St George Girls High School

Year 11

Common Test 2

June 2012

Mathematics

General Instructions

- Time: 75 minutes
- Write using blue or black pen
- Start each question in a new booklet.
- Calculators may be used
- Total marks (64)
- Attempt all questions 1 – 8

Question 1 – Functions and their Graphs – Start a New Booklet (8 Marks) Marks

Multiple Choice – For Parts a), b) and c) write down the letter matching the correct answer in your answer booklet.

a) Which of the following is not a function? 1

(A) $y = x^2 + 2$
(B) $y = \frac{5}{x} - 5$
(C) $y = 2^x - 3$
(D) $y = \pm\sqrt{4 - x^2}$

b) The centre and radius of the circle $(x+3)^2 + (y+2)^2 = 36$ 1

(A) centre $(3, 2)$, $r = 36$
(B) centre $(3, 2)$, $r = 6$
(C) centre $(-3, -2)$, $r = 36$
(D) centre $(-3, -2)$, $r = 6$

c) The function $F(x) = 8x^3 - 7x - 1$ is: 1

(A) even
(B) odd
(C) both even and odd
(D) neither even nor odd

d) Write down the new equation after the graph of $y = x^2$ is shifted 3 units to the right. 1

e) Sketch the following lines on the same set of axes. 2

(i) $x = 3$
(ii) $y = 2x - 3$

f) Find the domain and range of the function $y = -\sqrt{64 - x^2}$ 2

End of Question 1

Question 2 – Functions, Graphs and Inequations – Start a New Booklet (8 Marks) Marks

Using the function $f(x) = x^2 - 6x + 8$

a) Find the x-intercepts of the function. 2

b) Find the minimum value of the function 1

c) Sketch the graph of the function showing clearly the x-intercepts, y-intercept, and the point where the function has the minimum value. 2

d) Using the above graph or otherwise, solve $x^2 - 6x + 8 \leq 0$ 1

e) Shade the region where $y \geq x^2 - 6x + 8$ and $y < 2$ 2

End of Question 2

Question 3 – Graphs and Inequations – Start a New Booklet (8 Marks) Marks

a) Solve $3 - 2x \geq 7$ and graph the solution on a number line. 2

b) Solve the following inequality and graph the solution on a number line 3

$$|2 + 4x| > 6$$

c) Sketch the region $x^2 + y^2 \leq 9$ 3

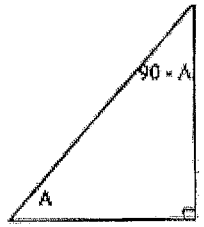
End of Question 3

Question 4 – Trigonometry – Start a New Booklet (8 Marks) Marks

Multiple Choice – For Part a) write down the letter matching the correct answer in your answer booklet.

a) If $\sin(90 - A) = 0.5235$, then A is closest to: 1

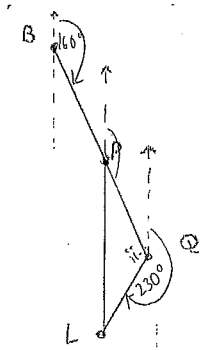
(A) 32°
(B) 59°
(C) 52°
(D) 35°



b) A boat sailing on a bearing of 160° at 12 km/h arrives at a point P due north of a lighthouse L at precisely noon. Forty minutes later the boat is at a point Q where the lighthouse is on a bearing of 230°.

(i) copy and Complete the diagram by marking in the angles of 160° and 230°. 2

Show the length (in km) of PQ on your diagram.

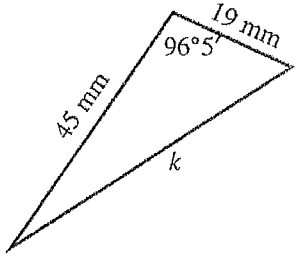


(ii) Calculate the size of angle PQL. 1

(iii) Hence calculate the distance PL to the nearest 100 m. 2

Question 4 (cont'd) Marks

c) In the given triangle find the value of the side marked k (correct to one decimal place). 2



End of Question 4

Question 5 – Trigonometry – Start a New Booklet (8 Marks) Marks

Multiple Choice – For Part a) write down the letter matching the correct answer in your answer booklet.

a) Simplify: $\tan\theta\cos\theta =$ 1

(A) $\sin\theta$
(B) $\cos\theta$
(C) $\sec\theta$
(D) $\cot\theta$

$\sin^2\theta + \cos^2\theta = 1$
$1 + \tan^2\theta = \sec^2\theta$
$1 + \cot^2\theta = \operatorname{cosec}^2\theta$
$\tan\theta = \frac{\sin\theta}{\cos\theta}$

b) Without the use of a calculator, simplify the following: 2

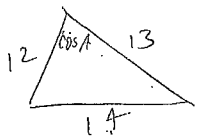
$$\tan^2 135° - \operatorname{cosec}^2 330° - \sin 240° \sec 30°$$

c) Find the exact value of: 2

(i) $\cos 180°$

(ii) $\cot(-120°)$

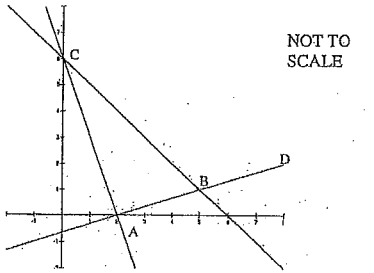
d) A triangle has sides 12 cm, 13 cm and 14 cm. Use the cosine rule to find the smallest angle (to the nearest minutes) and hence find the area of the triangle. 3



End of Question 5

Question 6 – Coordinate Geometry – Start a New Booklet (8 Marks) Marks

a) The points $A(2,0)$, $B(5,1)$ and $C(0,6)$ are shown on the Cartesian plane above. D is a point on the line AB.



(i) Show that the lines AB and AC are perpendicular 2

(ii) Show that the length of AB is $\sqrt{10}$ units 2

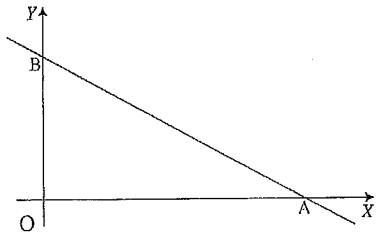
b) (i) Write down the general form of a line through the point of intersection of the two lines l_1: $2x - y + 6 = 0$ and l_2: $x + 2y - 5 = 0$. 2

(ii) Hence find the equation of the line through the point of intersection of the lines l_1 and l_2 and the point (3,0) 2

End of Question 6

Question 7 – Coordinate Geometry & Indices – Start a New Booklet (8 Marks) Marks

a)



B is the point $(0,6)$, A is a point on the x-axis and **O** is the origin.

(i) If $AB = 10$ units, find the coordinates of A. 1

(ii) Find the coordinates of M, the mid-point of AB. 1

(iii) Show that AB has equation $3x + 4y - 24 = 0$. 2

(iv) Find the shortest distance from O to the line AB. 1

b) The lines $ax + 2y = 6$ and $4y = bx - 9$ are parallel. Find the value of $\frac{a}{b}$. 2

Multiple Choice – For Part c) write down the letter matching the correct answer in your answer booklet.

c) If $49^x = \frac{1}{7}$, 1

(A) $x = \frac{1}{2}$

(B) $x = -\frac{1}{2}$

(C) $x = \frac{1}{7}$

(D) $x = -\frac{1}{7}$

End of Question 7

Question 8 – Indices and Logarithms – Start a New Booklet (8 Marks) Marks

Multiple Choice – For Part a) write down the letter matching the correct answer in your answer booklet.

a) $\log_2 2^4 =$ 1

(A) 4
(B) 16
(C) 256
(D) 65536

b) Solve $2^{x-3} = \frac{1}{4}$ 1

c) Find the exact value of $\frac{a^4 b}{c^4}$ where $a = \left(\frac{2}{3}\right)^3, b = \left(\frac{8}{3}\right)^2, c = \left(\frac{4}{3}\right)^4$ 2

d) Simplify: $\log_{10} 25 + \log_{10} 9$ 2

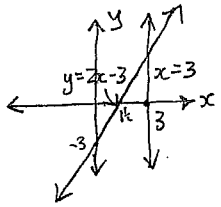
e) Find, correct to four significant figures, $\log_2 7$ 2

End of Paper

YEAR 11 COMMON TEST 2 - 2016

QUESTION 1

(a) D (b) D (c) D (d) $y = (x-3)^2$ (e)



(1 mark each graph)

(f) D: $-8 \leq x \leq 8$
R: $-8 \leq y \leq 0$ (1 mark each)

QUESTION 2

(a) x intercepts when $f(x) = 0$

$(x-4)(x-2) = 0$

$x = 2 \quad x = 4$

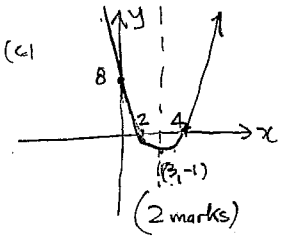
(1 mark each)

(b) When $x = 3$, $f(x) = -1$

Minimum value -1

(1 mark)

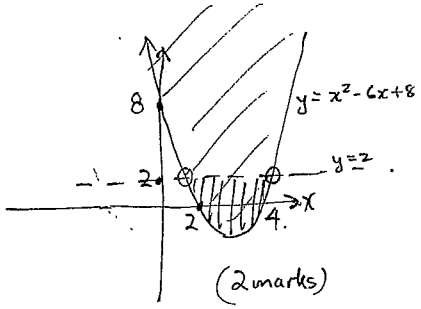
(c)



(2 marks)

(d) $2 \leq x \leq 4$ (1 mark)

(e)



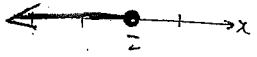
(2 marks)

QUESTION 3

(a) $3 - 2x \geq 7$

$-2x \geq 4$

$x \leq -2$

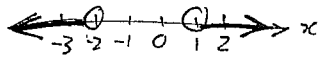


(b) $|2 + 4x| > 6$

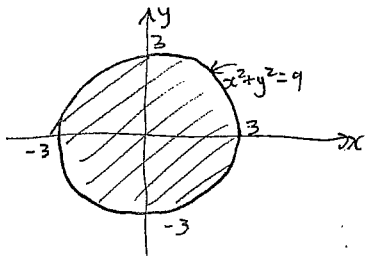
$2 + 4x > 6$ or $2 + 4x < -6$

$4x > 4 \qquad 4x < -8$

$x > 1 \qquad x < -2$



(c)



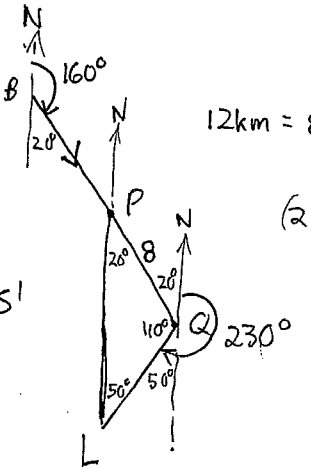
QUESTION 4

(a) $\cos A = 0.5235$

$A \approx 59°$

(B)

(b)



(i) 12km = 8km / 40min

(2 marks)

(ii) $\angle PQL = 110°$ (1 mark)

(iii) $\frac{PL}{\sin 110°} = \frac{8}{\sin 50°}$

$PL = \frac{8 \sin 110°}{\sin 50°}$ (2 marks)

$= 9.813\text{km}$

$\approx 9.8\text{km}$ (nearest 100m)

(c) (i) $k^2 = 45^2 + 19^2 - 2 \times 45 \times 19 \times \cos 96°51'$

$k^2 = 2567.216952$

$k = 50.7\text{ mm}$ (1 D.P)

(2 marks)

QUESTION 5

(a) A (1 mark)

(b) $\tan^2 135 - \operatorname{cosec}^2 330° - \sin 240° \sec 30° = (-1)^2 - (-2)^2 - -\frac{\sqrt{3}}{2} \times$

$\tan 135° = -\tan 45° = -1$

$\operatorname{cosec} 330° = \frac{1}{-\sin 30°} = -2$

$\sin 240° = -\sin 60° = -\frac{\sqrt{3}}{2}$

$\sec 30° = \frac{1}{\cos 30°} = \frac{2}{\sqrt{3}}$

$= 1 - 4 + 1$

$= -2$

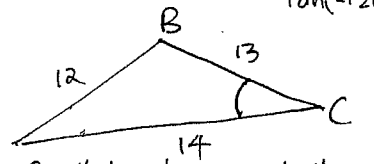
(3 marks)

(c) (i) $\cos 180° = -1$

(ii) $\cot(-120°) = \frac{1}{\tan(-120°)} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

(1 mark each)

(d)



(3 marks)

$\cos C = \frac{13^2 + 14^2 - 12^2}{2 \times 13 \times 14}$

$C = 52°37'$

Smallest angle is opposite the smallest side

Area $= \frac{1}{2} \times 13 \times 14 \times \sin 52°37'$

$= 72.3\text{ cm}^2$ (1 D.P)

QUESTION 6

(a) (i) $m_{AB} = \frac{1-0}{5-2} = \frac{1}{3} = m_1$

$m_{AC} = \frac{6-0}{0-2} = -3 = m_2$ (2 marks)

Since $m_1 m_2 = -1$ $AB \perp AC$

(ii) $AB = \sqrt{(5-2)^2+(1-0)^2}$

$= \sqrt{3^2+1^2}$

$= \sqrt{10}$ (2 marks)

(b) (i) $2x - y + 6 + k(x+2y-5) = 0$ (2 marks)

(ii) $6 - 0 + 6 + k(3 + 0 - 5) = 0$

$12 - 2k = 0$

$k = 6$ (2 marks)

$2x - y + 6 + 6(x + 2y - 5) = 0$

$2x - y + 6 + 6x + 12y - 30 = 0$

$\boxed{8x + 11y - 24 = 0}$

QUESTION 7

(a) (i) $(8, 0)$ (ii) $(4, 3)$ (1 mark each)

(iii) $y - 0 = \frac{-3}{4}(x - 8)$

$-4y = 3x - 24$ (2 marks)

$3x + 4y - 24 = 0$

(iv) $d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}} \right|$

$= \left| \frac{3 \times 0 + 4 \times 0 - 24}{\sqrt{3^2+4^2}} \right|$

$= \left| \frac{24}{5} \right|$ (1 mark)

$= 4\frac{4}{5}$

(b) $2y = -ax + 6$ $\quad 4y = bx - 9$

$m_1 = \frac{-a}{2}$ $\quad m_2 = \frac{b}{4}$ (2 marks)

$m_1 = m_2$

$-\frac{a}{2} = \frac{b}{4} \Rightarrow \frac{-a}{2} \div \frac{b}{4} = 1 \Rightarrow \frac{-2a}{b} = 1 \Rightarrow \frac{a}{b} = -\frac{1}{2}$

(c) B (1 mark)

QUESTION 8

(a) $\log_2 2^4 = 4\log_2 2 = 4$ (A) (1 mark)

(b) $2^{x-3} = 2^{-2}$

$x - 3 = -2$ (1 mark)

$x = 1$

(c) $\frac{a^4 b}{c^4} = \frac{\left(\frac{2}{3}\right)^{12}\left(\frac{8}{3}\right)^2}{\left(\frac{4}{3}\right)^{16}}$

$= \frac{2^{12} \times 8^2 \times 3^{16}}{3^{12} \times 3^2 \times 4^{16}}$

$= \frac{3^2 \times 2^{18}}{2^{32}}$

$= \frac{32}{2^{14}}$

$= \frac{9}{16384}$ (2 marks)

(d) $\log_{10} 25 + \log_{10} 9 = \log_{10} 225$

$= \log_{10} 15^2$

$= 2\log_{10} 15$ (2 marks)

(e) $\log_2 7 = \frac{\log_{10} 7}{\log_{10} 2} =$ (2 marks)

$= 2.807$ (4 sig. figs)