°$ (i) $\tan\frac{\theta}{2} = \frac{-1}{\sqrt{3}}$ (ii) $\sin(\theta - 15°) = \frac{1}{\sqrt{2}}$	2 2

End of Paper

YR 11 SOLUTIONS CT2 MATHEMATICS

Q1.a) C, F (1)

b) i) $f(x) = x^2 - 2x$

$f(-3) = (-3)^2 - 2(-3)$

$= 9 + 6$

$= 15$ (1)

ii) $f(a) = 0$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, 2$ (2)

iii) $f(k+1) = (k+1)^2 - 2(k+1)$

$= k^2 + 2k + 1 - 2k - 2$

$= k^2 - 1.$ (2)

c) i) $f(x) = 7x - 3$

D: all real x (1)

ii) $f(x) = \frac{2}{5-x}$

$5 - x \neq 0$

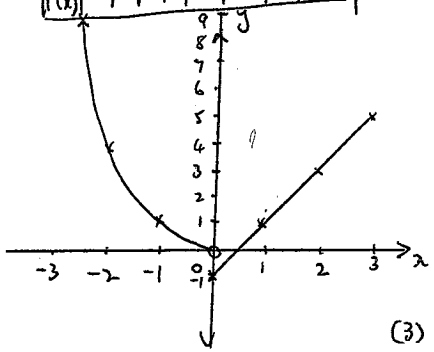
$-x = -5$

$x = 5$

D: all real except $x = 5$ (2)

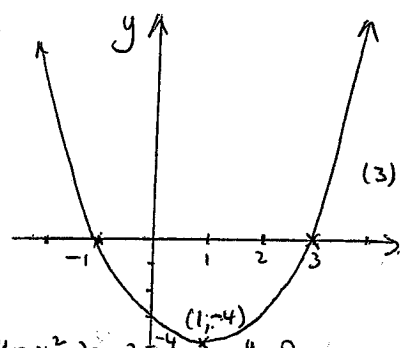
Q2.a) $f(x) = \begin{cases} 2x-1 & x \geq 0 \\ x^2 & x < 0. \end{cases}$

x	-3	-2	-1	0	1	2	3
f(x)	9	4	1	-1	1	3	5



(3)

b).



(3)

$y = x^2 - 2x - 3$

$= (x-3)(x+1)$

$y = 0$

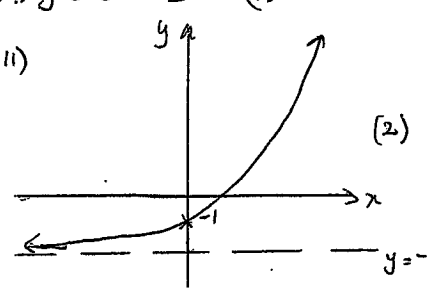
$x = -1, 3$

$x = 0, y = -3.$

vertex $(1, -4)$

c) i) $y = 3^x - 2$ (1)

ii)



(2)

Q3 a). Prove $\tan^2\beta \cos\beta + \cos\beta = \sec\beta$

LHS $= \tan^2\beta \cos\beta + \cos\beta$

$= \frac{\sin^2\beta}{\cos^2\beta} \cos\beta + \cos\beta.$

$= \frac{\sin^2\beta}{\cos\beta} + \frac{\cos\beta}{1}$

$= \frac{\sin^2\beta + \cos^2\beta}{\cos\beta}$

$= \frac{1}{\cos\beta}$

$= \sec\beta$

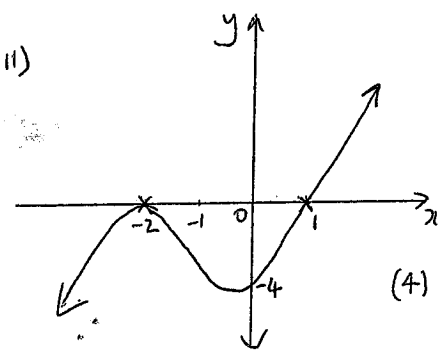
$=$ RHS (3)

b).

i)

x	-3	-2	-1	0	1	2
y	-4	0	-2	-4	0	16
sign	−		−	−		+

ii)



(4)

c). $f(a) = f(-a)$ (1)

Q4 a). i) $-4 < 1 - \frac{1}{3}x \leq 3.$

$-5 < -\frac{1}{3}x \leq 2.$

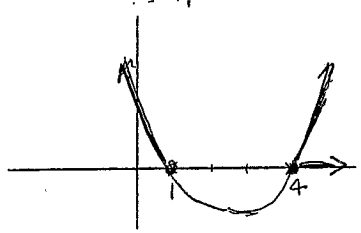
$15 > x \geq 6$

$6 \leq x < 15.$ (3)

ii). $x^2 - 5x + 4 \geq 0$

$(x-4)(x-1) \geq 0$

$= 4, 1.$



(3)

$x \leq 1 \quad x \geq 4.$

b) $|3x - 5| \leq 4$

$3x - 5 \leq 4 \qquad 3x - 5 \geq -4$

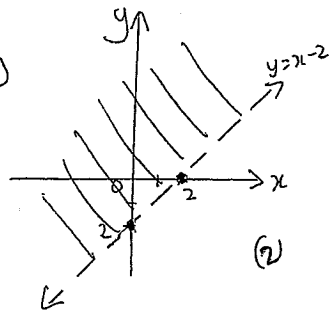
$3x \leq 9 \qquad 3x \geq 1$

$x \leq 3 \qquad x \geq \frac{1}{3}$

$\frac{1}{3} \leq x \leq 3.$ (3)

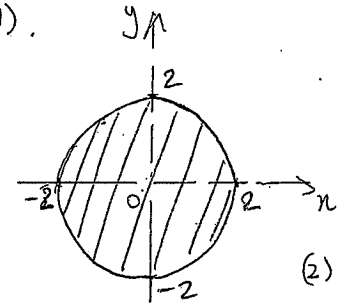
Q5.

a) i)



(2)

ii).

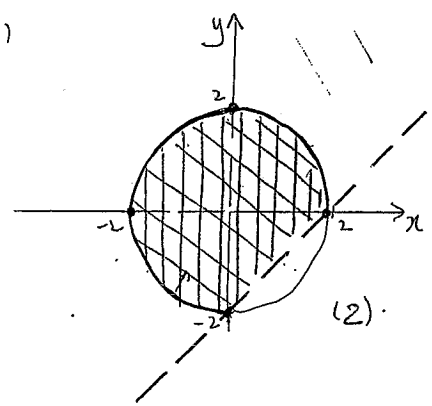


(2)

$y = x(x-3)$

$x = 0, 3$

iii)



(2).

b). $f(a) = \frac{a}{a^2-4}$

$f(-a) = \frac{-a}{(-a)^2-4}$

$= \frac{-a}{a^2-4}$

$f(a) = -f(-a)$.

6(a)

$y = -(x-1)(x+1)$

$0 < 1 \qquad y < 1-x^2$.

$\therefore x+y = 0.$ $\quad x+y \geq 0.$

$0+1 > 0 \quad$ or $y \geq -x$.

b) i) $\sin\theta = \frac{3}{4}$.

$\theta = 48°35'$ (1)

ii) $\sec\theta = 1.2230$

$\cos\theta = \frac{1}{1.2230}$

$\theta = 35°9'$ (2)

c). $\cos 16°24' = \frac{135}{x}$

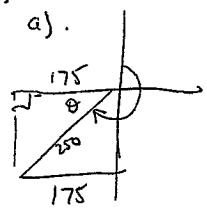
$x = \frac{135}{\cos 16°24'}$

$= 140.847$

$= 140.8$ (to 1dp)

Q7.

a).



$\theta = \frac{175}{250}$

$\theta = 46°$ (2)

(to nearest degree).

b). i) $\cos 225° = -\frac{1}{\sqrt{2}}$ (1)

ii) $\tan 330° = -\frac{1}{\sqrt{3}}$ (1)

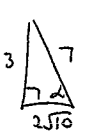
iii) $\cos 45°\sin 30° = \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{2}}{4}$ (2)

c) $\operatorname{cosec}\alpha = \frac{7}{3}$ $\quad \cos\alpha < 0$.

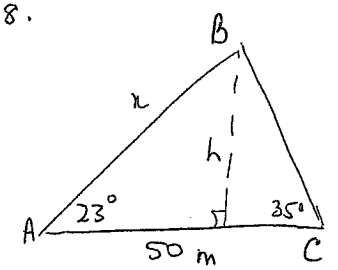
$\sin\alpha = \frac{3}{7}$ 2nd quadrant



$\therefore \tan\alpha = -\frac{3}{2\sqrt{10}}$

$= -\frac{3\sqrt{10}}{20}$ (3)

Q8.



$\hat{B} = 122$

$\frac{x}{\sin 35°} = \frac{50}{\sin 122°}$

$x = \frac{50\sin 35°}{\sin 122°}$

$= 33.8$ (2)

$\sin 23° = \frac{h}{33.8}$ $\quad h = 33.8\sin 23°$

$= 13.2$ m (2

b) i) $\tan\frac{\theta}{2} = -\frac{1}{\sqrt{3}}$ $\quad 0 \leq \theta \leq 36$

$0 \leq \frac{\theta}{2} \leq 18$

$\frac{\theta}{2} = 150°$

$\theta = 300°$ (2)

ii) $\sin(\theta - 15°) = \frac{1}{\sqrt{2}}$ $\quad 0 \leq \theta \leq 3$

$-15° \leq \theta - 15$

$\theta - 15 = 45, 135$

$\theta = 60°, 150°$.

(2)