St George Girls High School

Year 11 – Higher School Certificate Course

Assessment Task 1

2011

Mathematics

General Instructions

Total Number of Marks: 52

- Working time – 65 minutes.
- Reading time – 5 minutes.
- Write using black or blue pen.
- Attempt all 4 questions.
- Begin each question on a **new page**.
- All necessary working must be shown.
- All questions are of equal value.
- Marks will be deducted for careless work or poorly presented solutions

Question 1 – (13 Marks) – Start a New Page **Marks**

a) **Multiple Choice. Select the Correct Response.** Given that $f(x) = 2[(x+3)^2 - 4]$, the coordinates of the turning point of the graph of $y = f(x)$ are: 1

A. $(-3,-8)$ B. $(-3,-4)$ C. $(3,-8)$ D. $(3,-4)$

b) Differentiate the following with respect to x

(i) $y = 5x^3 - 7x^2 + x$ 1

(ii) $y = \sqrt{5x+3}$ 1

(iii) $y = x^2(3x+2)^3$ 2

(iv) $y = \frac{3x+1}{3x-1}$ $x \neq \frac{1}{3}$ 2

c) Sketch the locus of a point whose distance from the x-axis is 3 units and write down its equation. 2

d) (i) Let $A(-3,2)$ and $B(3,-4)$ be two fixed points and let P be the variable point (x,y). Find the equation of the locus of the point P which moves so that $PA = 2PB$ 2

(ii) Show that this equation is a circle and find its centre and radius. 2

Question 2 – (13 Marks) – Start a New Page **Marks**

a) **Multiple Choice. Select the Correct Response.** The equation of the tangent to the curve with equation $y = 2x^{\frac{3}{2}}$ at the point where $x = 4$ is 2

A. $y = -\frac{1}{6}x + \frac{50}{3}$ B. $y = 6x - 18$

C. $y = 6x - 8$ D. $y = 6x + 40$

b) Find the equation of the locus of a point that is always 5 units from the point $A(2, 3)$ 1

c) Show that the quadratic expression $x^2 - 6x + 12$ is positive definite. 2

d) Write down the coordinates of the focus and the equation of the directrix of $2y = -x^2$ 2

e) A curve has equation $y = 2x^3 + kx + p$. At the point $(1, 1)$ on the curve, the tangent to the curve is parallel to the line $y = x$. Find the values of k and p. 2

f) Find the set of values of k for which the quadratic equation 2

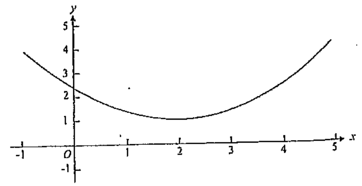
$$(k-1)\,x^2 + 2(k+1)\,x + 2k - 1 = 0 \quad \text{has real roots}$$

g) Find the equation of the locus of a point $P(x, y)$ which moves so that PA is perpendicular to PB, where the coordinates of A and B are $A(-3, 2)$ and $B(1, 4)$. 2

Question 3 – (13 Marks) – Start a New Page **Marks**

a) **Multiple Choice. Select the Correct Response.** The following shows part of the graph of the curve with equation $y = P(x - Q)^2 + 1$.

The values of P and Q respectively could be: 1



	P	Q
A.	$\frac{1}{3}$	-2
B.	$\frac{1}{3}$	2
C.	1	2
D.	3	2

b) Given $y = (7 - 4x)^5$, find $\frac{dy}{dx}$ 2

c) The quadratic equation $2x^2 + 4x + 1 = 0$ has roots α and β. Without solving the equation write down the values of:

(i) $\alpha + \beta$ 1

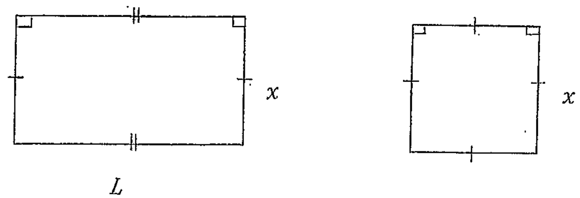
(ii) $\alpha\beta$ 1

(iii) $\alpha^2 + \beta^2$ 2

Question 3 (cont'd) **Marks**

d) Find the equation of the parabola that has focus $S(3,-8)$ and directrix the line $y=-2$. 2

e) A piece of wire 10 m long is broken into two parts, which are bent into the shape of a rectangle and a square as shown.



(i) Write down an algebraic expression for the length (L) of the rectangle, in terms of x. 1

(ii) Write down an algebraic expression for the combined area A of the square and rectangle. 1

(iii) Find the dimensions x and L that make the combined area a maximum. 2

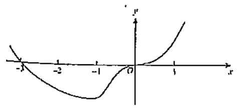
Question 4 – (13 Marks) – Start a New Page **Marks**

a) **Multiple Choice. Select the Correct Response.** A continuous function $f(x)$ has the following properties: 1

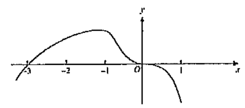
$$f(0)=0 \qquad f'(0)=0$$
$$f(-3)=0 \qquad f'(-1)=0$$
$$f'(x)>0 \text{ for } x<-1$$
$$f'(x)<0 \text{ for } x>-1$$

Which one of the following is most likely to represent the graph of the function $f(x)$ with rule $y=f(x)$?

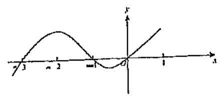
A.



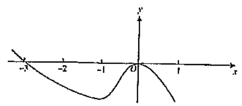
B.



C.



D.



b) Express $3x^2-8x+4$ in the form $A(x-1)^2+B(x-1)+C$ 3

Question 4 (cont'd) **Marks**

c) The normal to the curve $y = (x + 2)^2$ at the point $A(-3, 1)$ meets the curve again at B.

Find:

(i) the equation of the normal. 2

(ii) the coordinates of B. 3

d) Find any points on the curve $y = \frac{1}{3}x^3 - 5$ where the tangent has the angle of inclination of 45° 2

e) Write down the equations of two parabolas that have focal length 4, focus $(2, 1)$ and axis parallel to the x-axis. 2

End of Paper

Q1

a) $f(x) = 2\left((x+3)^2 - 4\right)$

$= 2(x^2 + 6x + 9 - 4)$

$= 2(x^2 + 6x + 5)$

$= 2x^2 + 12x + 10$

$f'(x) = 4x + 12$

$4x + 12 = 0$

$4x = -12$

$x = -3$

$2(-3)^2 + 12(-3) + 10 = -8$

$\therefore (-3, -8)$

$=$ A

b) (i) $y = 5x^3 - 7x^2 + x$

$y' = 3 \times 5x^2 - 7 \times 2x + 1$

$= 15x^2 - 14x + 1$

(ii) $y = \sqrt{5x+3} = (5x+3)^{\frac{1}{2}}$

$y' = \frac{1}{2}(5x+3)^{-\frac{1}{2}} \cdot 5$

$= \dfrac{5}{2\sqrt{5x+3}}$

(iii) $y = x^2(3x+2)^3$ Let $u = x^2$, $v = (3x+2)^3$

$u' = 2x$, $v' = 3(3x+2)^2 \times 3$

$= 9(3x+2)^2$

$y' = vu' + uv'$

$= 2x(3x+2)^3 + 9x^2(3x+2)^2$

$= x(3x+2)^2\left(2(3x+2) + 9x\right)$

$= x(3x+2)^2(6x + 4 + 9x)$

$= x(3x+2)^2(15x+4)$

(iv) $y = \dfrac{3x+1}{3x-1}$ Let $u = 3x+1$, $v = 3x-1$

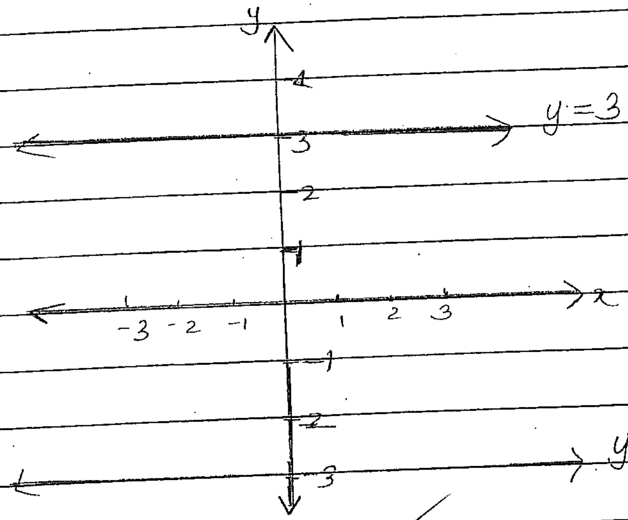
$u' = 3$, $v' = 3$

$y' = \dfrac{vu' - uv'}{v^2}$

$= \dfrac{3(3x-1) - 3(3x+1)}{(3x-1)^2}$

$= \dfrac{9x - 3 - 9x - 3}{(3x-1)^2} = \dfrac{-6}{(3x-1)^2}$

(c)



$\therefore y = \pm 3$

)(i) $A(-3,2)$ $B(3,-4)$ $P(x,y)$

$PA = \sqrt{(x+3)^2 + (y-2)^2}$ $\quad 2PB = 2\sqrt{(x-3)^2 + (y+4)^2}$

$PA = 2PB$

$\therefore (x+3)^2 + (y-2)^2 = 4\left((x-3)^2 + (y+4)^2\right)$ ✓

$x^2 + 6x + 9 + y^2 - 4y + 4 = 4(x^2 - 6x + 9) + 4(y^2 + 8y + 16)$

$x^2 + 6x + 9 + y^2 - 4y + 4 = 4x^2 - 24x + 36 + 4y^2 + 32y + 64$

$0 = 3x^2 - 30x + 3y^2 + 36y + 87$ ✓

$3x^2 - 30x + 3y^2 + 36y + 87 = 0$

$x^2 - 10x + y^2 + 12y + 29 = 0$ ⇐ Locus

ii) $x^2 - 10x + y^2 + 12y = -29$

$x^2 - 10x + 25 + y^2 + 12y + 36 = -29 + 25 + 36$

$(x-5)^2 + (y+6)^2 = 32$ ✓

$\therefore$ centre $(5,-6)$, radius $\sqrt{32}$

$= 4\sqrt{2}$ ✓

Q2

$x = 4 \quad y = 2(4)^{3/2}$

$= 16$

$\therefore (4,16)$

a) $y = 2x^{3/2}$

$y' = \frac{3}{\cancel{2}} \times \cancel{2} x^{1/2}$

$= \frac{3}{\sqrt{x}}$ at $x = 4$ $\quad m = \frac{3}{\sqrt{4}} = \frac{3}{2}$

$y - y_1 = m(x - x_1)$

$y - 16 = \frac{3}{2}(x - 4)$

$2y - 32 = 3x - 12$

$2y = \cancel{30}\ 3x + 20$

$y = \frac{3}{2}x + 10$?

$y = 2x^{\frac{3}{2}}$

$\frac{dy}{dx} = \frac{3}{\cancel{2}} \times \cancel{2} x^{\frac{1}{2}}$

$= 3x^{\frac{1}{2}} = 3\sqrt{x}$

at $x = 4$, $m = 3\sqrt{4} = 3 \times 2 = 6$

$y - y_1 = m(x - x_1)$

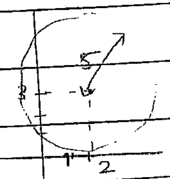
$y - 16 = 6(x - 4)$

$y - 16 = 6x - 24$

$y = 6x - 8 = C$

$\therefore$ C

b)



$\therefore (x-2)^2 + (y-3)^2 = 25$

(2)

c) $x^2 - 6x + 12$ $\quad \Delta = b^2 - 4ac$

$= (-6)^2 - 4(12)(1)$

$= 36 - 48$

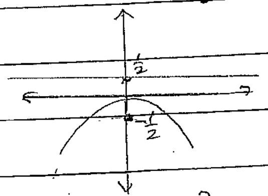
$= -12$

since $\Delta < 0$, there are no zeroes, ∴ it is a definite quadratic. since $a > 0$, it is always above the x axis. ∴ $x^2 - 6x + 12$ is positive definite.

d) $2y = -x^2$ $\quad 4a = 2$ $\quad$ ∴ vertex $(0, 0)$

$x^2 = -2y$ $\quad a = \frac{1}{2}$ $\quad$ focus $(0, -\frac{1}{2})$

directrix $y = \frac{1}{2}$



e) $y = 2x^3 + kx + p$ $\quad$ parallel to $y = x$ ∴ gradient 1

$\frac{dy}{dx} = 3 \times 2x^2 + k$

$= 6x^2 + k$ $\quad \cancel{6x^2 + k = 1}$

$= 1$ $\quad \cancel{6x^2 = k - 1}$

$6(1)^2 + k = 1$ $\quad \cancel{x^2 = \frac{k-1}{6}}$ $\quad \cancel{x = \sqrt{\frac{k-1}{6}}}$

$6 + k = 1$

$k = 1 - 6 = -5$

$y = 2x^3 + kx + p$ at $(1, 1)$

$1 = 2(1)^3 - 5(1) + p$

$1 = 2 - 5 + p$ $\quad$ ∴ $k = -5$

$1 = -3 + p$ $\quad p = 4$

$p = 4$

(f) $(k-1)x^2 + 2(k+1)x + (2k-1) = 0$ $\quad$ real roots $\Rightarrow \Delta \geq 0$

$\Delta = b^2 - 4ac$

$= (2(k+1))^2 - 4(2k-1)(k-1)$

$= 4(k^2 + 2k + 1) - 4(2k^2 - 2k - k + 1)$

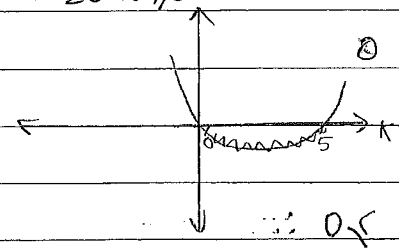
$= 4k^2 + 8k \cancel{+4} - 8k^2 + 12k \cancel{-4}$

$= -4k^2 + 20k$

$-4k^2 + 20k \geq 0$ $\quad 20k \geq 4k^2$

$0 \geq 4k^2 - 20k$

$4k(k-5) < 0$



∴ $0 \leq k \leq 5$ $\quad \frac{1}{2} + \frac{1}{2}$

∴ The equation has real roots when $0 < k < 5$

) $P(x, y)$ $\quad A(-3, 2)$ $\quad B(1, 4)$ $\quad \perp$ ∴ $m_{PA} \times m_{PB} = -1$

$m_{PA} = \frac{y-2}{x+3}$ $\quad m_{PB} = \frac{y-4}{x-1}$

$\frac{(y-2)(y-4)}{(x+3)(x-1)} = -1$

$y^2 - 4y - 2y + 8 = -(x^2 - x + 3x - 3)$

$y^2 - 6y + 8 = -x^2 - 2x + 3$

$x^2 + 2x + y^2 - 6y + 5 = 0$

$12\frac{1}{2}$

Q3

a) (B)

b) $y = (7-4x)^5$

$\frac{dy}{dx} = 5(7-4x)^4 \times -4$

$= -20(7-4x)^4$

c) $2x^2 + 4x + 1 = 0$

(i) $\alpha + \beta = \frac{-b}{a}$

$= \frac{-4}{2} = -2$

(ii) $\alpha\beta = \frac{c}{a} = \frac{1}{2}$

(iii) $\alpha^2 + \beta^2$ $\quad (\alpha+\beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$

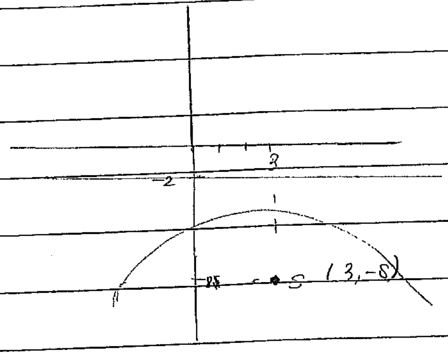
$\therefore \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (-2)^2 - 2\left(\frac{1}{2}\right)$

$= 4 - 1$

$= 3$

d)



~~vertex~~ ($8 - 8 + 2 = -6$

$-6 \div 2 = 3$

$\therefore a = 3$

vertex $(3, -5)$

$4a = 4 \times 3 = 12$

$\therefore$ equation: $(x-3)^2 = -12(y+5)$

$2x + 2L = 5$

e) $\cancel{2x + 2L = 10\,m}$

$\cancel{2(x+L) = 10}$

$\cancel{x + L = 5}$

$\cancel{L = 5 - x}$

$\cancel{A\ \text{Rectange} = x(5-x) = 5x - x^2}$ $\quad \cancel{A\ \text{square} = x^2}$

(i) $\cancel{2x + 2L = 5}$

$\cancel{2(x+L) = 5}$

$\cancel{x + L = \frac{5}{2}}$

$\cancel{L = \frac{5}{2} - x}$

$2x + 2L + 4x = 10$

$6x + 2L = 10$

$2L = 10 - 6x$

$L = 5 - 3x$

(ii) A rectangle $= x(5-3x)$ $\quad$ A square $= x^2$

$= 5x - 3x^2$

Combined area $A = 5x - 3x^2 + x^2$

$= 5x - 2x^2$

(iii) $\frac{dA}{dx} = 5 - 4x$ $\quad 5 - 4x = 0$

for maximum, $\frac{dy}{dx} = 0$ $\quad 4x = 5 \quad x = \frac{5}{4}$

$\therefore x = \frac{5}{4}$ $\quad L = 5 - 3x$

$= 5 - 3\left(\frac{5}{4}\right)$

$= \frac{5}{4}$

$\therefore x = \frac{5}{4}$

$L = \frac{5}{4}$

2

Q4

a) B

b) $3x^2 - 8x + 4 \equiv A(x-1)^2 + B(x-1) + C$

Let $x = 1$

$3 - 8 + 4 = C$

$C = -1$

$3x^2 - 8x + 5 = A(x-1)^2 + B(x-1)$

Let $x = 2$

$3(2)^2 - 8(2) + 5 = A(2-1)^2 + B(2-1)$

$1 = A + B \qquad A = 1 - B$ — (1)

Let $x = 0$

$5 = A(-1)^2 + B(-1)$

$5 = A - B$

(2) — $A = 5 + B$ sub (1) into (2)

$\therefore 1 - B = 5 + B$

$-4 = 2B$

$B = -2$

sub $B = -2$ into (1)

$A = 1 - -2$

$= 3$

$\therefore A = 3$

$B = -2$

$C = -1$

$\therefore 3x^2 - 8x + 4 \equiv 3(x-1)^2 - 2(x-1) - 1$

3

13/13

c) $y = (x+2)^2 \qquad \frac{dy}{dx} = 2(x+2)$

(i) $= 2x + 4 \qquad$ at $A(-3, 1)$

$m = 2(-3) + 4$

$= -2$

$m_{normal} = \frac{-1}{-2} = \frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{1}{2}(x + 3)$

$2y - 2 = x + 3$

$2y = x + 5 \qquad \therefore x - 2y + 5 = 0$ equation of normal

2

(ii) $x - 2y + 5 = (x+2)^2 - y$

$x - 2y + 5 = x^2 + 4x + 4 - y$

$x^2 + 3x + y - 1 = 0$ — (1)

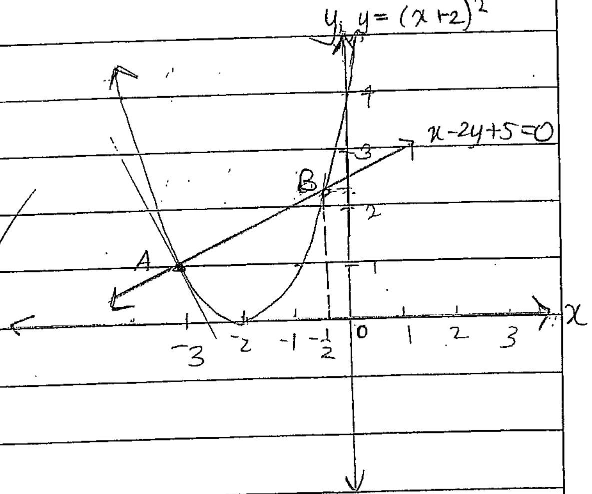
at B $x = -\frac{1}{2}$

$y = \frac{x + 5}{2}$

$= \frac{5 - \frac{1}{2}}{2}$

$= 2\frac{1}{4}$

$\therefore B\left(-\frac{1}{2}, 2\frac{1}{4}\right)$



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Test: sub into (1) $\left(-\frac{1}{2}\right)^2 + 3\left(-\frac{1}{2}\right) + 2\frac{1}{4} - 1 = \frac{1}{4} - \frac{3}{2} + \frac{9}{4} - 1$

$= 0$

$\therefore B\left(-\frac{1}{2}, 2\frac{1}{4}\right)$

i) $y = \frac{1}{3}x^3 - 5 \qquad \frac{dy}{dx} = \frac{1}{\cancel{3}} \times \cancel{3} x^2$

$= x^2$

gradient $= \tan \theta$

$= \tan 45$ $\qquad x = 1 \qquad$ 2 $\qquad x = -1$

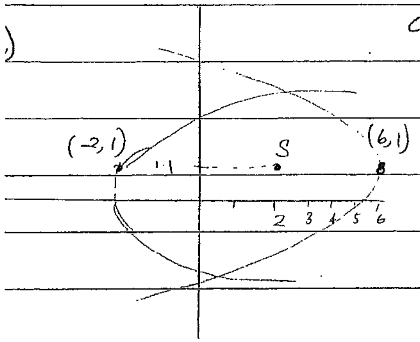
$= 1 \qquad y = \frac{1}{3}(1)^3 - 5 \qquad y = \frac{1}{3}(-1)^3 - 5$

$\therefore x^2 = 1 \qquad = \frac{-14}{3} \qquad = \frac{-16}{3}$

$x = \pm 1$

$\therefore$ The points are $\left(1, \frac{-14}{3}\right)$, and $\left(-1, -\frac{16}{3}\right)$

)



$a = 4 \quad 4a = 16$

$\cancel{(x+2)^2}$

$(y-1)^2 = 16(x+2)$

and

$(y-1)^2 = -16(x-6)$

2