St George Girls High School

Year 11

End of Preliminary Course Examination

2007

Mathematics

Time Allowed: 3 hours
(plus 5 minutes reading time)

Instructions
1. Attempt all 7 questions.
2. All questions are of equal value.
2. All necessary working must be shown.
3. Begin each question on a **new page**.
4. Marks will be deducted for careless work or poorly presented solutions.

Question 1 (17 marks) Start a new page **Marks**

a) Factorise

(i) $36-4x^2$ 2

(ii) x^3+2x^2+x+2 2

b) Solve for x

(i) $9-4x\le 1$ 2

(ii) $|2x-3|=9$ 2

(iii) $(x+2)(x-4)=7$ 3

(iv) $\frac{x+3}{2}-\frac{2}{3}=\frac{x}{6}+\frac{1}{4}$ 3

c) Solve by completing the square: $x^2-6x+3=0$ 3

Question 2 (17 marks) Start a new page **Marks**

a) Evaluate

(i) $\sqrt{\dfrac{45.23}{19.76-7.45}}$, correct to 2 decimal places. 1

(ii) $\left(4.93\times10^{13}\right)^2\div\left(3.26\times10^{-3}\right)$, giving your answer in scientific notation correct to 3 significant figures. 1

b) Express $\dfrac{4}{7-3\sqrt{3}}$ as a simplified fraction with rational denominator. 2

c) Simplify $\sqrt{18}+\sqrt{27}-\sqrt{50}+\sqrt{12}$ 2

d) Find the values of x and y if $\left(2\sqrt{7}+\sqrt{3}\right)^2=x+\sqrt{y}$, given that x and y are integers. 3

e) Write down the natural domain of

(i) $f(x)=\dfrac{1}{x+1}$ 1

(ii) $g(x)=\sqrt{1-x}$ 2

f) (i) Sketch the graph of $y=(x+1)(x-3)$, clearly showing the intercepts with the coordinate axes and the coordinates of the vertex. 3

Using your graph, or otherwise,

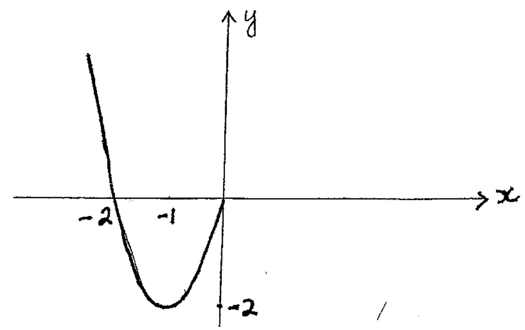
(ii) solve $(x+1)(x-3)\le 0$ 1

(iii) write down the range of $y=(x+1)(x-3)$ 1

Question 3 (17 marks) Start a new page **Marks**

a) Show that $f(x)=\dfrac{3x}{x^2+5}$ is an odd function. 2

b) Copy and complete the graph of $y=g(x)$, given that it is an even function. 2



c) Solve $|2x-5|\le 7$ 2

d) On separate diagrams sketch

(i) $y=\sqrt{4-x^2}$ 2

(ii) the intersection of the regions $x^2+y^2\le 4$ and $x+y\le 2$ 3

e) (i) On the same diagram draw neat sketches of $y=|x-4|$ and $y=4x+1$ 3

(ii) Solve the equation $|x-4|=4x+1$ 3

Question 4 (17 marks) Start a new page **Marks**

a) Without using your calculator show that **2**

$$\sin 60^\circ \tan 30^\circ + \sec 45^\circ \sin 45^\circ = \frac{3}{2}$$

b)

A is the top and C is the base of a vertical cliff that is 50m high.

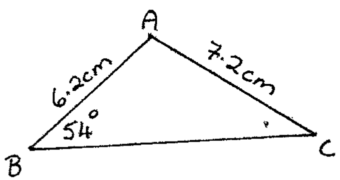
B is a boat 300m out to sea from C

(i) Copy the diagram and mark on it the angle of depression from A to B. **1**

(ii) Find this angle of depression, correct to the nearest minute. **2**

c) If $\cos\theta = \frac{2}{3}$ and $\sin\theta < 0$ find the exact value of $\tan\theta$ **3**

d)



(i) Find the size of $A\hat{C}B$, correct to the nearest minute. **2**

(ii) Find the area of ΔABC, correct to 2 significant figures. **3**

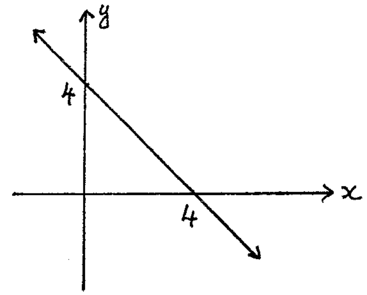
e) Prove the identity: $\sec\theta - \cos\theta = \sin\theta \tan\theta$ **2**

f) Solve $\sin 2\theta = \frac{1}{2}$ for $0^\circ < \theta < 360^\circ$ **2**

Question 5 (17 marks) Start a new page **Marks**

a) Write down the equation of the line

(i) **2**



(ii) that passes through the points $(3, 2)$ and $(3, -2)$ **1**

b) $P(-2, 2)$, $Q(2, 5)$ and $R(7, 1)$ are 3 points on the number plane.

(i) Find the coordinates of the midpoint of interval PR. **1**

(ii) Find the exact value of the length of interval QR. **2**

(iii) Find, in general form, the equation of PQ **3**

(iv) Find the angle of inclination of PQ. **2**

(v) If $PQRS$ is a parallelogram find the coordinates of S, clearly stating the property (or properties) of parallelograms that you use. **2**

c) Show that $y = 3x - 2$ and $2x + 6y + 9 = 0$ are perpendicular. **2**

d) Show that $A(-1, 2)$, $B(2, 1)$ and $C(8, -1)$ are collinear points. **2**

Question 6 (17 marks) Start a new page **Marks**

a) Show that $36, -9, \frac{9}{4}$ are successive terms of a geometric sequence. 1

b) For the arithmetic series $16+21+26+....$

(i) find an expression for t_n, the n^{th} term 2

(ii) hence find the 31^{st} term 1

c) Find the value of $\sum_{k=3}^{6} k^2$ 2

d) The third term of a geometric series is 36 and the sixth term is 972.

(i) Find the common ratio, r, and the first term, a 2

(ii) Hence find the sum of the first 10 terms of the series. 2

e) For what values of m does the geometric series
$1+(2m-3)+(2m-3)^2+(2m-3)^3+...$ have a limiting sum. 3

f) By expressing $0.\dot{4}\dot{2}$ as an infinite geometric series write $0.\dot{4}\dot{2}$ as a simplified fraction. 2

g) Find the number of terms in the arithmetic series $30 + 26 + 22 + \ldots$ that give a sum of 96. 3

Question 7 (17 marks) Start a new page **Marks**

a) Differentiate with respect to x:

(i) $y=3x^2-6$ 1

(ii) $y=x^2(3x-2)$ 1

(iii) $y=x^2\sqrt{x}$ 2

(iv) $y=\frac{4x^3+7x^2+1}{x^2}$ 2

b) If $f(x)=\frac{3}{x}$ find the value of $f'(2)$ 2

c) Find the point on the curve $y=x^2+3x+2$ where the tangent has gradient -1. 3

d) Solve for x:

(i) $36^x=\frac{1}{6}$ 1

(ii) $\log_2(x+1)+\log_2(x+3)=3$ 3

d) Solve $1.005^n>100$, giving your answer correct to 3 significant figures. 2

End of Paper

YR 11 Ext'l Prelim Course '07

Q1 (a) (i) $4(9-x^2)$

$= 4(3-x)(3+x)$

(ii) $x^2(x+2) + (x+2)$

$= (x+2)(x^2+1)$

(b) (i) $-4x \le -8$

$x \ge 2$

(ii) $2x-3=-9$ or $2x-3=9$

$2x=-6$ $\quad$ $2x=12$

$\underline{x=-3}$ $\quad$ $\underline{x=6}$

(iii) $x^2-2x-8=7$

$x^2-2x-15=0$

$(x-5)(x+3)=0$

$\therefore x=5$ or $x=-3$

(iv) $\frac{x+3}{2} = \frac{x}{6} + \frac{11}{12}$ $\times 12$

$6(x+3) = 2x+11$

$6x+18 = 2x+11$

$4x = -7$

$x = -\frac{7}{4}$

(c) $x^2-6x = -3$

$x^2-6x+9 = -3+9$

$(x-3)^2 = 6$

$x-3 = \pm\sqrt{6}$

$x = 3 \pm \sqrt{6}$

$x = 3-\sqrt{6}$ or $x = 3+\sqrt{6}$

Q2

(a) (i) 1.92 (two dec pl.)

(ii) 7.46×10^{29}

(b) $\frac{4}{7-3\sqrt{3}} \times \frac{7+3\sqrt{3}}{7+3\sqrt{3}}$

$= \frac{4(7+3\sqrt{3})}{49-27}$

$= \frac{4(7+3\sqrt{3})}{22}$

$= \frac{2(7+3\sqrt{3})}{11}$

(c) $3\sqrt{2}+3\sqrt{3}-5\sqrt{2}+2\sqrt{3}$

$= 5\sqrt{3}-2\sqrt{2}$.

(d) $28+4\sqrt{21}+3 = x+\sqrt{y}$

$31+4\sqrt{21} = x+\sqrt{y}$

$31+\sqrt{336} = x+\sqrt{y}$

$\therefore x=31,\ y=336$

(e) (i) Domain: $x+1 \ne 0$

$\therefore x \ne -1$

All real x, with $x \ne -1$

(ii) Domain: $1-x \ge 0$

$-x \ge -1$

$\{x : x \le 1\}$

(f) (i) $y = x^2-2x-3$

y-intercept is -3

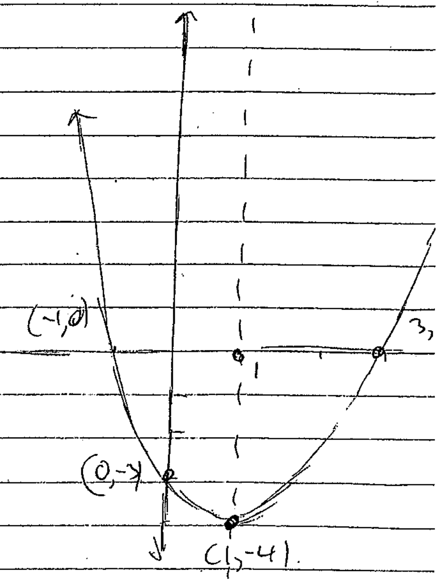
x intercept are -1 & 3

axis $x = \frac{2}{2}$

$\boxed{x=1}$

Vertex $(1,\ 1-2-3)$

$= (1,-4)$



(ii) $-1 < x \le 3$ from graph.

(iii) Range: $\{y : y \ge -4\}$

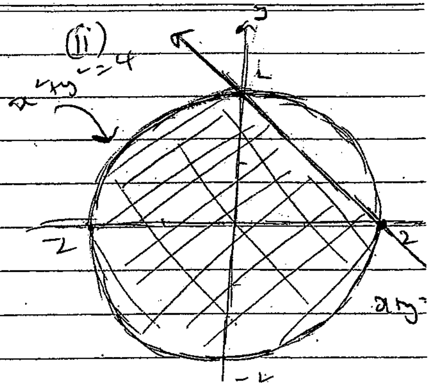
Q3 (a) $f(-x) = \dfrac{3x - x}{(-x)^2+5}$

$= \dfrac{-3x}{x^2+5}$

$= -[f(x)]$

Satisfies defn of odd funct $f(-x) = -[f(x)]$

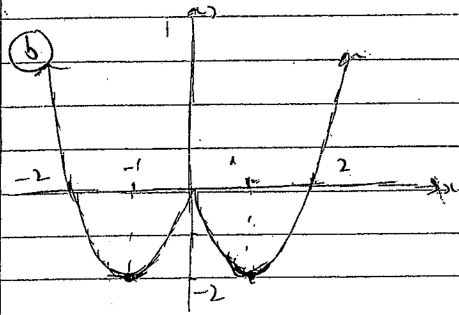
(ii)



Test $(0,0)$: $0^2+0^2 \leq 4$ ✓

$0+0 \leq 2$ ✓

(b)

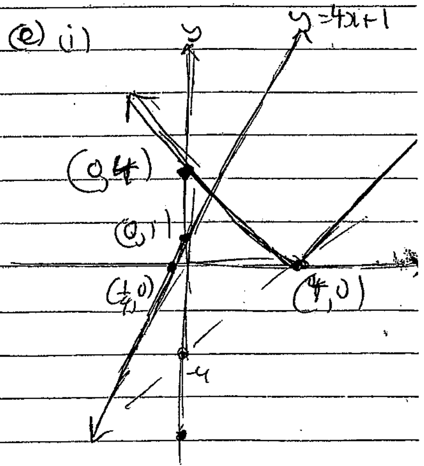


(c) $-7 \leq 2x-5 \leq 7$

$-2 \leq 2x \leq 12$

$-1 \leq x \leq 6$

(e) (i)



(ii) from diagram

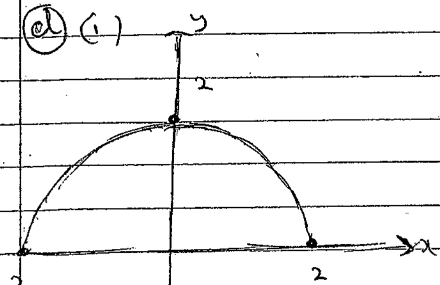
$-(x-4) = 4x+1$

$-x+4 = 4x+1$

$3 = 5x$

$\therefore x = \frac{3}{5}$

(d) (i)



$y = \sqrt{4-x^2}$

~~Alt~~ $-(x-4) = 4x+1$

$x = \frac{3}{5}$

$x-4 = 4x+1$ Check: $| \; | \neq -ve$ Test $\left|\frac{3}{5}-4\right| = 3\frac{2}{5}$; $4\times\frac{3}{5}+1$

$-5 = 3x$

Q4

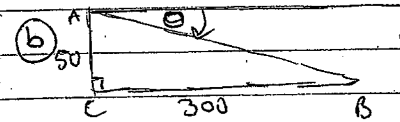
(a) $\sin 60^\circ \times \tan 30^\circ + \sec 45^\circ \times \sin 45$

$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} + \frac{\sqrt{2}}{1} \times \frac{1}{\sqrt{2}}$

$= \frac{1}{2} + 1$

$= \frac{3}{2}$

(b)



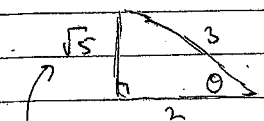
(i) θ is angle of depression

(ii) Alt $\angle$'s on // lines equal.

$\therefore \tan\theta = \frac{50}{300}$

$\theta =$

(c) $\left.\begin{array}{l}\cos\theta > 0 \\ \sin\theta < 0\end{array}\right\}$ θ in 4th Quadrant



Pythagoras

$\tan\theta = -\frac{\sqrt{5}}{2}$

(d) (i) $\frac{\sin \hat{C}}{6.2} = \frac{\sin 54}{7.2}$

$\sin C = \frac{6.2 \times \sin 54^\circ}{7.2}$

thus $C = 44^\circ 10'$ ~~or~~

[since $6.2 < 7.2$

$C < 54^\circ$]

(ii) Area $= \frac{1}{2} \times 6.2 \times 7.2 \times \sin 81^\circ$

$= 22.09$

Area is 22 cm² (2 sig figs)

(e) LHS

$\frac{1}{\cos\theta} - \cos\theta$

$= \frac{1-\cos^2\theta}{\cos\theta}$

$= \frac{\sin^2\theta}{\cos\theta}$

$= \sin\theta . \tan\theta$ as req

(f) Domain $0 < 2\theta < 720^\circ$

Related angle is 30°

$\therefore 2\theta = 30^\circ, 150^\circ, 390^\circ, 5$

$\theta = 15^\circ, 75^\circ, 195^\circ, 2$

Q5

(a) (i) $y = -x + 4$

or $x + y - 4 = 0$

(ii) $m = \frac{2-2}{3-3}$

undefined

Vertical line

$x = 3$

(b) (i) Midpt PR $(x, y) = \left(\frac{-2+7}{2}, \frac{2+1}{2}\right)$

$= \left(\frac{5}{2}, \frac{3}{2}\right)$

(ii) $d_{QR} = \sqrt{(2-7)^2 + (5-1)^2}$

$= \sqrt{25 + 16}$

$= \sqrt{41}$

(iii) $\frac{y-2}{x-2} = \frac{5-2}{2-2}$

$4y - 8 = 3x + 6$

$\therefore 3x - 4y + 14 = 0$

(iv) $m_{PR} = \frac{3}{4}$

so if θ is angle

$\tan\theta = \frac{3}{4}$

$\theta = 36°52'$

(v) Let $S(x, y)$ be vertex.

The midpt of PR same as midpt of QS!

& diagonals of a Parallelogram bisect.

$\left(\frac{x+2}{2}, \frac{y+5}{2}\right) = \left(\frac{5}{2}, \frac{3}{2}\right)$

$x + 2 = 5$ and $y + 5 = 3$

$x = 3$ $\quad y = -2$

$S(3, -2)$

(c) $y = 3x - 2$; $m_1 = 3$

$y = -\frac{1}{3}x - \frac{3}{2}$; $m_2 = -\frac{1}{3}$

$m_1 \times m_2 = -1$ Perp. property satisfied by 3 & $-\frac{1}{3}$ gradient

(d) Gradient AB; $m_1 = \frac{2-1}{-1-2}$

$= -\frac{1}{3}$

Grad AC; $m_2 = \frac{2-1}{-1-8}$

$= -\frac{1}{3}$

AB common & grad. same, the A, B, C are collinear.

Q6

(a) $\frac{T_2}{T_1} = \frac{-9}{36}$ & $\frac{T_3}{T_2} = \frac{\frac{9}{4}}{-9}$

$= -\frac{1}{4}$ $\quad = -\frac{1}{4}$

By defn common ratio $-\frac{1}{4}$ exists. $36, -9, \frac{9}{4}$ is G.S.

Geom mean

$\sqrt{36 \times \frac{9}{4}}$

$= \sqrt{81}$

$= \pm 9$

(b) (i) $a = 16$, $d = 5$ $\quad t_n = a + (n-1)d$

$= 16 + (n-1)5$

$t_n = 11 + 5n$

(ii) $t_{31} = 11 + 5 \times 31$

$= 166$ is 31st term.

(c) $9 + 16 + 25 + 36$

$= 86.$

(d) (i) $ar^2 = 36$ .. (A)

$ar^5 = 972$ .. (B)

$\frac{(B)}{(A)}$ $\quad r^3 = \frac{972}{36}$

$r^3 = 27$

$\therefore r = 3$

So $9a = 36$

$a = 4$

First term $a = 4$ & ratio $r = 3$.

(ii) $S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{10} = \frac{4(3^{10} - 1)}{3 - 1}$

$= 2(3^{10} - 1)$

(e) limiting sum only if $|r| < 1$

Here $r = (2m-3)^2$

$\therefore -1 < 2m - 3 < 1$

$2 < 2m < 4$

$1 < m < 2$

(f) $\frac{42}{100} + \frac{42}{10000} + \frac{42}{1000000} + \ldots$

$a = \frac{42}{100}$, $r = \frac{1}{100}$ & $|r| < 1$

So $S_\infty = \frac{\frac{42}{100}}{1 - \frac{1}{100}}$

$= \frac{42}{100} \times \frac{100}{99}$

$= \frac{14}{33}$

(g) $a = 30$

$d = -4$

$S_n = 96$

So $\frac{n}{2}[60 + (n-1) \times -4] = 96$

$\frac{n}{2}[64 - 4n] = 96$

$-2n^2 + 32n - 96 = 0$

ie $n^2 - 16n + 48 = 0$

$(n-4)(n-12) = 0$

$n = 4$ or $n = 12$

Sum of four terms or sum of twelve term gives 96.

Question 7

(a) (i) $\frac{dy}{dx} = 6x$

(ii) $\frac{dy}{dx} = (3x-2) \times 2x + x^2 \times 3$

$= 6x^2 - 4x + 3x^2$

$= 9x^2 - 4x$

or $y = 3x^3 - 2x^2$

$y' = 9x^2 - 4x$

(iii) $y = x^{\frac{5}{2}}$

$\frac{dy}{dx} = \frac{5}{2} x^{\frac{3}{2}}$

(iv) $y = 4x + 7 + x^{-2}$

$\frac{dy}{dx} = 4 - 2x^{-3}$

$= 4 - \frac{2}{x^3}$

(b) $f(x) = 3x^{-1}$

$f'(x) = -3x^{-2}$

$= \frac{-3}{x^2}$

$f'(2) = -\frac{3}{4}$

(c) $\frac{dy}{dx} = 2x + 3$

let $2x + 3 = -1$

$2x = -4$

$x = -2$

then $y = (-2)^2 + 3 \times -2 + 2$

$y = 0$

Point $(-2, 0)$

(d) (i) $(6^2)^x = 6^{-1}$

$6^{2x} = 6^{-1}$

$\therefore 2x = -1$

$x = -\frac{1}{2}$

(ii) $\log_2 [(x+1) \times (x+3)] = 3$

$x^2 + 4x + 3 = 2^3$

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x = -5$ or $x = 1$

$x > -1$ since domain

$x+1 > 0$, $x+3 > 0$

$\therefore$ $x = 1$ is soln.

(e) $n \log_{10} 1.005 > \log_{10} 10$

$n \log_{10} 1.005 > 2$

$\therefore n > \frac{2}{\log_{10} 1.005}$

$n > 923$

(three sig fig).