**St George Girls High School**

**Year 11**

**HSC Assessment Task 1**

**November 2011**

# Mathematics Extension 1

***Time Allowed: 65 minutes***
***(plus 5 minutes reading time)***

**Marks: 60**

**Instructions:**

- Use only black or blue pens.
- Start each question on a new page
- Set out work clearly
- All necessary working must be shown in all questions.
- Board approved calculators may be used.
- All six questions may be attempted.



**Question 1 – (10 marks) – Start a new page** **Marks**

a) *Multiple Choice Question. Write down the letter corresponding to the correct answer.*

If $\int_1^4 f(x)dx = 2$, then $\int_1^4 (2f(x)+3)dx$ is equal to

A 4 C 10

B 7 D 13 **1**

b) Evaluate $\int_{-3}^{4}(3x^2 - 7x - 5)dx$ **2**

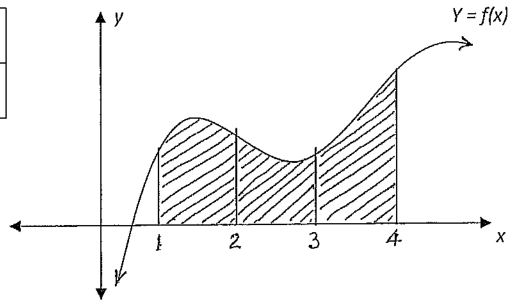
c) Find the indefinite integrals:

(i) $\int \left(\frac{1}{x\sqrt{x}} + \frac{2}{x^3}\right)dx$ **2**

(ii) $\int (1-x)^8 dx$ **2**

d) Use the trapezoidal rule to approximate the area of the region shown below using all the function values given: **2**

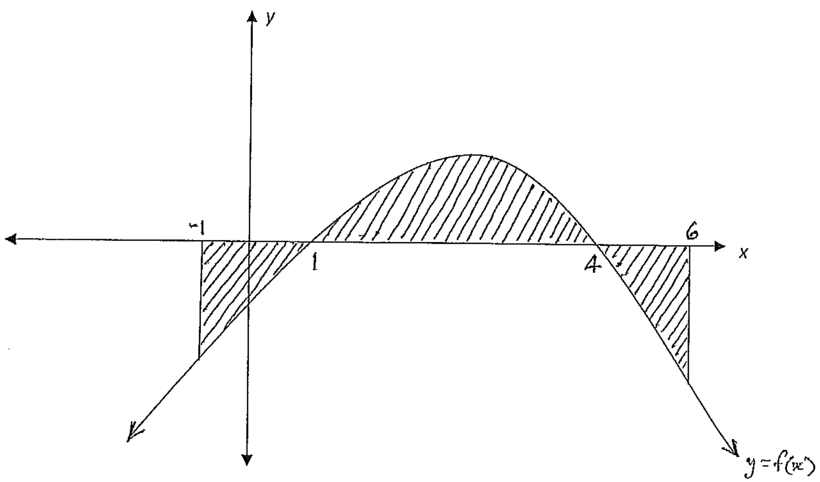
| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $f(x)$ | 4 | 6 | 4 | 9 |




e) Evaluate $\log_7 12$ correct to 3 significant figures. **1**

**Question 2** – **(10 marks) – Start a new page** **Marks**

a) *Multiple Choice Question. Write down the letter corresponding to the correct answer.*




The total area of the shaded regions in the diagram is given by: **1**

A $\int_{-1}^{6} f(x)dx$

B $\int_{1}^{4} f(x)dx + 2\int_{4}^{6} f(x)dx$

C $-\int_{-1}^{1} f(x)dx + \int_{1}^{4} f(x)dx - \int_{4}^{6} f(x)dx$

D $-\int_{1}^{-1} f(x)dx + \int_{1}^{4} f(x)dx - \int_{6}^{4} f(x)dx$

b) Evaluate:

$$\int_{0}^{1} x^2(x^3-2)^4 dx$$

**2**



**Question 2 continued** **Marks**

c)

(i) Sketch the region enclosed by the curves $y = 4x - x^2$ and $y = 2x$ **1**

(ii) Using your sketch or otherwise find the area of the region enclosed by the curves $y = 4x - x^2$ and $y = 2x$ **2**

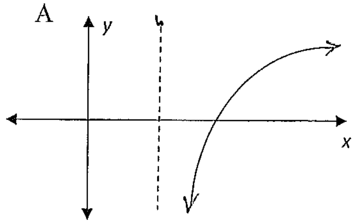
(iii) If this area is rotated about the x axis, find the volume of the solid formed **4**

**Question 3 – (10 marks) – Start a new page** **Marks**

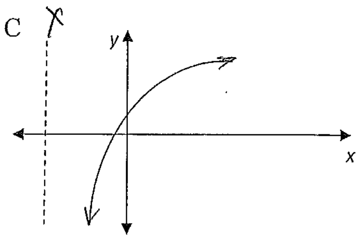
a) *Multiple Choice Question. Write down the letter corresponding to the correct answer.*

Which of the following could be the graph of $y = a\log_e(x-b)$ where $a < 0$ and $b > 0$ **1**

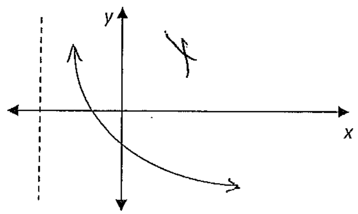
A



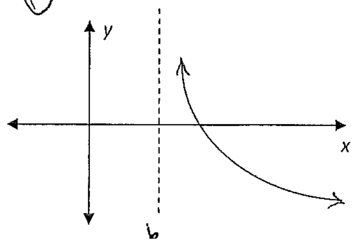
C



B



D



b) Find the derivative of:

(i) $\log_e\left(\frac{x+1}{x-1}\right)$ **2**

(ii) $\log_e\sqrt{x^2+1}$ **2**

c) Given that $\log_a b = 7$ and $\log_c a = 3$ find the value of $\log_b c$ **2**

d) Using two applications of Simpsons Rule calculate an approximation of: **3**

$\int_1^5 \log_e x\, dx$ (correct to 2 decimal places)



**Question 4 – (10 marks) – Start a new page** **Marks**

a) *Multiple Choice Question. Write down the letter corresponding to the correct answer.*

Consider the region bounded by the x axis, the y axis, the line with equation $y = 3$ and the curve with equation $y = \log_e(x-1)$.
The exact value of the area of this region is: **1**

A $e^{-3} - 1$ C $16 + 3\log_e 2$

B $e^3 + 2$ D $3e^2$

b) Find:.

(i) $\int_1^3 e^{7-5x}dx$ **2**

(ii) $\int_1^2 x^2e^{x^3}dx$ **2**

c) The growth rate per hour, $\frac{dP}{dt}$, of a population of bacteria, $P$, is 15% of the population at that time, t. Initially the population is 70 000:

(i) Show that the population in any hour can be calculated by the model $P = P_0e^{0.15t}$ **2**

(ii) Sketch the curve of the population against time **1**

(iii) Determine the population after 5 hours. **2**

**Question 5 – (10 marks) – Start a new page** **Marks**

a) *Multiple Choice Question. Write down the letter corresponding to the correct answer.*
If $y = 3a^{2x} + b$ then $x$ is equal to: **1**

A $\frac{1}{6}\log_a(y-b)$ C $\frac{1}{6}\log_a\left(\frac{y}{b}\right)$

B $\frac{1}{2}\log_a\left(\frac{y-b}{3}\right)$ D $\frac{1}{2}\log_a\left(\frac{3y}{b}\right)$

b) Differentiate with respect to x:

(i) $e^{x^2+4x}$ **1**

(ii) $\frac{x^2}{e^x}$ **2**

c) Consider the function $y = e^{kx+1}$

(i) Find $\frac{dy}{dx}$ **1**

(ii) Find $\frac{d^2y}{dx^2}$ **1**

(iii) Determine the values of $k$ for which $y = e^{kx+1}$ satisfies the equation: **2**

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$$

d) Find the minimum value of $(x-2)e^x$ **2**



**Question 6 – (10 marks) – Start a new page** **Marks**

a) *Multiple Choice Question. Write down the letter corresponding to the correct answer.*
Let $k = \int_{-2}^{-1} \frac{1}{x}\,dx$ , then $e^k$ is equal to : **1**

A $\frac{1}{2}$ C 1

B 2 D $\log_e 2$

b)
(i) State the domain of the function $y = x^3 \ln x$ **1**

(ii) Find any stationary points of $y = x^3 \ln x$ and determine their nature. **3**

(iii) Show that there is a point of inflexion at $x = e^{-\frac{5}{6}}$ **2**

(iv) Evaluate $\lim\limits_{x \to 0} x^3 \ln x$ **1**

(v) Sketch the graph of $y = x^3 \ln x$ **2**

**END OF PAPER**

YR 11 EXT 1 TASK 1 2011 (HSC 2012)

QUESTION 1

a) $\int_1^4 (2f(x)+3)\,dx = \int_1^4 2f(x)\,dx + \int_1^4 3\,dx$

$= 2\times 2 + [3x]_1^4$

$= 4 + 12 - 3$

$= 13$

$\therefore$ D

b) $\int_{-3}^4 (3x^2-7x-5)\,dx = \left[x^3 - \frac{7}{2}x^2 - 5x\right]_{-3}^4$

$= [64 - 56 - 20] - [-27 - \frac{63}{2} + 15]$

$= -12 - [-\frac{87}{2}]$

$= \frac{63}{2}$

c) i) $\int \left(\frac{1}{x\sqrt{x}} + \frac{2}{x^3}\right)dx = \int (x^{-3/2} + 2x^{-3})\,dx$

$= -2x^{-1/2} - x^{-2} + c$

$= \frac{-2}{\sqrt{x}} - \frac{1}{x^2} + c$

ii) $\int (1-x)^8\,dx = -\frac{1}{9}(x-8)^9 + c$

d) $A \approx \frac{1}{2}(4+6) + \frac{1}{2}(6+4) + \frac{1}{2}(4+9)$

$= 5 + 5 + 13/2$

Area is approx $\frac{33}{2}$ sq. units

e) $\log_7 12 = \frac{\log_e 12}{\log_e 7}$

$= 1.277$ (3 sign fig)

Question 2

a) C

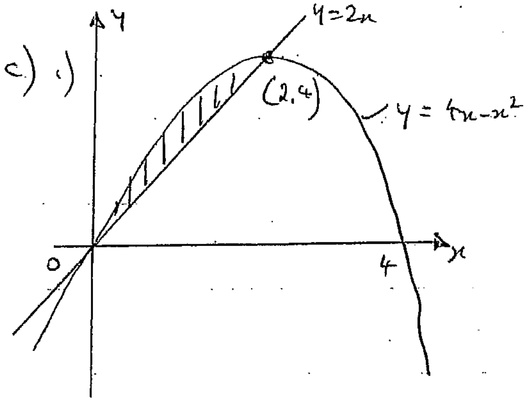
b) $\frac{1}{3}\int_0^1 3x^2(x^3-2)^4\,dx = \frac{1}{3}\left[\frac{1}{5}(x^3-2)^5\right]_0^1$

$= \frac{1}{15}\left[(1-2)^5 - (0-2)^5\right]$

$= \frac{1}{15}\left[-1 - (-32)\right]$

$= \frac{31}{15}$

c) i)




$2x = 4x - x^2$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 2$ or $0$.

ii) $A = \int_0^2 (4x - x^2 - 2x)\,dx$

$= \int_0^2 (2x - x^2)\,dx$

$= \left[x^2 - \frac{x^3}{3}\right]_0^2$

$= [(4 - \frac{8}{3}) - (0-0)]$

$= \frac{4}{3}$

Area is $4/3$ sq. units

iii) $V = \pi\int_0^2 [(4x-x^2)^2 - (2x)^2]\,dx$

$= \pi\int_0^2 (16x^2 - 8x^3 + x^4 - 4x^2)\,dx$

$$= \pi \int_0^2 (12x^2 - 8x^3 + x^4)\, dx$$

$$= \pi \left[ 4x^3 - 2x^4 + \frac{x^5}{5} \right]_0^2$$

$$= \pi \left[ \left(32 - 32 + \frac{32}{5}\right) - (0 - 0 - 0) \right]$$

$$= \frac{32\pi}{5}$$

Volume of solid of revolution is $\frac{32\pi}{5}$ $u^3$

## Question 3

a) D

b) i) $y = \log_e \left( \frac{x+1}{x-1} \right)$

$$y' = \log_e (x+1) - \log_e (x-1)$$

$$= \frac{1}{x+1} - \frac{1}{x-1}$$

$$= \frac{-2}{x^2-1}$$

ii) $y = \log_e \sqrt{x^2+1}$

$$= \log_e (x^2+1)^{1/2}$$

$$= \frac{1}{2} \log_e (x^2+1)$$

$$y' = \frac{1}{2} \cdot \frac{1}{x^2+1} \cdot 2x$$

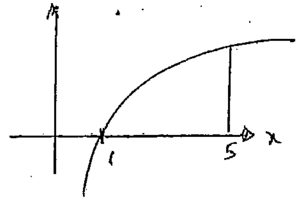
$$= \frac{x}{x^2+1}$$

c) $\log_b c = \dfrac{\log_a c}{\log_a b}$

$$= \frac{\frac{1}{\log_c a}}{\log_a b}$$

$$= \frac{1/3}{7}$$

$$= \frac{1}{21}$$

d) 

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\ln x$ | 0 | $\ln 2$ | $\ln 3$ | $\ln 4$ | $\ln 5$ |

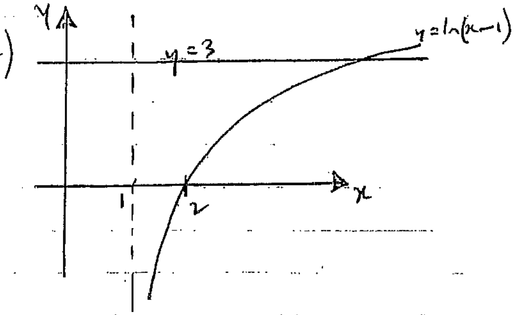
$$\int_1^5 \log_e x \, dx \approx \frac{2}{6}\left[\ln 1 + 4\ln 2 + \ln 3\right] + \frac{2}{6}\left[\ln 3 + 4\ln 4 + \ln 5\right.$$

$$= \frac{1}{3}\left[ \ln 48 + \ln 11250 \right]$$

$$= \frac{1}{3} \ln 540000$$

$$= 4.40 \quad (2\ dp)$$

## Question 4

a) 


$$A = \int_0^3 (e^y + 1)\, dy$$

$$= e^3 + 2$$

$\therefore$ B

b) i) $\int_1^3 e^{7-5x}\,dx = \left[-\frac{1}{5}e^{7-5x}\right]_1^3$

$= -\frac{1}{5}\left[e^{-8} - e^{2}\right]$

$= \frac{1}{5}\left[e^{2} - e^{-8}\right]$

ii) $\int_1^2 x^2 e^{x^3}\,dx = \left[\frac{1}{3}e^{x^3}\right]_1^2$

$= \frac{1}{3}\left[e^{8} - e\right]$

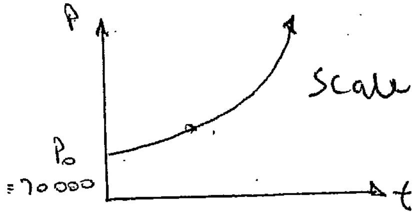
c) $\frac{dP}{dt} = 0.15P$

(i) $P = P_0 e^{0.15t}$

$\frac{dP}{dt} = P_0 \cdot (0.15)\, e^{0.15t}$

$= 0.15\left(P_0 e^{0.15t}\right)$

$= 0.15\,P$ as required

ii) 


When $t=0$ $P = 70\,000$

$\therefore P_0 = 70\,000$

iii) $t=5$ $P = 70\,000\, e^{0.15\times 5}$

$= 148\,190$

$\therefore$ Population after 5 hours 148 190 bacteria

QUESTION 3

a) $y = 3a^{2x} + b$

$y - b = 3a^{2x}$

$\frac{y-b}{3} = a^{2x}$

$\log_a\left(\frac{y-b}{3}\right) = 2x$

$\therefore x = \frac{1}{2}\log_a\left(\frac{y-b}{3}\right)$ So B.

b) i) $\frac{d}{dx}\left(e^{x^2+4x}\right) = (2x+4)\,e^{x^2+4x}$

ii) $\frac{d}{dx}\left(\frac{x^2}{e^x}\right) = \frac{2x\cdot e^x - e^x\cdot x^2}{(e^x)^2}$

$= \frac{x e^x(2-x)}{(e^x)^2}$

$= \frac{x(2-x)}{e^x}$

c) $y = e^{kx+1}$

i) $\frac{dy}{dx} = k\,e^{kx+1}$

ii) $\frac{d^2y}{dx^2} = k^2 e^{kx+1}$

iii) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = k^2 e^{kx+1} + 3k\,e^{kx+1} + 2e^{k}$

$= e^{kx+1}\left[k^2+3k+2\right]$

is equal to 0 when $k^2+3k+2=0$ as $e^{kx+1}$

$\therefore k = -1$ or $-2$

d) $\frac{d}{dx}(x-2)e^x$

$= \frac{d}{dx}(xe^x - 2e^x)$

$= 1.e^x + xe^x - 2e^x$

$= xe^x - e^x$

$= e^x(x-1)$

$\therefore$ derivative equals 0 at $x=1$

$\therefore$ critical point

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $(x-1)e^x$ | $-1$ | 0 | $e^2>0$ |

\ — /

$\therefore$ Minimum value at $x=1$ will be $\underline{\underline{-e}}$

## Question 6

a) $k = \int_{-2}^{-1} \frac{1}{x}\,dx$

$= -\int_{1}^{2} \frac{1}{x}\,dx$

$= -[\ln x]_1^2$

$= -[\ln 2 - \ln 1]$

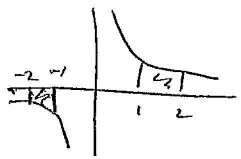
$= -\ln 2$

$= \ln \frac{1}{2}$

$e^k = e^{\ln \frac{1}{2}}$

$= \frac{1}{2}$

$\therefore$ A




OR $\int_{-2}^{-1} \frac{1}{x}\,dx$

$= [\ln|x|]_{-2}^{-1}$

$= \ln|-1| - \ln|-2|$

$= 0 - \ln 2$

$= \ln 2^{-1}$

$= \ln \frac{1}{2}$

1

5) i) $y = x^3 \ln x$ 1

D: $x > 0$

ii) $y' = 3x^2 \ln x + x^3 \cdot \frac{1}{x}$

$= 3x^2 \ln x + x^2$

$= x^2(3\ln x + 1)$

$y' = 0$ when $x^2 = 0$

$x = 0$

No solution

Not in domain

$3\ln x + 1 = 0$

$3\ln x = -1$

$\ln x = -\frac{1}{3}$

$x = e^{-1/3}$ 1

$y'' = 2x(3\ln x + 1) + x^2 \cdot \left(\frac{3}{x}\right)$

$= 6x \ln x + 2x + 3x$

$= x(6\ln x + 5)$

at $x = e^{-1/3}$ 1

$y'' = 2.15$

$> 0$ $\cup$ min t.p $\left(e^{-1/3}, \frac{-1}{3e}\right)$ 1

iii) $y'' = 0$ at $x = 0$ Not in domain or $6\ln x + 5 = 0$

$6\ln x = -5$

$\ln x = -\frac{5}{6}$

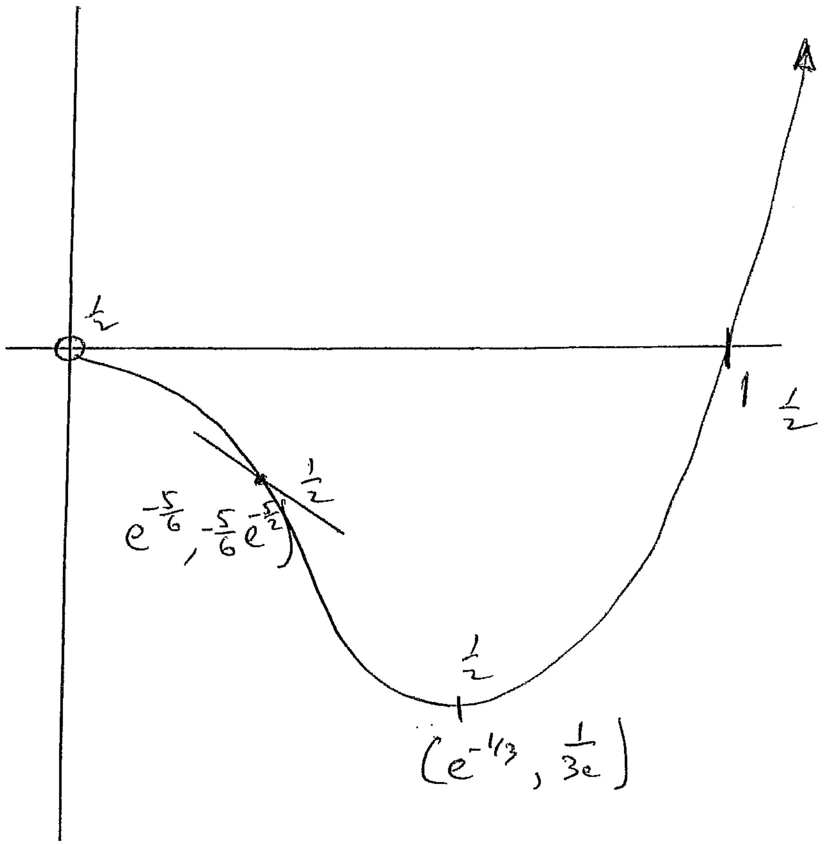
$x = e^{-5/6}$ 1

| $x$ | $e^{-1}$ | $e^{-5/6}$ | $e^0$ |
|---|---|---|---|
| $y''$ | $-0.4$ | 0 | 5 |

1 $\therefore$ pt of inflexion at $x = e^{-5/6}$

iv) $\lim_{x \to 0^+} x^3 \ln x = 0$

1



$\frac{1}{2}$
$\frac{1}{2}$
$\left(e^{-\frac{5}{6}}, -\frac{5}{6}e^{-\frac{5}{2}}\right)$
$\frac{1}{2}$
$\left(e^{-1/3}, \frac{1}{3e}\right)$
1
$\frac{1}{2}$