St George Girls High School

Year 11

Common Test – 1

2013

Mathematics Extension 1

Marks: 65

Instructions
1. Working time – 65 minutes
2. All questions should be attempted.
3. Show all working.
4. **Start each question in a new booklet.**
5. Marks will be deducted for careless work or poorly presented solutions.
6. On the cover sheet of the answer booklets clearly show:

   a) your name
   b) your mathematics class and teacher

7. Section I – parts a) to e) are multiple choice. Use the multiple choice answer sheet provided.
8. Section II – questions 6 – 10 write each question on a separate booklet showing all relevant mathematical reasoning.

Section I: (5 Marks) Marks

Multiple Choice – Parts a) to e) are multiple choice questions. Use the multiple choice answer sheet provided.

1.) The function $y+1=2(x-3)^2$ has a range 1

(A) $y \geq \frac{1}{2}$
(B) $y \geq -1$
(C) $y \geq 1$
(D) all real x

2. If $3^x = q$ then 1

(A) $\log_q x = 3$
(B) $\log_3 x = q$
(C) $\log_q 3 = x$
(D) $\log_3 q = x$

3. $\frac{x^{-1}+1}{x^{-1}-x}$ when written using only positive indices is: 1

(A) $\frac{1}{1-x}$ (B) $\frac{1-x^2}{x}$ (C) $\frac{1-x}{x}$ (D) $\frac{1+x}{x}$

4. The fraction $\frac{37}{13}$ can be written in the form $2+\cfrac{1}{x+\cfrac{1}{y+\frac{1}{z}}}$ where x, y and z are respectively 1

(A) 11, 2, 5
(B) 1, 5, 2
(C) 5, 2, 11
(D) 1, 2, 5

5. If $f(x)=4^x$ then $f(x+1)-f(x)$ equals 1

(A) 4
(B) $2\,f(x)$
(C) $3\,f(x)$
(D) $4\,f(x)$

Section II

Question 6: (12 Marks) – Start A New Booklet **Marks**

a) Expand and simplify $\left(x - \frac{1}{x}\right)^2$ **2**

b) Simplify $\frac{3^{2n} - 3^{n-1}}{3^n - 3^{-1}}$ **2**

c) Simplify

(i) $\frac{x^3+8}{x^3+x^2} \div \frac{x^2+4x+4}{4x^2+4x}$ (ii) $\frac{2}{3x-x^2} + \frac{3}{x^2-9}$ **3, 3**

d) Factorise $y^4 - 5y^2 - 36$ **2**

Question 7: (12 Marks) – Start A New Booklet **Marks**

a) Solve

(i) $4 > 2 + \frac{x}{3}$ **2**

(ii) $7x - (2x - 4) = 4 - (8 - x)$ **2**

(iii) $y^2 = 60 + 11y$ **2**

b) Find the exact solutions of $x = \frac{13}{6} - \frac{1}{x}$ **3**

c) Solve simultaneously **3**

$$x = 2y - 1$$
$$3x^2 = x + 2y^2$$

Question 8: (12 Marks) – Start A New Booklet **Marks**

a) Write $1.\dot{2}3\dot{4}$ as a fraction in simplest form **2**

nplify $4\sqrt{27} - 3\sqrt{8} + 2\sqrt{32}$ **2**

c) Expand and simplify **2**

$$(3\sqrt{2} - 2\sqrt{7})(3\sqrt{2} + 2\sqrt{7})$$

d) Express as a single fraction with a rational denominator **3**

$$\frac{1}{\sqrt{5} + \sqrt{2}} + \frac{3}{3\sqrt{2} - \sqrt{5}}$$

e) Solve for x: $\sqrt{7x} - \sqrt{3x} = 4$ **3**

Question 9: (12 Marks) – Start A New Booklet Marks

a) State the largest possible domain of $f(x) = 3 - \sqrt{4-x}$ 1

b) Given 1

$$f(x) = \begin{cases} 1 - x^2 & , f(x) \geq 0 \\ 1 - x & , f(x) < 0 \end{cases}$$

evaluate $f(-4)$

c) Draw a neat sketch of the following functions showing all relevant features

(i) $y = \frac{1}{x-1}$ 1

(ii) $y = \sqrt{2x+1}$ 1

d) Determine by algebraic methods the inverse of $f(x) = \frac{1}{2}x - 1$ and hence show $f\left(f^{-1}(3)\right) = 3$ 3

e) Using the graph of $y = f(x)$ provided at the end of the paper draw a neat and accurate sketch of the following:

(i) $y = -f(x)$ 1

(ii) $y = f(-x)$ 1

(iii) $y = 2f(x) + 1$ 1

f) Given $f(x) = x^2 - 3x + 5$ find the value of $\frac{f(x+h)-f(x)}{h}$ 2

Question 10: (12 Marks) – Start A New Booklet Marks

a) Solve $4^x = 20$ to 2 decimal places. 2

b) If $\log_a 4 = 0.66$ find $\log_a 2a^2$ 2

c) Given the recurrence relationship of a sequence is 2

$$T_1 = 5 \qquad T_n = T_{n-1} + 3n$$

write down the first 3 terms of the sequence

d) An arithmetic sequence is defined by $T_n = 127 - 3n$. How many positive terms does this sequence have? 2

~~e) Find the nth term of the sequence~~ 2

~~360, 180, 90, ...~~

f) Find a and d for an A.P. given $T_3 + T_4 = 39$ and $T_7 + T_8 = 63$ 2

End of Paper

Multiplechoice

1 B
2 D
3 A
4 B
5 C

Question 6

(a) $\left(x-\frac{1}{x}\right)^2 = x^2 - 2 + \frac{1}{x^2}$

(b) $\frac{3^{2n}-3^{n-1}}{3^n-3^{-1}} = \frac{3^n(3^n-3^{-1})}{1(3^n-3^{-1})}$

$= 3^n \quad n \neq -1$

c) $\frac{x^3+8}{x^3+x^2} \div \frac{x^2+4x+4}{4x^2+4x}$

$= \frac{\cancel{(x+2)}(x^2-2x+4)}{x^2\cancel{(x+1)}} \times \frac{4x\cancel{(x+1)}}{\cancel{(x+2)}(x+2)}$

$= \frac{4(x^2-2x+4)}{x(x+2)}$

OR

$\frac{4x^2-8x+16}{x^2+2x}$

(ii) $\frac{2}{x(3-x)} + \frac{3}{(x-3)(x+3)}$

$= \frac{-2(x+3)+3x}{x(x-3)(x+3)}$

$= \frac{-2x-6+3x}{x(x-3)(x+3)}$

$= \frac{x-6}{x(x-3)(x+3)}$

d) y^4-5y^2-36

$=(y^2-9)(y^2+4)$

$=(y-3)(y+3)(y^2+4)$

Question 7

(i) $4 > 2 + \frac{x}{3}$

$4-2 > \frac{x}{3}$

$2 > \frac{x}{3}$

$\frac{x}{3} < 2$

$x < 6$

(ii) $7x - 2x + 4 = 4 - 8 + x$

$5x + 4 = x - 4$

$4x = -8$

$x = -2.$

(iii) $y^2 - 11y - 60 = 0$

$(y-15)(y+4) = 0$

$y = 15$ or -4

(b) $x = \frac{13}{6} - \frac{1}{x} \quad x \neq 0$

multiply throughout by $6x$

$6x^2 = \frac{13 \times \cancel{6}x}{\cancel{6}} - \frac{6\cancel{x}}{\cancel{x}}$

$6x^2 = 13x - 6$

$6x^2 - 13x + 6 = 0$

add -13, mult 36) $-9, -4$

$6x^2 - 9x - 4x + 6 = 0$

$3x(2x-3) - 2(2x-3) = 0$

$(2x-3)(3x-2) = 0$

$2x - 3 = 0 \quad 3x - 2 = 0$

$x = \frac{3}{2}$ or $\frac{2}{3}$.

(c) $x = (2y-1)$ ——— ①

$3x^2 = x + 2y^2$ ——— ②

sub ① in ②

$3x^2 = x + 2(y^2)$

$3(2y-1)^2 = 2y - 1 + 2y^2$

$3[4y^2 - 4y + 1] = 2y - 1 + 2y^2$

$12y^2 - 12y + 3 = 2y^2 + 2y - 1$

$10y^2 - 14y + 4 = 0$

$5y^2 - 7y + 2 = 0$

$(5y-2)(y-1) = 0$

$y = \frac{2}{5}$ or $y = 1$

When $y = 1$, substituting in ①

$x = 2(1) - 1$

$= 1$

$\therefore x = 1, y = 1$ is a solution.

When $y = \frac{2}{5}$, substituting in ①

$x = 2\left(\frac{2}{5}\right) - 1$

$= \frac{4}{5} - 1$

$= -\frac{1}{5}$

$\therefore x = -\frac{1}{5}, y = \frac{2}{5}$ is also a solution

$\therefore$ 2 sol'ns are ...

$(x=1, y=1)$ and $\left[x = -\frac{1}{5}, y = \frac{2}{5}\right]$

Question 8

a) $1.\dot{2}3\dot{4}$ as a fraction

let $x = 1.234234\ldots$ ——— ①

$1000x = 1234.234\ldots$ ——— ②

$x = 1.234\ldots$

②−① $999x = 1233$

$x = \frac{1233}{999}$

$x = 1\frac{26}{111}$ or $\frac{137}{111}$

(b) $4\sqrt{27} - 3\sqrt{8} + 2\sqrt{32}$

$= 4\times\sqrt{9}\times\sqrt{3} - 3\sqrt{4}\times\sqrt{2} + 2\sqrt{16}\times\sqrt{2}$

$= 4\times3\sqrt{3} - 3\times2\sqrt{2} + 2\times4\sqrt{2}$

$= 12\sqrt{3} - 6\sqrt{2} + 8\sqrt{2}$

$= 12\sqrt{3} + 2\sqrt{2}$

(c) $[(3\sqrt{2}) - 2\sqrt{7}][3\sqrt{2} + 2\sqrt{7}]$

$= (3\sqrt{2})^2 - (2\sqrt{7})^2$

$= 9.2 - 4.7$

$= 18 - 28$

$= -10$

(d) $\frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} + \frac{3}{3\sqrt{2}-\sqrt{5}} \times \frac{3\sqrt{2}+\sqrt{5}}{3\sqrt{2}+\sqrt{5}}$

$= \frac{\sqrt{5}-\sqrt{2}}{5-2} + \frac{9\sqrt{2}+3\sqrt{5}}{18-5}$

$= \frac{\sqrt{5}-\sqrt{2}}{3} + \frac{9\sqrt{2}+3\sqrt{5}}{13}$

$= \frac{13(\sqrt{5}-\sqrt{2}) + 9[3\sqrt{2}+\sqrt{5}]}{39}$

$= \frac{13\sqrt{5} - 13\sqrt{2} + 27\sqrt{2} + 9\sqrt{5}}{39}$

$= \frac{22\sqrt{5} + 14\sqrt{2}}{39}$

(e) $\sqrt{7}\sqrt{x} - \sqrt{3}\sqrt{x} = 4$

$\sqrt{x}[\sqrt{7} - \sqrt{3}] = 4$

$\sqrt{x} = \frac{4}{\sqrt{7}-\sqrt{3}}$

$x = \left(\frac{4}{\sqrt{7}-\sqrt{3}}\right)^2$

$= \frac{16}{7+3-2\sqrt{21}}$

$= \frac{16}{10-2\sqrt{21}}$

$= \frac{8}{5-\sqrt{21}} \times \frac{5+\sqrt{21}}{5+\sqrt{21}}$

$= \frac{40+8\sqrt{21}}{25-21}$

$= \frac{4[10+2\sqrt{21}]}{4}$

$= 10 + 2\sqrt{21}$

OR

$\sqrt{x}[\sqrt{7} - \sqrt{3}] = 4$

$\sqrt{x}[\sqrt{7} - \cancel{\sqrt{3}}] = [\sqrt{7} - \cancel{\sqrt{3}}][\sqrt{7} + \sqrt{3}]$

$x = (\sqrt{7}+\sqrt{3})^2$

$= 7 + 2\sqrt{21} + 3$

$= 10 + 2\sqrt{21}$

7mm A4

Question 9

a) $f(x) = 3 - \sqrt{4-x}$

$4 - x \geq 0$

$4 \geq x$

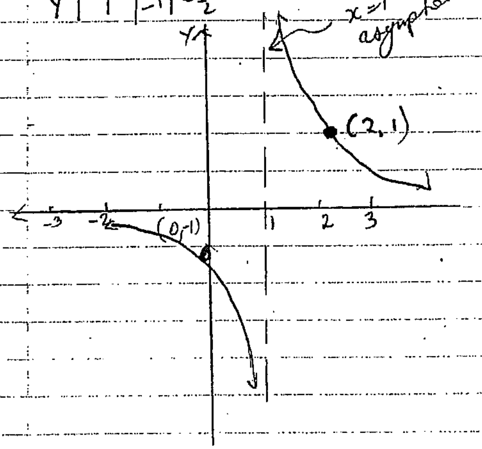
$x \leq 4$

(b) $f(-4) = 1 - -4$

$= 5$

(c) $y = \frac{1}{x-1}$ hyperbola

x	2	0	−1
Y	1	−1	$-\frac{1}{2}$



(d) $f(x) = \frac{x}{2} - 1$

let $y = \frac{x}{2} - 1$

swapping x & y get

$x = \frac{y}{2} - 1$

$2x = y - 2$

$y = 2x + 2$

$\therefore f^{-1}(x) = 2x + 2.$

(e) $f^{-1}(3) = 2(3) + 2$

$= 8$

$\therefore f[f^{-1}(3)] = f(8)$

$= \frac{8}{2} - 1$

$= 3.$

(f) $f(x) = x^2 - 3x + 5$

$f(x+h) = (x+h)^2 - 3(x+h) + 5$

$= x^2 + 2xh + h^2 - 3x - 3h + 5$

$\therefore \frac{f(x+h) - f(x)}{h}$

$= \frac{x^2 + 2xh + h^2 - 3x - 3h + 5 - [x^2 - 3x + 5}{h}$

$= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{3x} - 3h + \cancel{5} - \cancel{x^2} + \cancel{3x} - \cancel{5}}{h}$

$= \frac{2xh + h^2 - 3h}{h}$

$= \frac{\cancel{h}[2x + h - 3]}{\cancel{h}}$

$= 2x + h - 3.$

(g) $y = \sqrt{2x+1}$

x	$-\frac{1}{2}$	4	0
Y	0	3	1

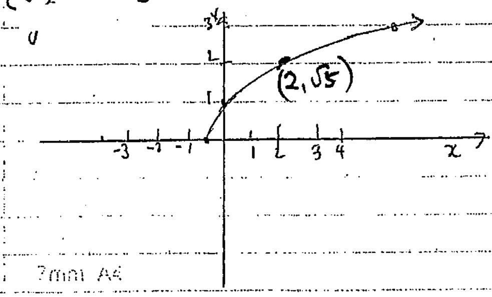
If $2x + 1 \geq 0$

$2x \geq -1$

$x \geq -\frac{1}{2}$

Squaring we can see it is '$\frac{1}{2}$'

$[y^2 = 2x + 1]$ a parabola.



7mm A4

Question 10

a) $4^x = 20$
$\log_a 4^x = \log_{10} 20$
$x \log_a 4 = \log_{10} 20$
$x = \frac{\log_{10} 20}{\log_{10} 4}$
$x = 2\cdot16$
or
$x = \log_4 20$
$= \frac{\log_{10} 20}{\log_{10} 4}$ change of base formula
$= 2\cdot16$

b) $\log_a 4 = 0\cdot66$
$\log_a 2^2 = 0\cdot66$
$2 \log_a 2 = 0\cdot66$
$\therefore \log_a 2 = 0\cdot33$

So... $\log_a (2a^2)$
$= \log_a [2 \times a^2]$
$= \log_a 2 + \log_a a^2$
$= \log_a 2 + 2 [\log_a a]$
$= \log_a 2 + 2 \times 1$
$= 0\cdot33 + 2$
$= 2\cdot33$

(c) $T_n = T_{n-1} + 3n$ $T_1 = 5$
$T_2 = T_1 + 3(2)$
$= 5 + 6$
$= 11$
$T_3 = T_2 + 3(3)$
$= 11 + 9$
$= 20$
$\therefore$ 1st 3 terms of sequence are
5, 11 and 20

d) $T_n = 127 - 3n$
this sequence decreases as the terms go on.
If $T_n = 0$,

$127 - 3n = 0$
$3n = 127$
$n = \frac{127}{3}$
$n = 42\frac{1}{3}$
$\therefore$ It has 42 positive terms.

(e) graph

(f) $T_3 + T_4 = 39$ —— ①
$T_7 + T_8 = 63$ —— ②

① $\Rightarrow$ $a + 2d + a + 3d = 39$
$2a + 5d = 39$ —— ①
② $\Rightarrow$ $a + 6d + a + 7d = 63$
$2a + 13d = 63$ —— ②
② − ① $2a + 5d = 39$
$8d = 24$
$\therefore d = 3$
Substitute $d = 3$ in $2a + 5d = 39$
$2a + 15 = 39$
$2a = 24$
$a = 12$

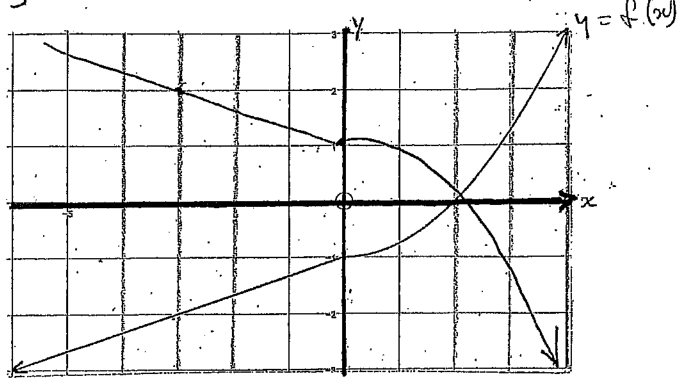
$\therefore$ $a = 12$, $d = 3$

7mm ruled

Question 5

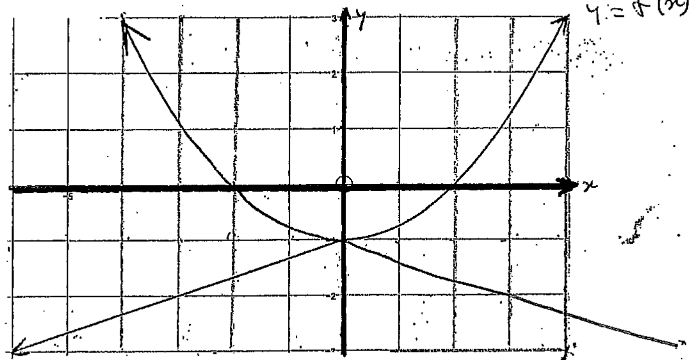
e) (i)

Draw
$y = -f(x)$
reflect about x axis



(ii)

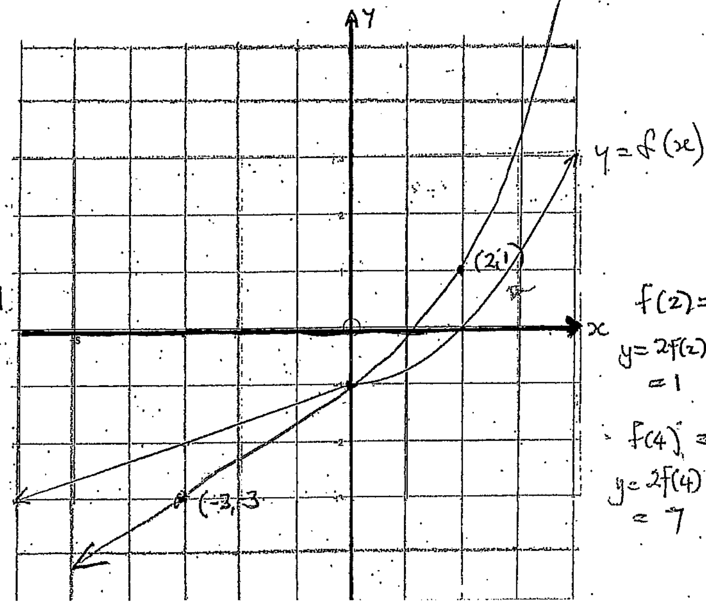
Draw
$y = f(-x)$
reflect about y axis



(iii)

Draw
$y = 2f(x) + 1$

$f(0) = -1$
$y = 2f(0) + 1$
$= -2 + 1$
$= -1$
$f(-3) = -2$
$y = 2f(-3) + 1$
$= -4 + 1$
$= -3$



$f(2) = 0$
$y = 2f(2) + 1$
$= 1$

$f(4) = 3$
$y = 2f(4) + 1$
$= 7$