St George Girls High School

Year 11

Common Test – 1

2012

Mathematics Extension 1

Marks: 60

Instructions

1. Working time – 70 minutes
2. All questions should be attempted.
3. Show all working.
4. **Start each question on a new page.**
5. Marks will be deducted for careless work or poorly presented solutions.
6. On the cover sheet of the answer booklet clearly show:

   a) your name
   b) your mathematics class and teacher

Question 1: (10 Marks) – Start A New Page **Marks**

a) Suppose $A = \{3, 5, 7, 9, 11, 13\}$

and $B = \{1, 4, 8, 12\}$

with universal set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$

List the members in **3**

(i) $A \cup B$

(ii) $A \cap B$

(iii) $\overline{A \cap B}$

b) If $n(A) = 21$, $n(B) = 16$ and $n(A \cup B) = 30$

Find $n(A \cap B)$ **1**

c) Solve simultaneously **4**

$$a - b + c = 7$$

$$a + 2b - c = -4$$

$$3a - b - c = 3$$

d) (i) What is the natural domain of the function **1**

$$y = \frac{1}{\sqrt{x-4}}$$

(ii) What is the range of the function **1**

$$y = \frac{1}{\sqrt{x-4}}$$

Question 2: (10 Marks) – Start A New Page **Marks**

a) If $\log_a 2 = A$, $\log_a 3 = B$, $\log_a 5 = C$

Find an expression in terms of a, b and c

(i) $\log_a 45$ 2

(ii) $\log_a 0.4$ 2

(iii) $\log_a \left(\frac{5}{\sqrt{a}}\right)$ 2

b) Find the common difference and a formula for the nth term of the Arithmetic progression $\log_3 2, \ \log_3 4, \ \log_3 8, \ldots$ 2

c) Solve $3^x = 12$ giving your answer correct to 3 decimal places. 2

Question 3: (10 Marks) – Start A New Page **Marks**

a) Find the inverse function $y = f^{-1}(x)$ if 2

$$f(x) = \frac{3x+1}{x+2}$$

b) Given the functions $F(x) = 2^x$ and $G(x) = 3x + 1$ 2

Evaluate $G(F(2))$

c) Find the number of terms in the series 2

$$2 + 14 + 98 + \cdots + 235298$$

d) Find the value of x for which the geometric progression 2

$$1\text{-}(3x+2) + (3x+2)^2 + \cdots$$

has a limiting sum.

e) Factorise fully: $a^2 - b^2 + 2b - 1$ 2

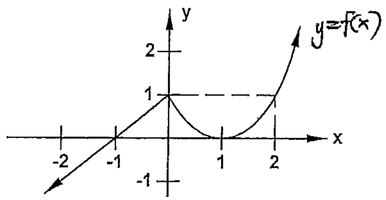
Question 4: (10 Marks) – Start A New Page — Marks

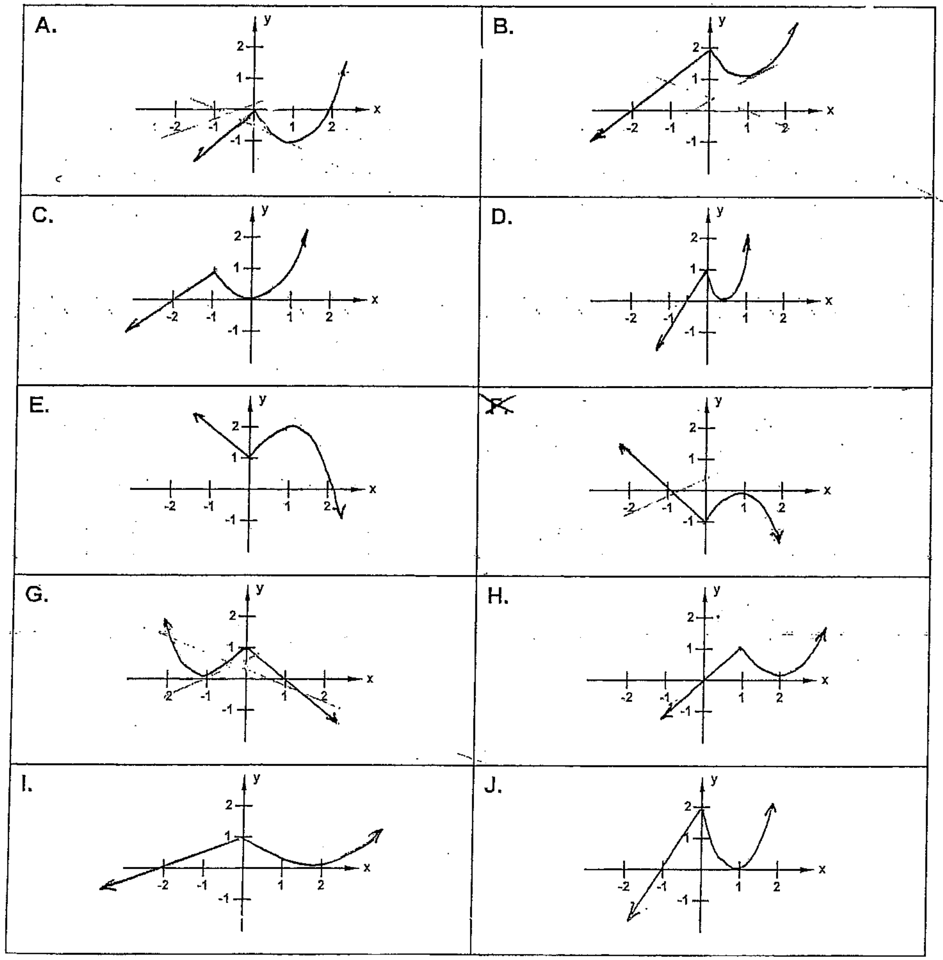
a) What is the natural domain of the function $y = \log_2(4 - x)$? — 1

b) Solve $\log_3(x+3) + \log_3(x-5) = 2$ — 3

c) This question is to be answered in the table provided on page 7

The graph $y = f(x)$ is given — 6





Question 5: (10 Marks) – Start A New Page — Marks

a) Evaluate — 2

$$\sum_{k=1}^{4} (-1)^k k^2$$

b) Consider the geometric series

$$1 + 6 + 36 + 216 + \cdots$$

(i) How many terms are below 1 000 000? — 2

(ii) Find the sum of these terms. — 2

c) An Arithmetic series has 32 as its fifth term and the difference between the ninth and tenth term is -8.

(i) Find the first term a — 2

(ii) Calculate the sum of the first 5 terms — 2

Question 6: (10 Marks) – Start A New Page **Marks**

a) Simplify $3^{x+\log_3 x}$ **2**

b) (i) Write down the expansion of $(x-a)^3$ and hence complete the cube in **2**

$$x^3 - 6x^2 + \cdots - \cdots = (x - \cdots)^3$$

(ii) Hence use a suitable substitution to change the equation **3**

$$x^3 - 6x^2 + 14x - 5 = 0$$

into a cubic equation of the form $u^3 + cu + d = 0$

c) If $3^a = 5^b = 45^c$ show that $a = \dfrac{2bc}{b-c}$ **3**

TABLE – which of the diagrams on page 5 represent the equations listed below.

	EQUATION	GRAPH (A → J)
(i)	$y = f(-x)$	
(ii)	$y = -f(x)$	
(iii)	$y = f(x-1)$	
(iv)	$y = f(x) - 1$	
(v)	$y = 2xf(x)$	
(vi)	$y = f(2x)$	

End of Paper

Question 1

a) (i) $A \cup B = \{1,3,4,5,7,8,9,11,12,13\}$

(ii) $A \cap B = \phi$

(iii) $\overline{A \cap B} = \{1,2,3,4,5,6,7,8,9,10,11,12,13\}$
$= E$

b) $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 21 + 16 - 30$

$= 7$

c) $a - b + c = 7$ ①

$a + 2b - c = -4$ ②

$3a - b - c = 3$ ③

① + ②

$2a + b = 3$ ④

① + ③

$4a - 2b = 10$

$2a - b = 5$ ⑤

④ + ⑤

$4a = 8$

$\boxed{a = 2}$

Sub $a = 2$ into ⑤

$4 - b = 5$

$\boxed{b = -1}$

Sub $a = 2, b = -1$ into ①

$2 - -1 + c = 7$ $\boxed{c = 4}$.

d) (i) $\sqrt{x-4} > 0$

$x - 4 > 0$

D: $x > 4$

ii) $\mathbb{R}$ $y > 0$

Question 2

a) (i) $\log_a 45 = \log_a (3 \times 3 \times 5)$

$= \log_a 3 + \log_a 3 + \log_a 5$

$= 2B + C$

ii) $\log_a 0.4 = \log_a \left(\frac{2}{5}\right)$

$= \log_a 2 - \log_a 5$

$= A - C$

iii) $\log_a \left(\frac{5}{\sqrt{a}}\right) = \log_a 5 - \log_a a^{\frac{1}{2}}$

$= \log_a 5 - \frac{1}{2}\log_a a$

$= C - \frac{1}{2}$

b) $T_2 - T_1 = \log_3 4 - \log_3 2$

$= \log_3 2^2 - \log_3 2$

$= 2\log_3 2 - \log_3 2$

$= \log_3 2$

$\therefore d = \log_3 2$

$a = \log_3 2$

$T_n = a + (n-1)d$

$= \log_3 2 + n\log_3 2 - \log_3 2$

$= n\log_3 2$

c) $3^x = 12$

$x = \log_3 12$

$x = \frac{\log_{10} 12}{\log_{10} 3}$

$= 2.261859\ldots$

$\doteqdot 2.262$

Question 3

a) let $y = \frac{3x+1}{x+2}$

Inverse $x = \frac{3y+1}{y+2}$

$yx + 2x = 3y + 1$

$yx - 3y = 1 - 2x$

$y(x-3) = 1 - 2x$

$y = \frac{1-2x}{x-3}$

$f^{-1}(x) = \frac{1-2x}{x-3}$

or

$f^{-1}(x) = \frac{2x-1}{3-x}$

b) $G(F(2)) = 3(2^2) + 1$

$= 3 \times 4 + 1$

$= 13$

c) $a = 2$

$r = \frac{T_2}{T_1} = \frac{14}{2} = 7$

$T_n = ar^{n-1}$

$235298 = 2 \times 7^{n-1}$

$7^{n-1} = 117649$

$n-1 = \log_7 117649$

$n-1 = \frac{\log_{10} 117649}{\log_{10} 7}$

$= 6$

$n = 7$

$\therefore$ there are 7 terms

d) For limiting sum $-1 < r < 1$

so $-1 < -(3x+2) < 1$

$1 > 3x+2 > -1$

$-1 > 3x > -3$

$-\frac{1}{3} > x > -1$

e) $a^2 - b^2 + 2b - 1$

$= a^2 - (b^2 - 2b + 1)$

$= a^2 - (b-1)^2$

$= [a - (b-1)][a + (b-1)]$

$= (a - b + 1)(a + b - 1)$

Question 4

a) $y = \log_2(4-x)$

$4 - x > 0$

D: $4 > x$

or $x < 4$

b) $\log_3(x+3) + \log_3(x-5) = 2$

$\log_3[(x+3)\times(x-5)] = 2$

$\log_3(x^2 - 2x - 15) = 2$

$x^2 - 2x - 15 = 3^2$

$x^2 - 2x - 24 = 0$

$(x-6)(x+4) = 0$

$\therefore x = 6$

($x \neq 4$, log of negative)

c) i G
ii F
iii H
iv A
v J
vi D

Question 5

a) $\sum_{k=1}^{4} (-1)^k k^2$

$= -1 + 4 - 9 + 16$

$= 10$

b) $a = 1, r = 6$

i) $T_n < 1\,000\,000$

$1 \times 6^{n-1} < 1\,000\,000$

$\log_{10} 6^{n-1} < \log_{10} 1\,000\,000$

$(n-1)\log_{10} 6 < 6$

$n - 1 < \frac{6}{\log_{10} 6}$

$n < 8.7105\ldots$

$\therefore$ 8 terms are below 1000000

ii) $S_8 = \frac{1(6^8 - 1)}{6 - 1}$

$= \frac{6^8 - 1}{5}$

$= 335923$

c) $T_5 = 32 \Rightarrow a + 4d = 32$ ①

i) $T_{10} - T_9 = -8 \Rightarrow d = -8$ ②

Sub ② into ①

$a + 4(-8) = 32$

$a = 64$

ii) $S_5 = \frac{5}{2}(64 + 32)$

$= 240$

Question 6

a) $3^{x + \log_3 x} = 3^x \times 3^{\log_3 x}$

$= 3^x \times x$

b) i) $(x-a)^3 = x^3 - 3ax^2 + 3a^2x - a^3$

So

$x^3 - 6x^2 + 12x - 8$

$= (x-2)^3$

ii)

$x^3 - 6x^2 + 14x - 5 = 0$

$x^3 - 6x^2 + 12x - 8 + 2x + 3 = 0$

$(x-2)^3 + 2x + 3 = 0$

$(x-2)^3 + 2x + 4 - 1 = 0$

$(x-2)^3 + 2(x-2) - 1 = 0$

c) $3^a = 45^c$

$\log_{10} 3^a = \log_{10} 45^c$

$a \log_{10} 3 = c(\log_{10}(3 \times 3 \times 5))$

$a \log_{10} 3 = c(\log_{10} 3 + \log_{10} 3 + \log_{10} 5)$

$a \log_{10} 3 = c \times 2\log_{10} 3 + c \log_{10} 5$

$a \log_{10} 3 - 2c \log_{10} 3 = c \log_{10} 5$

$a - 2c(\log_{10} 3) = c \log_{10} 5$

$$\boxed{\frac{a-2c}{c} = \frac{\log_{10} 5}{\log_{10} 3}}$$

$5^b = 45^c$

$b \log_{10} 5 = c(\log_{10} 3 + \log_{10} 3 + \log_{10} 5)$

$b \log_{10} 5 = 2c \log_{10} 3 + c \log_{10} 5$

$2c \log_{10} 3 = b - c(\log_{10} 5)$

$$\boxed{\frac{2c}{b-c} = \frac{\log_{10} 5}{\log_{10} 3}}$$

$\therefore \frac{a-2c}{c} = \frac{2c}{b-c}$

$a - 2c = \frac{2c^2}{b-c}$

$a = \frac{2c^2}{b-c} + 2c$

$= 2c\left(\frac{c}{b-c} + 1\right)$

$= 2c\left(\frac{\cancel{c} + b - \cancel{c}}{b-c}\right)$

$\frac{2cb}{b-c}$

Question 6 c) - Solution #2

$3^a = 5^b = 45^c$

$a = \log_3 45^c$

$= c(\log_3 3 + \log_3 3 + \log_3 5)$

$a = 2c + c\log_3 5$

$\frac{a-2c}{c} = \log_3 5$ ①

$\log_3 5^b = \log_3 45^c$

$b\log_3 5 = c(\log_3 3 + \log_3 3 + \log_3 5)$

$b\log_3 5 = 2c + c\log_3 5$

$(b-c)\log_3 5 = 2c$

$\log_3 5 = \frac{2c}{b-c}$ ②

so ① = ②

$\frac{a-2c}{c} = \frac{2c}{b-c}$

$ab - 2bc - ac + 2c^2 = 2c^2$

$ab - ac = 2bc$

$a = \frac{2bc}{b-c}$