St George Girls High School

Year 11

Common Test 2

June 2007

Mathematics Extension 1

General Instructions

1. Working time – 70 minutes
2. Use only black or blue pens.
3. Board-approved calculators may be used.
4. All necessary working must be shown in all questions.
5. Start each question on a new page.
6. All 5 questions may be attempted.

Question 1 – 14 marks – (Start a new page) **Marks**

a) Find the equation of the line through the point $(2, \sqrt{3})$ and perpendicular to a line making an angle of 30° with the positive direction of the x-axis. 3

b) If $\log_a 2 = m$ and $\log_a 3 = n$ find each of the following in terms of m and n 4

(i) $\log_a 6$

(ii) $\log_a\left(\frac{2}{9}\right)$

c) Solve for θ where $0^\circ \le \theta \le 360^\circ$ 3

$$2\cos\frac{\theta}{2} = 1$$

d) Solve for x, correct to 2 decimal places $3^x = 6$ 2

e) Sketch $y = 1 - \sin x$ for $0^\circ \le x \le 360^\circ$ 2

Question 2 – 14 marks – (Start a new page) **Marks**

a) (i) Find the coordinates of the point C which divides the join of $A(-1,-4)$ to $B(3,2)$ externally in the ratio 1:5. 3

(ii) In what ratio does B divide CA? 2

b) Given the sequence $\log 2x, \log 4x, \log 8x...$

(i) Show that this sequence is arithmetic. 2

(ii) Find an expression for the nth term. 2

c) Prove that $\dfrac{\sin\theta+\sin^2\theta}{\cos^3\theta}=\dfrac{\tan\theta}{1-\sin\theta}$ 3

d) Show that the points $A(-2,-3)$, $B(4,1)$ and $C(7,3)$ are collinear. 2

Question 3 – 14 marks – (Start a new page) **Marks**

a) Find the value(s) of p if the line $5x-12y+p=0$ is a tangent to the circle $x^2+y^2=9$ 3

b) Eliminate θ from 3

$$x=\cos\theta+1$$
$$y=1-\sin\theta$$

c) For the geometric sequence $\dfrac{2}{3}, \dfrac{4}{15}, \dfrac{8}{75}...$

(i) Find an expression for the nth term 2

(ii) Find the first term less than 0.000001 in this sequence 2

d) Find the equation of the line through the point of intersection of $l_1: 2x-3y+5=0$ and $l_2: 5x+y+1=0$ which is parallel to the line with equation $3x+2y-1=0$. 4

Use the equation $2x-3y+5+k(5x+y+1)=0$ as the starting point for your solution.

Question 4 – 14 marks – (Start a new page) **Marks**

a) Give, in exact form, the distance between the parallel lines $3x-5y+7=0$ and $3x-5y-2=0$ **2**

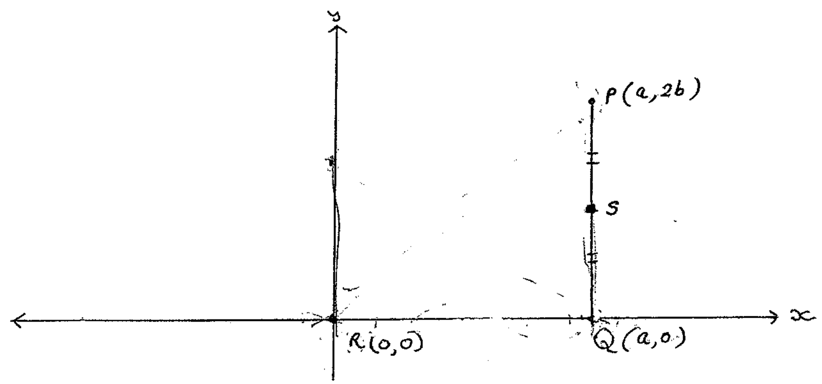
b) Find all possible values of θ (to the nearest minute) if **3**

$$2\cos^2\theta-5\cos\theta+2=0 \text{ and } 0^\circ\le\theta\le360^\circ$$

c) Simplify: $\dfrac{6^m+10^m}{9^m+15^m}$ **2**

d) Find x if: $x,\ 2x+6,\ 5x+8$ form a geometric sequence. **3**

e)



(i) Copy this diagram into your answer booklet. By choosing appropriate coordinates for P complete the diagram so that ΔPQR is right angled at Q and S is the midpoint of PQ.

Mark on the diagram the point on QR produced to T such that $RT=QR$ **2**

(ii) Prove that $PR^2-ST^2=3\left(PS^2-RT^2\right)$ **2**

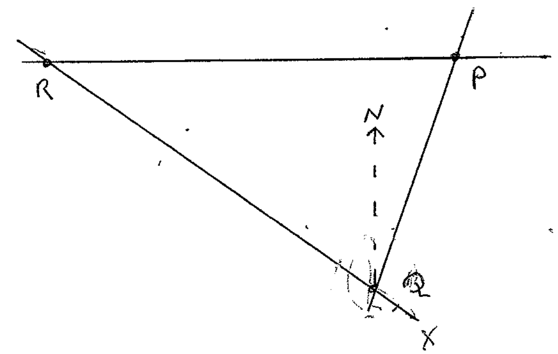
Question 5 – 14 marks – (Start a new page) **Marks**

a) Given $\cos A=\dfrac{-3}{7}$ and $\sin A>0$. Find the exact value of $\operatorname{cosec}A$ **3**

b) Write down the five terms of the arithmetic sequence with $T_1=2$ and $T_5=28$ **3**

c) In ΔABC, $\angle CAB=40^\circ$, $a=6.4$ and $b=9.8$. Find the size of $\angle ABC$ (correct to the nearest degree). **3**

d)



[not to scale]

PQ, QR and PR are straight roads. PR runs from East to West. From Q, P is 12km on a bearing of 12° and R is 14km on a bearing of 312°.

Two cars C_1 and C_2 leave Q at the same time. C_1 travels along QR then RP at a speed of 90km/h. C_2 travels along QP at 40km/h. **5**

(i) Show that $\angle RQP=60^\circ$

(ii) Find which car reaches P first, and by how many seconds does it do so.

End of Paper

Jenny Q1

YR 11 EXT 1 CT2 SOLUTIONS 2007.

Q1

a) $\tan 30 = \frac{1}{\sqrt{3}}$ $(2, \sqrt{3})$ (3)

$m_2 = -\sqrt{3}$ 1

$y - y_1 = m(x - x_1)$

$y - \sqrt{3} = -\sqrt{3}(x-2)$ 1

$y = -\sqrt{3}x + 2\sqrt{3} + \sqrt{3}$ 1

$= -\sqrt{3}x + 3\sqrt{3}$ $y = -1.73x + 5.20$

b) $\log_a 2 = m$ $\log_a 3 = n$ (4)

i) $\log_a 6 = \log_a(2\times3)$

$= \log_a 2 + \log_a 3$ 1

$= m + n$ 1

ii) $\log_a\left(\frac{2}{9}\right) = \log_a 2 - \log_a 9$ 1

$= m - 2\log_a 3$

$= m - 2n$ 1

1 for $2\log_a 3$

c) $2\cos\frac{\theta}{2} = 1$ $0 \le \theta \le 360$ (3)

$\cos\frac{\theta}{2} = \frac{1}{2}$ 1 $0 \le \frac{\theta}{2} \le 180$ 1

$\frac{\theta}{2} = 60$

$\theta = 120$ 1

d) $3^x = 6$ (2)

$\log_{10} 3^x = \log_{10} 6$ 1

$x\log_{10} 3 = \log_{10} 6$

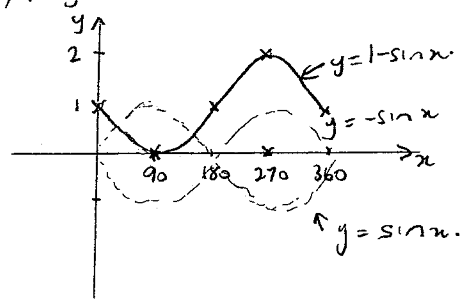
$x = \frac{\log_{10} 6}{\log_{10} 3}$ 1

$= 1.630929\ldots$

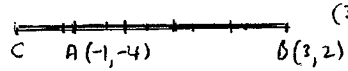
$= 1.63$ (to 2dp) $-\frac{1}{2}$

1✓ 2✓ 3✓ 4✓ 5✓

e) $y = 1 - \sin x$ (2)



Q2 a) i) (3)



$-1 : 5$.

$x = \frac{5\times-1 + -1\times3}{-1+5}$ $y = \frac{5\times-4 + -1\times2}{-1+5}$

$= -\frac{8}{4}$ $= -\frac{22}{4}$

$= -2$ $= -5\frac{1}{2}$

$C\left(-2, -5\frac{1}{2}\right)$

ii) $5 : -4$ (2)

b) $\log 2x, \log 4x, \log 8x$

$\log x + \log 2, \log x + 2\log 2, \log x + 3\log 2$

i) $T_3 - T_2 = \log x + 3\log 2 - (\log x + 2\log 2)$

$= \log 2$

$T_2 - T_1 = \log x + 2\log 2 - (\log x + \log 2)$

$= \log 2$

∴ Sequence is arithmetic (2)

ii) $T_n = a + (n-1)d$ (2)

$= \log x + \log 2 + (n-1)\log 2$

$= \log x + \cancel{\log 2} + n\log 2 - \cancel{\log 2}$

$= \log x + n\log 2$

2

2c) $LHS = \frac{\sin\theta + \sin^2\theta}{\cos^3\theta}$ (3)

$= \frac{\sin\theta(1+\sin\theta)}{\cos\theta \times \cos^2\theta}$

$= \tan\theta \frac{(1+\sin\theta)}{1-\sin^2\theta}$

$= \tan\theta \frac{\cancel{(1+\sin\theta)}}{\cancel{(1+\sin\theta)}(1-\sin\theta)}$

$= \frac{\tan\theta}{1-\sin\theta}$

$= RHS$

d) $A(-2,-3)$ $B(4,1)$ $C(7,3)$ (2)

$m_{AB} = \frac{1+3}{4+2}$ $m_{BC} = \frac{3-1}{7-4}$

$= \frac{2}{3}$ $= \frac{2}{3}$

∴ $m_{AB} = m_{BC}$

∴ A,B,C are collinear

Q3 a) $x^2+y^2=9$ circle centre $(0,0)$ radius 3 (3)

$d = \frac{|5\times0 - 12\times0 + p|}{\sqrt{5^2+12^2}}$

$3 = \frac{|p|}{13}$

$|p| = 39$

$p = \pm 39$

b) $x = \cos\theta + 1 \Rightarrow \cos\theta = x-1$ (3)

$y = 1 - \sin\theta$ $\sin\theta = 1-y$

$\cos^2\theta = (x-1)^2$

$\sin^2\theta = (1-y)^2$

$\sin^2\theta + \cos^2\theta = (x-1)^2 + (1-y)^2$

$(x-1)^2 + (1-y)^2 = 1$

or $(x-1)^2 + (y-1)^2 = 1$.

c) $\frac{2}{3}, \frac{4}{15}, \frac{8}{75}$

$a = \frac{2}{3}$ $r = \frac{4}{15} \div \frac{2}{3}$

$= \frac{2}{5}$

i) $T_n = ar^{n-1}$ (2)

$= \frac{2}{3}\left(\frac{2}{5}\right)^{n-1}$

$= \frac{2^n}{3\times5^{n-1}}$

ii) $T_n < 0.000001$ (2)

$\frac{2}{3}\left(\frac{2}{5}\right)^{n-1} < 0.000001$

$\left(\frac{2}{5}\right)^{n-1} < 1.5\times10^{-6}$

$(n-1)\log_{10}\left(\frac{2}{5}\right) < \log_{10}(1.5\times10^{-6})$

$n-1 > \frac{\log_{10}(1.5\times10^{-6})}{\log_{10}\left(\frac{2}{5}\right)}$

$n > 14.635\ldots + 1$

$n > 15.635\ldots$

$n = 16$

$T_{16} = \frac{2^{16}}{3\times5^{15}}$

d) $l_1: 2x - 3y + 5 = 0$ $l_2: 5x + y + 1 = 0$

$3x + 2y - 1 = 0$ $m = -\frac{3}{2}$ (4)

$2x - 3y + 5 + k(5x + y + 1) = 0$

$2x - 3y + 5 + 5kx + ky + k = 0$

$(2+5k)x + (k-3)y + 5 + k = 0$.

$\frac{-2-5k}{k-3} = -\frac{3}{2}$

$-4 - 10k = -3k + 9$

$7k = -13$

$k = -\frac{13}{7}$

∴ $2x - 3y + 5 - \frac{13}{7}(5x + y + 1) = 0$

$14x - 21y + 35 - 65x - 13y - 13 = 0$

$51x + 34y + 22 = 0$.

Q4a) $3x-5y+7=0 \quad 3x-5y-2=0$

pt $(1,2)$ (2)

$$d=\frac{|3\times1-5\times2-2|}{\sqrt{3^2+5^2}}$$

$$=\frac{9}{\sqrt{34}} \text{ or } \frac{9\sqrt{34}}{34}$$

b) $2\cos^2\theta-5\cos\theta+2=0 \quad 0\le\theta\le360$

$(2\cos\theta-1)(\cos\theta-2)=0$ (3)

$2\cos\theta-1=0$

$2\cos\theta=1$ $\quad\quad$ $\cos\theta=2$

$\cos\theta=\frac{1}{2}$ $\quad\quad$ no solution

$\theta=60,300$

c) $$\frac{6^m+10^m}{9^m+15^m}=\frac{3^m\times2^m+5^m\times2^m}{3^m\times3^m+5^m\times3^m}$$ (2)

$$=\frac{2^m\cancel{(3^m+5^m)}}{3^m\cancel{(3^m+5^m)}}$$

$$=\left(\frac{2}{3}\right)^m$$

d) $x,\ 2x+6,\ 5x+8$ (3)

$$\frac{5x+8}{2x+6}=\frac{2x+6}{x}$$

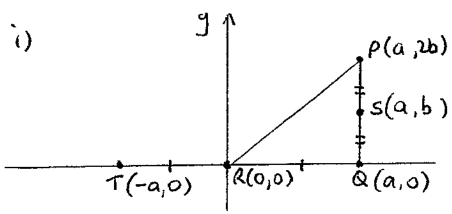
$5x^2+8x=4x^2+24x+36$

$x^2-16x-36=0$

$(x-18)(x+2)=0$

$x=18,-2$

e) i) (2)



LHS $=PR^2-ST^2$

$=(2b)^2+a^2-[b^2+(2a)^2]$

$=4b^2+a^2-b^2-4a^2$

$=3b^2-3a^2$

$=3(b^2-a^2)$

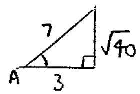
RHS $=3(PS^2-RT^2)$

$=3(b^2-a^2)$

$=$ LHS

(2)

Q5a) $\cos A=-\frac{3}{7} \quad \sin A>0$ (3)



$\operatorname{cosec} A=\frac{7}{\sqrt{40}}$

or $\frac{7\sqrt{10}}{20}$

b) $T_1=2 \quad T_5=28$ (3)

$a=2$ —① $\quad a+4d=28$ —②

② − ①

$4d=26$

$d=\frac{13}{2}$

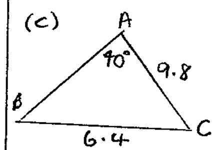
$T_1=2$, $T_2=2+\frac{13}{2}$ $\quad T_3=8\frac{1}{2}+\frac{13}{2}$

$=8\frac{1}{2}$ $\quad\quad =15$

$T_4=15+\frac{13}{2}$ $\quad\quad T_5=28$.

$=21\frac{1}{2}$

(c)



$$\frac{\sin\hat{B}}{9.8}=\frac{\sin40}{6.4}$$ (3)

$$\sin\hat{B}=\frac{9.8\sin40}{6.4}$$

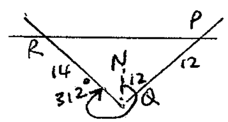
$=0.98$

$\therefore A\hat{B}C=79.8$

$=80,100$

(to nearest degree)

d)



(5)

i) $R\hat{Q}P=N\hat{Q}P+N\hat{Q}R$

$=12+360-312$

$=60$

ii) $RP=\sqrt{14^2+12^2-2\times14\times12\cos60}$

$=\sqrt{172}$

$=13.1$

$T_1=\frac{D}{S}$ $\quad\quad\quad T_2=\frac{12}{40}$

$=\frac{14+13.1}{90}$ $\quad\quad =18$ min

$=18$ min 4 sec.

C_2 reaches P first by 4 seconds.