

2012



Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Begin each question in a new booklet
- Write your student number on each booklet.
- Board-approved calculators may be used.
- A table of standard integrals is provided at the back of this paper. Detach.
- Multiple choice Answer sheet is at the back of this paper. Detach.
- Show all necessary working in Questions 11 – 16.
- Diagrams are not to scale.
- The mark allocated for each question is listed at the side of the question.

Total Marks – 100

Section I – Pages 2 – 4

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – Pages 5 – 12

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours 45 minutes for this section

TABLE OF STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

Note $\ln x = \log_e x, \quad x > 0$

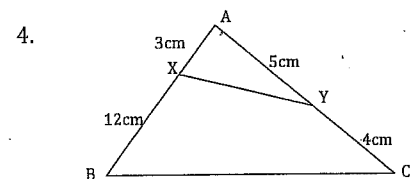
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Section I – (10 marks)

Marks

Answer this section on the answer sheet provided at the back of this paper.
 Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

- The angle which the straight line $3x + 5y + 2 = 0$ makes with the positive direction of the x -axis is closest to:
 A. 31° B. 59° C. 121° D. 149°
- Janet works out the sum of n terms of a given arithmetic series. Her answer, which is correct, could be:
 A. $S_n = 2(2^n - 1)$
 B. $S_n = 9 - 2n$
 C. $S_n = 8n - n^2$
 D. $S_n = 7 \times 2^{n-1}$
- The values of x for which $y = 2x^3 - 12x^2 + 18x + 7$ is increasing are:
 A. $x < 2$ B. $x > 2$ C. $1 < x < 3$ D. $x < 1$ or $x > 3$

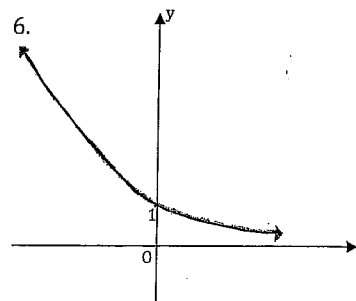


If ΔABC has area 36 cm^2 then the area of ΔAXY is:

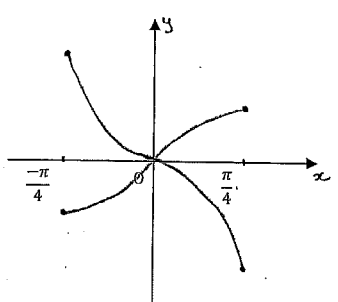
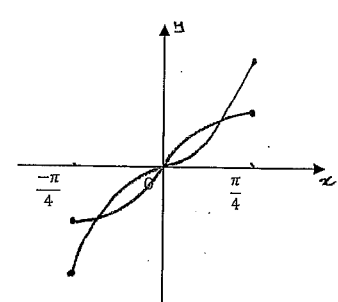
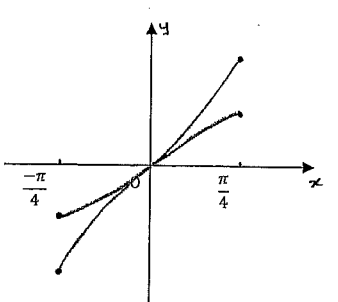
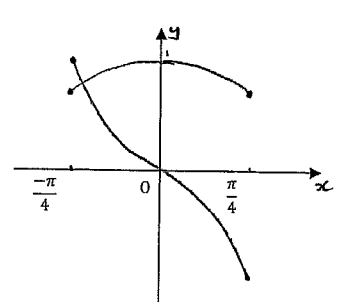
- When the curve of equation $y = e^x$ is rotated about the x -axis between $x = -2$ and $x = 2$ the volume of the solid generated is given by:
 A. $\pi \int_{-2}^2 e^x dx$ B. $2\pi \int_0^2 e^{x^2} dx$
 C. $\pi \int_{-2}^2 e^{x^2} dx$ D. $\pi \int_{-2}^2 e^{2x} dx$

Section I (cont'd)

Marks



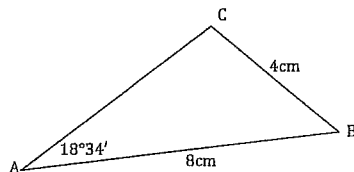
The graph illustrated could be:

- The quadratic function, $Q(x) = 5x^2 - 4x + 3$, has roots for $Q(x) = 0$ of α and β . Hence, $\alpha^2 + \beta^2 =$
 A. $\frac{46}{25}$ B. $\frac{29}{25}$ C. $\frac{-11}{25}$ D. $\frac{-14}{25}$
- The graphs of $y = \sin x$ and $y = \tan x$ for $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ are represented in:
 A. 
 B. 
 C. 
 D. 

Section I (cont'd)

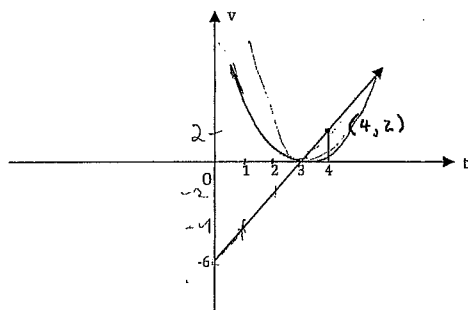
Marks

9. A possible answer to the size of $\angle C$ in the triangle below is:



- A. $140^\circ 27'$ B. $0^\circ 10'$ C. $37^\circ 8'$ D. None of these answers

10.



The graph shows velocity expressed as a function of time. The distance travelled by the particle in the first 4 seconds is:

- A. 8 units B. 10 units C. $4\sqrt{5}$ units D. 12 units

Section II - Show all working

Question 11 - Start A New Booklet - (15 marks)

Marks

- a) Write the answer to $\sqrt{\frac{4.83 \times 10.86}{17.83 - 5.92}}$ correct to 3 significant figures. 2
- b) Solve $|2x - 3| \leq 5$ 2
- c) If $\log_a 2 = 0.36$ and $\log_a 5 = 0.83$ evaluate $\log_a \sqrt{10}$ 2
- d) Differentiate each of the following with respect to x
- (i) $\cos 7x$ 1
- (ii) $\sqrt{e^{2x} + 4}$ 2
- (iii) $x \ln x$ 2
- e) Find:
- (i) $\int (3 - 2x)^4 dx$ 1
- (ii) $\int \frac{1}{\sqrt{x}} dx$ 1
- (iii) $\int \cos x^\circ dx$ 2

Question 12 - Start A New Booklet - (15 marks)

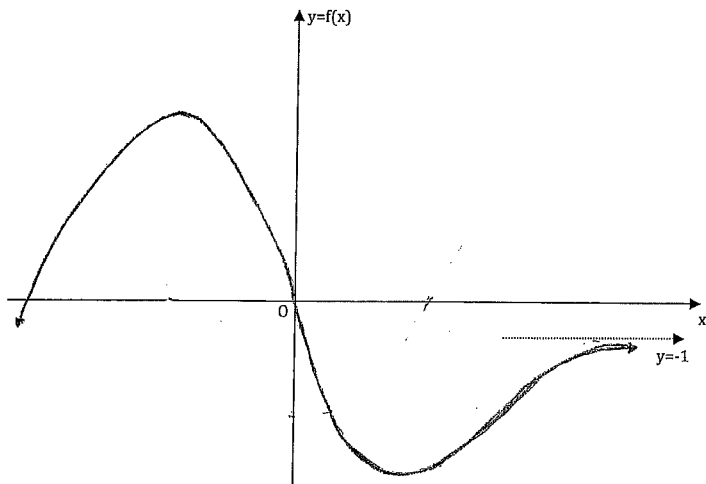
Marks

a) Graph the region on the number plane given by $y > \log_e(x - 1)$ 2

b) Copy this graph carefully onto your own paper. The graph shows $y = f(x)$.

On your graph draw the graph of $y = f'(x)$ making it clear which graph is your answer.

2



c) Initially a particle, travelling in straight line, is at rest at the origin. It is given an acceleration of $(6t + 4)$ cm/sec².

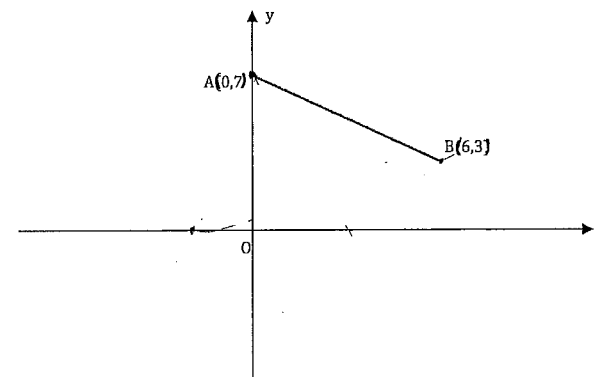
3

Find the motion equation for displacement.

Question 12 - (cont'd)

Marks

d)



$A(0, 7)$ and $B(6, 3)$ are points on the number plane and the equation of AB is $2x + 3y - 21 = 0$

(i) Find the length of AB 1

(ii) Find the gradient of AB 1

(iii) Show that the equation of the perpendicular from $D(-2, 0)$ to AB is $3x - 2y + 6 = 0$ 2

(iv) Find the perpendicular distance from D to AB . 2

(v) Find the coordinates of C such that $ABCD$ is a parallelogram. 1

(vi) Find the area of parallelogram $ABCD$. 1

Question 13 - Start A New Booklet - (15 marks)

Marks

a) $20 + 10 + 5 + \dots$ is a geometric series. Find which term of the series will be just less than 0.0001.

3

b) If $\cos \theta = \frac{-8}{17}$ and $\tan \theta < 0$, find the exact value for $\sin \theta$.

2

c) Sketch the graph of $y = -3 \sin 2x$ for $0 \leq x \leq 2\pi$

3

d) Copy the table of values into your writing booklet and supply the missing numbers, for $f(x) = x \sin x$, writing each correct to 3 decimal places.

3

x	1	1.5	2	2.5	3
$f(x) = x \sin x$	0.841				

Use Simpson's Rule with 5 function values to find an approximation for

$$\int_1^3 x \sin x \, dx$$

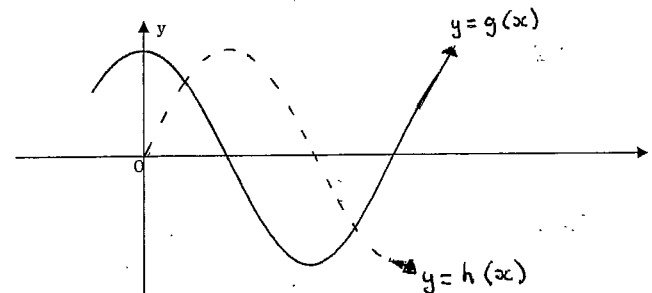
e) Find the volume formed when the area enclosed between $y = x^2$ and $y = 4x - x^2$ is rotated about the x -axis.

4

Question 14 - Start A New Booklet - (15 marks)

Marks

a)



A, B, C, D, E, F and G are the areas of the regions in which they are given.

Using these letters, write an expression for:

(i) $\int_0^4 h(x) \, dx$

(ii) $\int_1^4 g(x) \, dx$

1,2

(b)

Solve $\tan 3\theta = 1$ for $0 \leq \theta \leq 2\pi$

3

c) Find the equation of the parabola with vertex $(-1, 1)$ and focus $(-3, 1)$

3

d) (i) Differentiate

$$y = \log_e \left(\frac{x-1}{x+1} \right)$$

2

(ii) Hence, or otherwise, find

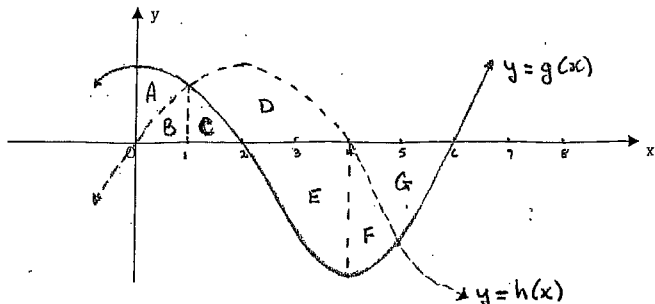
$$\int \frac{1}{x^2 - 1} \, dx$$

1

e) Given $y = -4x - 20$ is the equation of a tangent to $y = x^3 - 4x^2 - 7x + 10$ and $x > 0$, find the coordinates of the point of contact.

3

a)



A, B, C, D, E, F and G are the areas of the regions in which they are given.

** This is the diagram to be used for Q 14 (a)

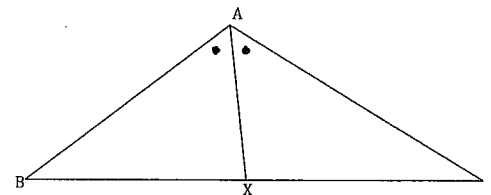
Question 15 – Start A New Booklet – (15 marks)

a) Simplify:

$$\frac{\sin^2 \theta}{\tan \theta \sin(90 - \theta)}$$

2

b)



Copy the diagram carefully onto your paper.

X is a point on the side BC of $\triangle ABC$ and AX bisects $\angle BAC$.

(i) Draw the line through X parallel to BA to meet AC at L.

This construction gives $\frac{BX}{XC} = \frac{AL}{LC}$

1

(ii) Prove that $\triangle ALX$ is isosceles.

2

(iii) Given that $\triangle CAB \parallel \triangle CLX$ (Do not prove this) prove that $\frac{BX}{XC} = \frac{AB}{AC}$

2

c) The equation of motion of a particle is $x = te^{-t}$

where x is in centimetres

t is in seconds.

(i) Find the time when the particle is at rest.

3

(ii) Find the equation of motion for acceleration and the acceleration when $v = 0$.

2

(iii) Find the time when acceleration is zero.

1

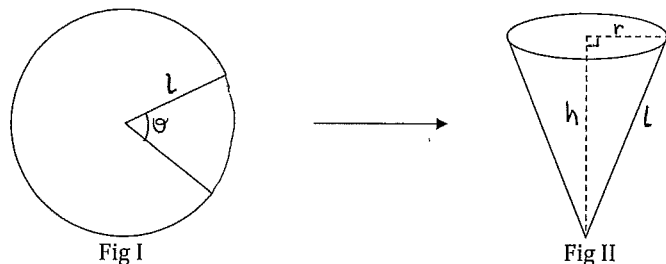
(iv) Using the answers from parts (i) to (iii) and other necessary information, sketch the displacement-time function $x = te^{-t}$. Show all important features clearly.

2

Question 16 – Start A New Booklet – (15 marks)

Mark

- a) An open cone, of radius r cm, and height, h cm is made from a sector of a circle. The area of the sector used is 300 cm².



(i) Show from Figure I that slant height l is given by $l^2 = \frac{450}{\pi}$ 2

(ii) Show from Figure II that $h = \sqrt{l^2 - r^2}$ 1

(iii) Hence or otherwise show that the volume of the cone is given by 1

$$V = \frac{1}{3}r^2\sqrt{450\pi - \pi^2r^2}$$

(iv) Show that $\frac{dv}{dr} = \frac{300\pi r - \pi^2r^3}{\sqrt{450\pi - \pi^2r^2}}$ 2

(v) Find the value of r for the volume of the cone to be a maximum. 2

Question 16 – (cont'd)

Marks

- b) Kando, the mathematical kangaroo always hops (i.e. jumps) according to mathematical rules. One day, Kando decides to go hopping according to the following rules:

- The length of odd number hops (1st, 3rd, 5th hop etc), in metres, is given by the arithmetic series $t_n = 4 - (n - 1)$, where $n = 1, 3, 5, \dots$ is an odd number;
- The length of even number hops (2nd, 4th, 6th hop etc), in metres, is given by the geometric series $T_N = \frac{192}{63} \left(\frac{1}{2}\right)^{\frac{N-2}{2}}$, where $N = 2, 4, 6, \dots$ is an even number;
- If the length of a hop is negative according to the relevant series, Kando hops the prescribed distance *backwards*.

(i) Write down the first term and common difference for the series t_n . 1

(ii) Write down the first term and common ratio for the series T_n . 1

(iii) Find where Kando is relative to her starting point after 12 hops. 3

(iv) Find the total distance travelled backwards in the first 16 hops. 2

Mathematics Trial 2012 Solutions

Multiple Choice

- 1. D 6. C
- 2. C 7. D
- 3. D 8. C
- 4. A 9. A
- 5. D 10. B

(iii) $y = x \ln x$

$$y' = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$$

(e) (i) $\int (3-2x)^4 dx = \frac{(3-2x)^5}{-2 \times 5} + C = -\frac{1}{10}(3-2x)^5 + C$

(ii) $\int \frac{1}{\sqrt{x}} dx = \int x^{-\frac{1}{2}} dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{x} + C$

(iii) $\int \cos x^\circ dx = \int \cos \frac{\pi x}{180} dx = \frac{1}{\frac{\pi}{180}} \sin \frac{\pi x}{180} + C = \frac{180}{\pi} \sin \frac{\pi x}{180} + C$

Question 11

a) $\sqrt{\frac{4.83 \times 10.86}{17.83 - 5.92}} = 2.0986 \approx 2.10$

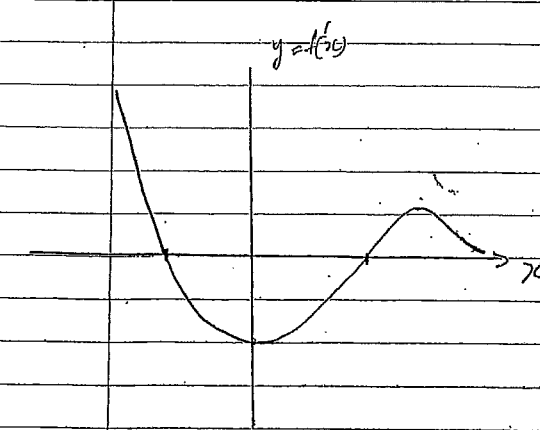
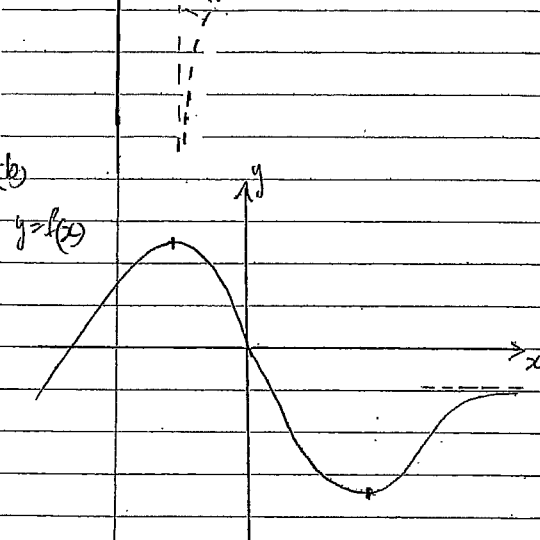
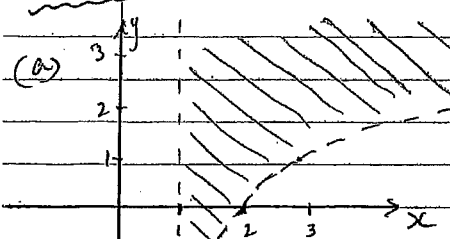
b) $|2x-3| \leq 5$
 $-5 \leq 2x-3 \leq 5$
 $-2 \leq 2x \leq 8$
 $-1 \leq x \leq 4$

c) $\log_a \sqrt{10} = \log_a 10^{\frac{1}{2}} = \frac{1}{2} \log_a 10 = \frac{1}{2} (\log_a 5 + \log_a 2) = \frac{1}{2} (0.83 + 0.36) = 0.595$

b) (i) $y = \cos 7x$
 $y' = -7 \sin 7x$

(ii) $y = \sqrt{e^{2x} + 4}$
 $y = (e^{2x} + 4)^{\frac{1}{2}}$
 $y' = \frac{1}{2} (e^{2x} + 4)^{-\frac{1}{2}} \times 2e^{2x} = \frac{e^{2x}}{\sqrt{e^{2x} + 4}}$

Question 12



(c) $t=0 \quad v=0 \quad x=0 \quad \therefore c=0$
 $a = 6t+4$
 $v = \int 6t+4 dt = 3t^2+4t$
 $x = \int 3t^2+4t dt = t^3+2t^2$

(d) (i) $d = \sqrt{(6-0)^2 + (3-7)^2} = \sqrt{36+16} = \sqrt{52}$

(ii) $m = \frac{3-7}{6-0} = -\frac{2}{3}$

(iii) $m = \frac{3}{2} \quad (-2, 0)$
 $y-0 = \frac{3}{2}(x+2)$
 $2y = 3x+6$
 $3x-2y+6=0$

(iv) $d = \frac{|2x_1+3y_1-21|}{\sqrt{2^2+3^2}} = \frac{|2 \cdot 2 + 3 \cdot 0 - 21|}{\sqrt{13}} = \frac{25}{\sqrt{13}}$

(v) $C = (4-4)$

(vi) Area = base \times height
 $= \sqrt{52} \times \frac{25}{\sqrt{13}} = \sqrt{4} \times 25 = 50 \text{ sq units.}$

Question 13

$n=20$ $r=0.5$

$T_n = ar^{n-1}$
 $\frac{20(0.5)^{n-1}}$

$20(0.5)^{n-1} < 0.0001$

$(0.5)^{n-1} < 0.000005$

$(n-1) \log(0.5) < \log(0.000005)$

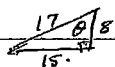
$(n-1) > \frac{\log(0.000005)}{\log(0.5)}$

$n-1 > 17.6$

$n > 18.6$

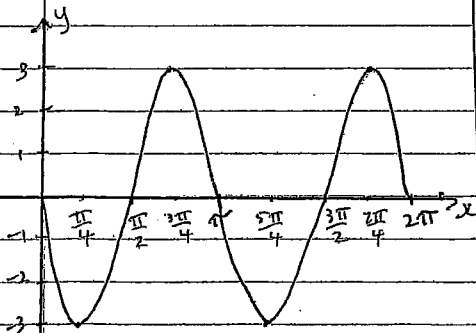
\therefore 19th term

i) $\cos \theta = \frac{-8}{17}$



2nd Quadrant.

$\therefore \sin \theta = \frac{15}{17}$



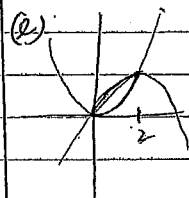
x	1	1.5	2	2.5	3
$\sin x$	0.841	1.496	1.819	1.496	0.423

$h=0.5$

$\int_1^3 x \sin x dx \approx \frac{0.5}{3} \{0.841 + 4 \times 1.496 + 6 \times 1.819\}$

$+ \frac{0.5}{3} \{1.819 + 4 \times 1.496 + 6 \times 0.423\}$

≈ 2.812



$x^2 = 4x - x^2$
 $0 = 4x - 2x^2$
 $0 = 2x - x^2$
 $0 = x(2-x)$

$x=0, 2$

Vol = $\pi \int_0^2 (4x-x^2)^2 dx - \pi \int_0^2 x^4 dx$

$= \pi \int_0^2 (16x^2 - 8x^3 + x^4) dx - \pi \int_0^2 x^4 dx$

$= \pi \int_0^2 (16x^2 - 8x^3 + x^4 - x^4) dx$

$= \pi \int_0^2 (16x^2 - 8x^3) dx$

$= \pi \left[\frac{16x^3}{3} - \frac{8x^4}{4} \right]_0^2$

$= \pi \left[\left(\frac{16 \cdot 2^3}{3} - \frac{8 \cdot 2^4}{4} \right) - (0) \right]$

$= \pi \cdot \frac{32}{3}$

$= \frac{32\pi}{3}$

Question 14

(i) $\int_0^h f(x) dx = B+C+D$

(ii) $\int_1^4 g(x) dx = C-E$

b) $\tan 3\theta = 1$ $0 \leq \theta \leq 2\pi$
 $0 \leq 3\theta \leq 6\pi$

Q1 & Q3

$3\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{17\pi}{4}, \frac{21\pi}{4}$

$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{21\pi}{12}$ (3, -20)

(c) Vertex = (-1, 1) focus = (-3, 1)

$a = -2$ $x^2 = 4ay$
 $(x+1)^2 = 4 \cdot (-2)(y-1)$
 $(x+1)^2 = -8(y-1)$

dy (i) $y = \ln \frac{(x-1)}{(x+1)}$
 $g = \ln(x-1) - \ln(x+1)$

$g' = \frac{1}{x-1} \cdot 1 - \frac{1}{x+1} \cdot 1$

$y' = \frac{1}{x-1} - \frac{1}{x+1}$

(ii) $\int \frac{1}{x^2-1} dx = \frac{1}{2} \int \frac{2}{x^2-1} dx$
 $= \frac{1}{2} \ln \frac{(x-1)}{(x+1)} + C$

(c) $y = x^3 - 4x^2 - 7x + 10$
 $y' = 3x^2 - 8x - 7$

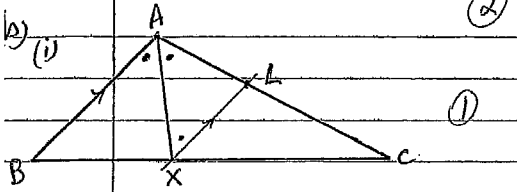
$m = -4$
 $\therefore -4 = 3x^2 - 8x - 7$
 $0 = 3x^2 - 8x - 3$
 $(3x+1)(x-3)$
 $x > 0 \therefore x = 3$
 $y = 3^3 - 4 \cdot 3^2 - 7 \cdot 3 + 10$
 $= -20$

Question 15

$$\sin^2 \theta = \sin^2 \theta$$

$$\frac{\tan \theta \sin(90-\theta)}{\cos \theta} = \frac{\sin \theta \cdot \cos \theta}{\cos \theta}$$

$$= \frac{\sin^2 \theta}{\sin \theta} = \sin \theta$$



i) $\hat{BAX} = \hat{AXL}$ (alternate angles on parallel lines)

$\therefore \Delta ALX$ isosceles (base angles equal)

$\therefore AL = LX$ (2)

i) $\Delta CAB \cong \Delta CLX$

$$\frac{AB}{LX} = \frac{AC}{LC}$$

$$\frac{AB}{AC} = \frac{LX}{LC}$$

$$\frac{AB}{AC} = \frac{AL}{LC} \quad (AL = LX)$$

but $\frac{AL}{LC} = \frac{BX}{XC}$ (given)

$$\frac{AB}{AC} = \frac{BX}{XC}$$

(c) $x = te^{-t}$

$$\dot{x} = 1 \cdot e^{-t} + t \cdot e^{-t} = e^{-t}(1+t)$$

(i) $\dot{x} = 0$

$$(1+t)e^{-t} = 0$$

$$1+t = 0 \quad t = -1$$

(ii) $\ddot{x} = -e^{-t}(1+t) + -1 \cdot e^{-t}$

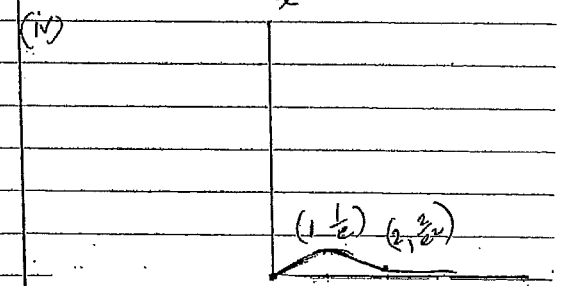
$$= -e^{-t}(-1-t-1) = (t+2)e^{-t}$$

$t = 1 \quad \ddot{x} = (1+2)e^{-1} = 3e^{-1} = \frac{3}{e}$

(iii) $\ddot{x} = 0$

$$(t+2)e^{-t} = 0$$

$$t = -2$$



(2)

Question 16

(i) $A = \frac{1}{2} r^2 \theta$ $\theta = 2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$

$$900 = 2\pi r^2 \cdot \frac{4\pi}{3}$$

$$\frac{450}{\pi} = r^2$$

$$r = \sqrt{\frac{450}{\pi}}$$

(ii) $r^2 + h^2 = l^2$

$$h = \sqrt{l^2 - r^2}$$

(iii) $V = \frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} \pi r^2 \sqrt{l^2 - r^2}$$

$$= \frac{1}{3} \pi r^2 \sqrt{\frac{450}{\pi} - r^2}$$

$$= \frac{1}{3} r^2 \sqrt{450\pi - \pi^2 r^2}$$

(iv) $V = \frac{1}{3} r^2 (450\pi - \pi^2 r^2)^{\frac{1}{2}}$

$$V' = \frac{2}{3} r (450\pi - \pi^2 r^2)^{\frac{1}{2}} + \frac{1}{2} (450\pi - \pi^2 r^2)^{-\frac{1}{2}} \cdot -2\pi^2 r^{\frac{1}{2}}$$

$$= \frac{2}{3} r (450\pi - \pi^2 r^2)^{\frac{1}{2}} - \frac{\pi^2 r^3}{(450\pi - \pi^2 r^2)^{\frac{1}{2}}}$$

$$= \frac{2r}{3} (450\pi - \pi^2 r^2) - \frac{\pi^2 r^3}{3}$$

$$= 300r\pi - \frac{2}{3}\pi^2 r^3 - \frac{1}{3}\pi^2 r^3$$

$$= (450\pi - \pi^2 r^2)^{\frac{1}{2}}$$

$$= \frac{300\pi r - \pi^2 r^3}{\sqrt{450\pi - \pi^2 r^2}}$$

(v) $V' = 0$

$$\frac{300\pi r - \pi^2 r^3}{\sqrt{450\pi - \pi^2 r^2}} = 0$$

$$300\pi r - \pi^2 r^3 = 0$$

$$300\pi - \pi r^2 = 0$$

$$r(\sqrt{300} + \sqrt{\pi}r)(\sqrt{300} - \sqrt{\pi}r) = 0$$

$\therefore r = 0, -\sqrt{\frac{300}{\pi}}, \sqrt{\frac{300}{\pi}}$

$0, -\sqrt{\frac{300}{\pi}}$ are not valid solutions

$$\therefore r = \sqrt{\frac{300}{\pi}}$$

r	9	$\sqrt{\frac{300}{\pi}}$	10
V'	37.8	0	-13.4

$\therefore r = \sqrt{\frac{300}{\pi}}$ gives max. volume.

Question 16

$$a = \frac{192}{63} \quad r = \frac{1}{2} \quad S_n = \frac{a(1-r^n)}{1-r}$$

i) $T_n = 4 - (n-1)$

$$T_1 = 4 \quad \therefore a = 4$$

$$T_2 = 2 \quad d = -2$$

$$T_3 = 0$$

$$S_6 = \frac{192}{63} \cdot \frac{(1 - (\frac{1}{2})^6)}{1 - \frac{1}{2}}$$

$$= \frac{192}{63} \cdot \frac{(1 - \frac{1}{64})}{\frac{1}{2}}$$

ii) $T_N = \frac{192}{63} \cdot (\frac{1}{2})^{\frac{N-2}{2}}$

$$T_1 = \frac{192}{63} \cdot (\frac{1}{2})^0$$

$$= \frac{192}{63}$$

$$= \frac{384}{63} \cdot (1 - \frac{1}{64})$$

$$= \frac{384}{63} \cdot \frac{63}{64}$$

$$= \frac{384}{64}$$

$$= 6$$

$$T_2 = \frac{192}{63} \cdot (\frac{1}{2})^1$$

$$= \frac{192}{63} \cdot \frac{1}{2}$$

$$S_n + S_N$$

$$= -6 + 6$$

$$= 0$$

$$T_3 = \frac{192}{63} \cdot (\frac{1}{2})^2$$

\therefore Kando is at the starting point.

$$\therefore a = \frac{192}{63} \quad r = \frac{1}{2}$$

(iv) Total distance backwards.

i) $S_n + S_N \quad n=6 \quad N=6$

$$\text{is } -2 - 4 - 6 - 8 - 10 = -30$$

$n=4 \quad d=-2$

\therefore 30 metres backwards.

$$S_6 = \frac{6}{2} (2 \times 4 + 5 \times -2)$$

$$= 3(8 - 10)$$

$$= -6$$