St George Girls High School

Year 11 – Higher School Certificate Course

Assessment Task 1

2009

Mathematics

General Instructions

Total Number of Marks: 65

- Reading time – 5 minutes.
- Working time – 70 minutes.
- Write using black or blue pen.
- Attempt all questions.
- Start each question on a new page.
- Show ALL working.
- Marks for each question are shown in right column
- Complete cover sheet clearly showing
  - your name
  - mathematics class and teacher

Question 1 – (13 Marks) – Start a New Page Marks

a) Complete the sentence: 2

If at $x = a$ $f'(a) = 0$ and $f''(a) > 0$, then at $x = a$ there exists

a ______________________

b) (i) Sketch the locus of a point whose distance from the x –axis is 2 units. 2

(ii) Write down the equation of the locus. 2

c) Write down the centre and the radius of the circle 3

$$x^2 + 12x + y^2 + 14y = -60$$

d) Sketch the parabola 4

$$(y + 5)^2 = -8(x - 3)$$

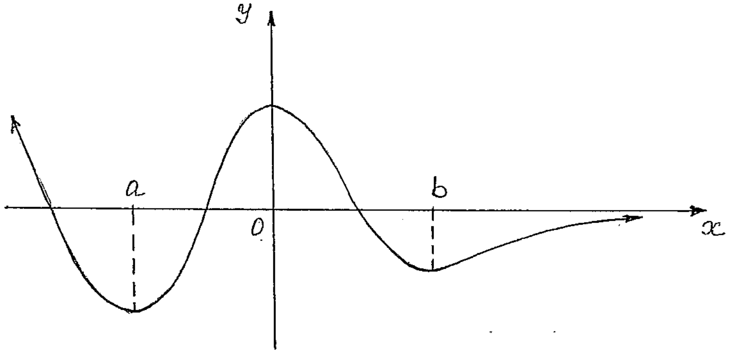
clearly indicating the coordinates of the vertex and focus and the equation of the directrix.

Question 2 – (13 Marks) – Start a New Page **Marks**

a) For what values of x is the function, which has derivative $\frac{dy}{dx} = -5x$ increasing? 1

b) Given below is the sketch of the function $y = f(x)$. 4

Copy it into your booklet and sketch a possible curve to represent $y = f'(x)$.
(Use a separate set of axes)



c) Find the equation of the locus of a point $P(x, y)$ which moves in the plane so that it is equidistant from the point $(3, 2)$ and the line $y = -2$ 4

d) Factorise completely $3x^3 + 81$ 2

e) If $y = \frac{1}{(3-7x)^3}$ find 2

(i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

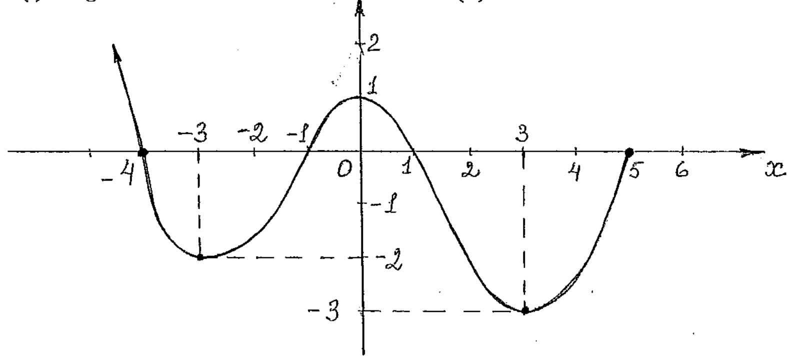
Question 3 – (13 Marks) – Start a New Page **Marks**

a) Find the primitive functions of 3

(i) $\frac{5}{x^3}$ (ii) $(4x-3)^7$

b) Use the graph below to identify the coordinates of the

(i) global minimum (ii) local maximum 2



c) Given the points $A(-1, 7)$ and $B(2, 3)$, find the equation of the locus of the point $P(x, y)$ so $< APB$ is $90°$. 4

d) Determine whether the function $y = x^4 - 2x^2 - x$ is odd, even or neither. 2

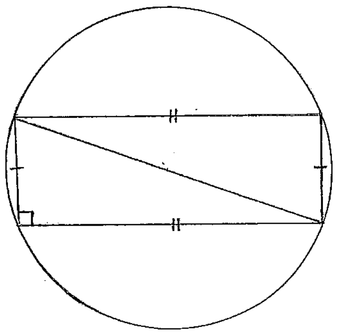
e) Differentiate and express the derivative without negative powers: 2

$$3x + \frac{1}{x} - 5\sqrt{x} + 4$$

Question 4 – (13 Marks) – Start a New Page

Marks

a) A rectangle of sides x and y cm is constructed in a circle of diameter 24 cm as shown in the diagram.



(i) Show that the area, A, of the rectangle is given by $A = x\sqrt{576 - x^2}$ 2

(ii) Show that $\frac{dA}{dx} = \frac{576-2x^2}{\sqrt{576-x^2}}$ 3

(iii) Find the area of the largest rectangle which can be drawn in this circle. 4

b) Find the equation of the tangent to the curve $y = x^2 + 2x - 5$ at the point $A(2, 3)$ 4

Question 5 – (13 Marks) – Start a New Page

Marks

Consider the function $f(x) = x^4 + \frac{8}{3}x^3 + 1$

(i) Find any stationary points and determine their nature. 4

(ii) Find all points of inflection. 3

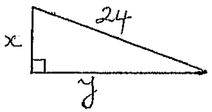
(iii) Find the values of the function at the ends of the interval $-3 \le x \le 1$. 2

(iv) Sketch the curve for $-3 \le x \le 1$ indicating all important features. 4

End of Paper

Q4.

(a) (i)



$24^2 = x^2 + y^2$ (Pythagoras' Rule)

$\therefore 576 = x^2 + y^2$

$y^2 = 576 - x^2$ (1)

$\therefore y = +\sqrt{576 - x^2}$ (>0) (1)

$\therefore A = L \times B$ (rectangle)

$= x \cdot y$ (1)

$= x\sqrt{576 - x^2}$ (as required)

[2]

(ii) Note $A = u \cdot v$ (product)

$= x(576 - x^2)^{\frac{1}{2}}$

$\therefore \frac{dA}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ (PRODUCT RULE)

$= x \cdot \frac{1}{2}(576 - x^2)^{-\frac{1}{2}}(-2x) + (576 - x^2)^{\frac{1}{2}} \cdot 1$ (1)

$= \frac{-x^2}{\sqrt{576 - x^2}} + \sqrt{576 - x^2}$ (1)

$= \frac{-x^2 + 576 - x^2}{\sqrt{576 - x^2}}$ (1)

$= \frac{576 - 2x^2}{\sqrt{576 - x^2}}$ (as req'd)

[3]

(iii) Maximum occurs when

$\frac{dA}{dx} = 0$

$\therefore 576 - 2x^2 = 0$ from (ii)

$\therefore x^2 = \frac{576}{2}$

$= 288$

$\therefore x = \pm\sqrt{288}$

$= \pm 12\sqrt{2}$

We consider only $x > 0$

$\therefore x = 12\sqrt{2}$ (2)

Test for max

x	12	$12\sqrt{2}$	24
y'	$+288$ >0	0	-576 <0

$+ / \overset{0}{-} \backslash -$

$\therefore$ Maximum when $x = 12\sqrt{2}$ (1)

Hence

A_{max} from (i)

$= 12\sqrt{2} \cdot \sqrt{576 - 288}$

$= 12\sqrt{2} \times 12\sqrt{2}$

$= 288 \ u^2$ (1)

[4]

(b) $y = x^2 + 2x - 5$

$\frac{dy}{dx} = 2x + 2$ (1)

$\therefore$ If $x = 2$ $\frac{dy}{dx} = 2 \times 2 + 2$

$= 6$ (1)

$= m_T$

$\therefore$ Equation of tangent is

$y - y_1 = m_T(x - x_1)$

POINT SLOPE

$y - 3 = 6(x - 2)$ (1)

$y - 3 = 6x - 12$

or $y = 6x - 9$

or $6x - y - 9 = 0$

(1)

[4]

Q5.

(i) $f(x) = x^4 + \frac{8}{3}x^{+3} + 1$

$\therefore f'(x) = 4x^3 + 8x^2 = 0$ for stat points

$\therefore 4x^2(x + 2) = 0$ (1)

$\therefore x = 0$ or $x = -2$

$f''(x) = 12x^2 + 16x$ (1)

at $x = 0$ $f''(0) = 0$ + changes sign

$y = 1$

$\therefore (0, 1)$ is a HORIZONTAL INFLEXION (1)

at $x = -2$ $f''(-2) = 48 - 32 > 0$

$\therefore$ MINIMUM T.P

$y = 16 - \frac{8}{3} \times 8 + 1$

$= 16 - \frac{64}{3} + 1$

$= 17 - 21\frac{1}{3}$

$= -4\frac{1}{3}$ (1)

$\therefore$ MIN T.P is $\left(-2, -4\frac{1}{3}\right)$

[4]

(ii) INFLECTIONS

Note in (i) we have $(0, 1)$

Solving $f''(x) = 0$

$\therefore 12x^2 + 16x = 0$

$4x(3x + 4) = 0$ (1)

$x = 0$ (above)

+ $x = -\frac{4}{3}$ + changes sign

If $x = -\frac{4}{3}$ $y = \left(-\frac{4}{3}\right)^4 + \frac{8}{3}\left(-\frac{4}{3}\right)^3 + 1$

$= \frac{256}{81} - \frac{512}{81} + \frac{81}{81}$

$= -\frac{175}{81} = -2\frac{13}{81}$

$\therefore$ Inflexion is $\left(-\frac{4}{3}, -\frac{175}{81}\right)$

The 2 INFLEXIONS

(1) $(0, 1)$ + $\left(-1\frac{1}{3}, -2\frac{13}{81}\right)$

[3]

(iii) $f(-3) = (-3)^4 + \frac{8}{3}(-3)^3 + 1$

$= 81 - 72 + 1$

$= 10$

+ $f(1) = (+1)^4 + \frac{8}{3}(1)^3 + 1$

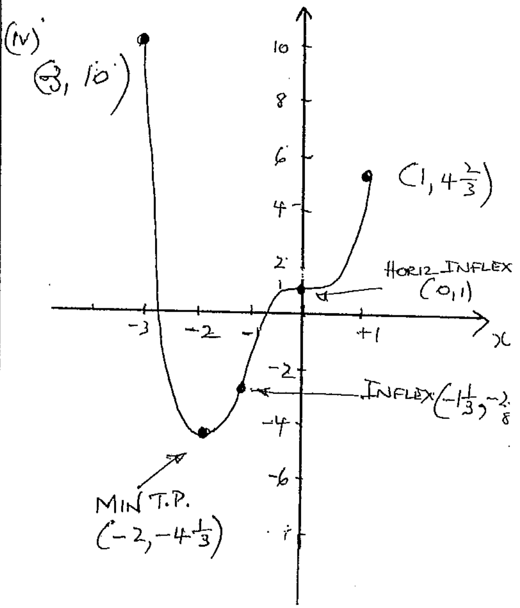
$= 1 + \frac{8}{3} + 1$

$= 4\frac{2}{3}$ (1) (1)

ie end points are $(-3, 10)$ + $\left(1, 4\frac{2}{3}\right)$

[2]

(iv)



5 IMPORTANT FEATURES

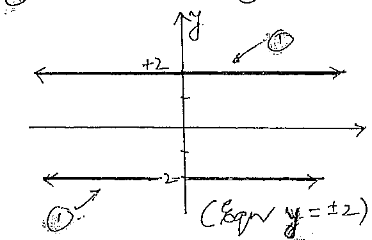
1 OFF EACH MISTAKE

[4]

Q1

(a) Minimum Turning Point ① ① [2]

(b) (i)



[2]

(ii) Equation is $y = \pm 2$ ① ①
(or $y = 2$, $y = -2$) [2]

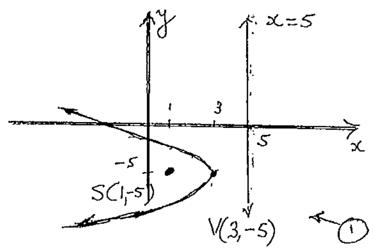
(c) $x^2 + 12x + (36) + y^2 + 14y + (49) = -60 + (36) + (49)$ on completing the square.

$\therefore (x+6)^2 + (y+7)^2 = 25$

$(x+6)^2 + (y+7)^2 = 5^2$ ①

Circle centre $(-6, -7)$ ①
radius 5 units ① [3]

(d)



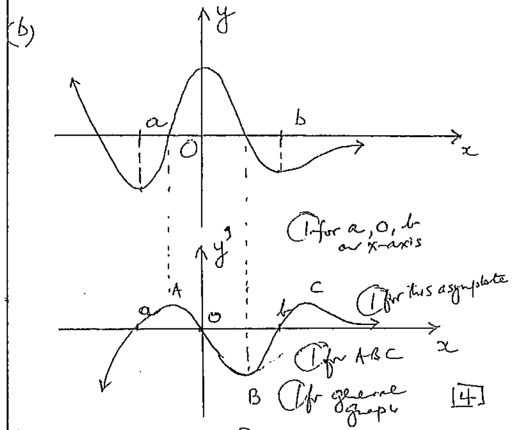
Note $4a = 8$
$a = 2$ (focal length)

Also note on diagram/graph
Vertex $(3, -5)$ ①
Focus $(1, -5)$ ①
Directrix $x = 5$ ① [4]

Q2 (a) For increasing $\frac{dy}{dx} = -5x > 0$

$\therefore x < 0$ [1]

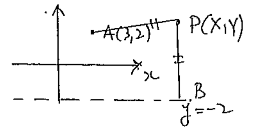
(b)



[4]

(c) NOTE TWO POSSIBLE SOLUTIONS

I "First Principles"
Let $P(X,Y)$ satisfy the locus
If point given is $A(3,2)$ + line is $y = -2$ with B as shown



We have $AP = PB$ or $AP^2 = PB^2$

① ⟶ $(X-3)^2 + (Y-2)^2 = (Y+2)^2$

① ⟶ $\therefore X^2 - 6X + 9 + Y^2 - 4Y + 4 = Y^2 + 4Y + 4$

② ⟷ OR $(X-3)^2 = 8Y$

Locus is $(x-3)^2 = 8y$ [4]

II Definition of Parabola implied in question with Focus $(3,2)$ + directrix $y = -2$ ①
Hence Vertex $(3,0)$ & $a = 2$ (focal length)
① $\therefore$ Equation is of form $X^2 = 4aY$ ②
or $(x-3)^2 = 8y$ ①

(d) $3x^3 + 81 = 3(x^3 + 3^3)$ Sum of 2 cubes ①
$= 3(x+3)(x^2 - 3x + 9)$ ① [2]

(e) $y = (3-7x)^{-3}$

$\therefore$ (i) $\frac{dy}{dx} = -7 \cdot -3(3-7x)^{-4}$ or $\frac{21}{(3-7x)^4}$ [1]

& (ii) $\frac{d^2y}{dx^2} = 21 \cdot -7 \cdot -4(3-7x)^{-5}$ or $\frac{588}{(3-7x)^5}$ [1]

Q3 [3]

(a) (i) Primitive of $5x^{-3}$ is $\frac{5}{-2}x^{-2} + C$

or $-\frac{5}{2x^2} + C$ ①

(ii) Primitive of $(4x-3)^7$ is $\frac{1}{4} \times \frac{1}{8}(4x-3)^8 + C$

or $\frac{1}{32}(4x-3)^8 + C$ ②

(b) (i) GLOBAL MIN is $(3, -3)$ ①

(ii) LOCAL MAX is $(0, 1)$ ①

(c) POSSIBLE TWO SOLUTIONS

I Using Pythagoras's Rule

$AB^2 = AP^2 + PB^2$ ②

$3^2 + 4^2 = (x+1)^2 + (y-7)^2 + (x-2)^2 + (y-3)^2$

$25 = x^2 + 2x + 1 + y^2 - 14y + 49 + x^2 - 4x + 4 + y^2 - 6y + 9$ ②

$25 = 2x^2 - 2x + 2y^2 - 20y + 63$

or $2x^2 - 2x + 2y^2 - 20y + 38 = 0$

$x^2 - x + (\frac{1}{4}) + y^2 - 10y + (25) = -19 + (25\frac{1}{4})$ on completing square

$(x - \frac{1}{2})^2 + (y-5)^2 = \frac{25}{4}$ or $6\frac{1}{4}$

II Using gradients $m_1 \times m_2 = -1$ [4]

$\frac{y-7}{x+1} \times \frac{y-3}{x-2} = -1$ ②

$(y-7)(y-3) = (2-x)(x+1)$

$y^2 - 10y + 21 = -x^2 + x + 2$ ②

$x^2 - x - 2 + y^2 - 10y + 21 = 0$

$x^2 - x + y^2 - 10y = -19$

(So continuing as in I)

$\Rightarrow (x - \frac{1}{2})^2 + (y-5)^2 = \frac{25}{4}$

No need to write in final format

(d) Given $f(x) = x^4 - 2x^2 - x$

$f(-x) = x^4 - 2x^2 + x \neq f(x)$

+ also $f(-x) \neq -f(x)$

$\therefore$ Function is NEITHER [2] (or 0.)

(e) $y = 3x + x^{-1} - 5x^{\frac{1}{2}} + 4$

$\therefore \frac{dy}{dx} = 3 - x^{-2} - \frac{1}{2} \times 5x^{-\frac{1}{2}} (+0)$ ①

$\therefore \frac{dy}{dx} = 3 - \frac{1}{x^2} - \frac{5}{2x^{\frac{1}{2}}}$ ①

(OR $\frac{dy}{dx} = 3 - \frac{1}{x^2} - \frac{5}{2\sqrt{x}}$) [2]