St George Girls High School

Year 12

Mid-HSC Course Examination

2013

Mathematics Extension 2

General Instructions

- Working time – 90 minutes
- Reading time – 5 minutes
- Write using blue or black pen.
- Board-approved calculators may be used.
- A multiple choice answer sheet is provided for Section I
- A set of templates for Question 8 c) is provided
- A table of standard integrals is provided.

Total marks – 72

- Attempt all questions in sections I and II
- Section I is multiple choice with answers to be recorded on the answer sheet provided
- Begin each of Questions 7 to 9 of Section II in a new booklet
- In all questions in Section II all necessary working should be shown.
- All drawn graphs should take up at least one third of a page.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n \neq -1;\quad x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\ x > 0$

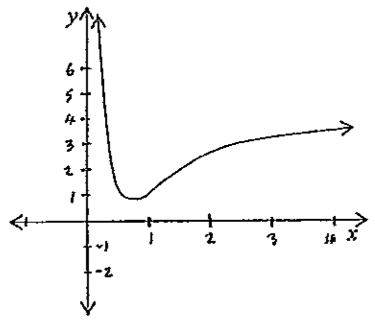
Section I:

Multiple Choice (Each question is worth 1 mark)

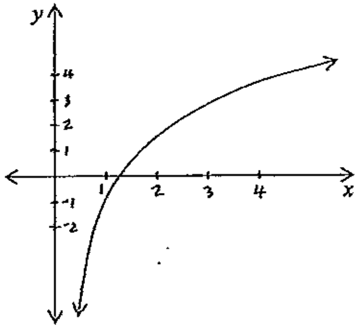
Use the multiple choice answer sheet to record your answers

1. Which of the following is the sketch of $y = \log_2 x + \frac{1}{x}$

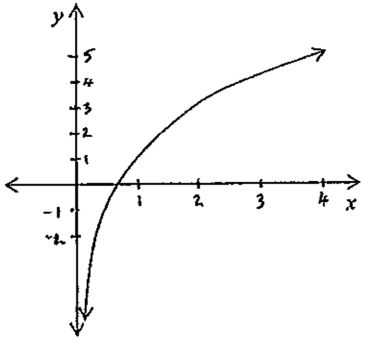
A



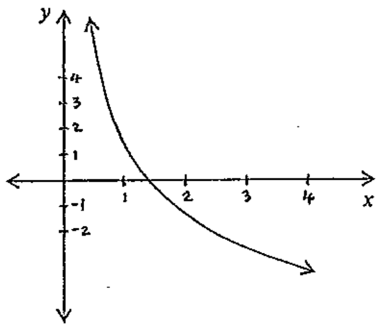
B



C



D



2. For the ellipse with equation $\frac{x^2}{4} + \frac{y^2}{3} = 1$, what is the eccentricity?

A $\frac{1}{4}$

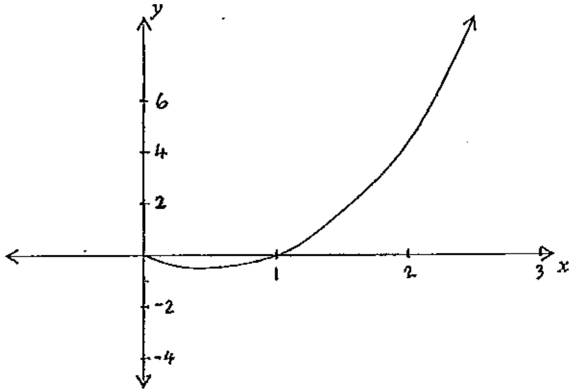
B $\frac{3}{4}$

C $\frac{1}{2}$

D $\frac{9}{16}$

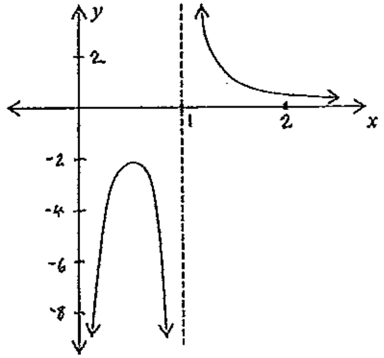
---

3. The diagram shows the graph of the function $y = f(x)$



Which of the following is the graph of $y = \frac{1}{f(x)}$?

A



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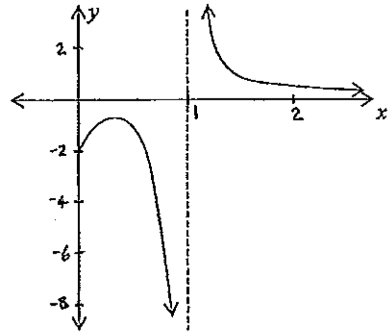
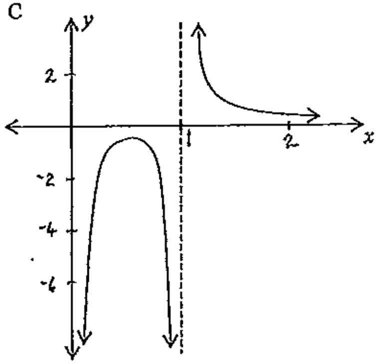
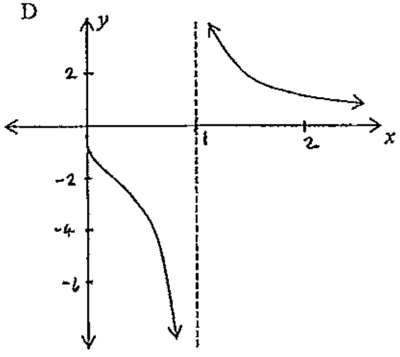


C



D



4. Consider the hyperbola with the equation $\frac{x^2}{144} - \frac{y^2}{25} = 1$
What are the equations of the directrices?

A $x = \pm\frac{13}{144}$

B $x = \pm\frac{144}{13}$

C $x = \pm\frac{25}{13}$

D $x = \pm\frac{13}{15}$

5. The points $P(acos\theta, bsin\theta)$ and $Q(acos\phi, bsin\phi)$ lie on the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

and the chord PQ subtends a right angle at (0,0). Which of the following is the correct expression?

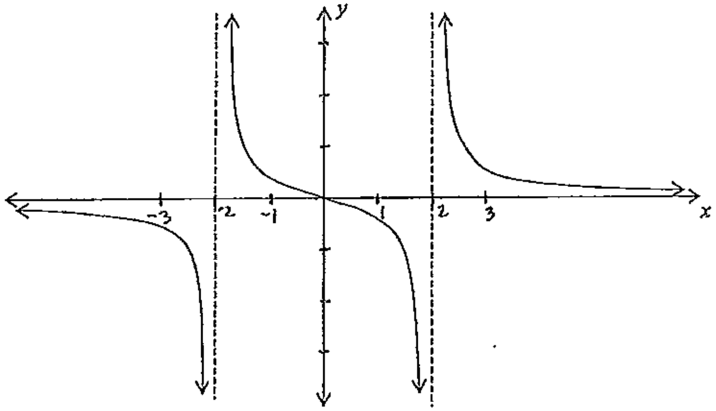
A $\tan\theta\tan\phi = -\frac{b^2}{a^2}$

B $\tan\theta\tan\phi = \frac{b^2}{a^2}$

C $\tan\theta\tan\phi = -\frac{a^2}{b^2}$

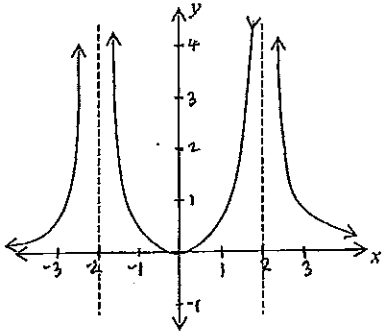
D $\tan\theta\tan\phi = \frac{a^2}{b^2}$

6. The diagram shows the graph of the function $y = f(x)$

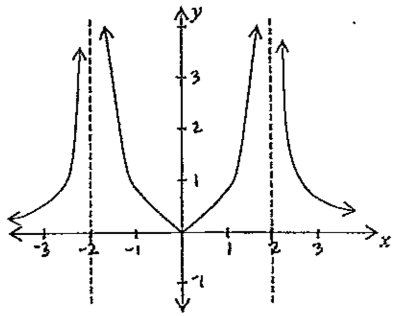


Which of the following is the graph of $y^2 = f(x)$?

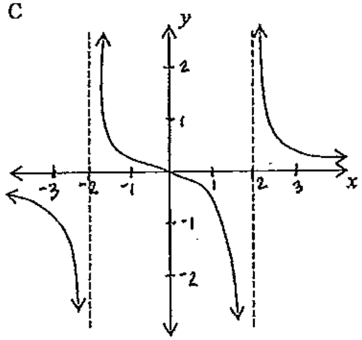
A



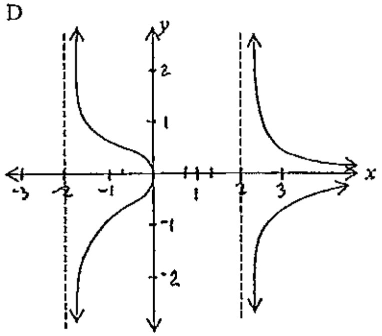
B



C



D



Section II:

Answer each question in a SEPARATE writing booklet.

In Questions 7, 8 and 9, your responses should include relevant mathematical reasoning and/or calculations.

Marks

Question 7 (22 marks) Start a new booklet

a) Sketch the graph of $y = \frac{x+3}{x+4}$ showing clearly the coordinates of any points of intersection with the *x* axis and the *y* axis, and the equations of any asymptotes. 2

b) Use the graph of $y = \frac{x+3}{x+4}$ in part a) to find:

(i) the largest possible domain of the function $y = \sqrt{\frac{x+3}{x+4}}$ 1

(ii) the set of values of *x* for which the function $y = x - \log_e(x+4)$ is increasing. 2

c) Use the graph of $y = \frac{x+3}{x+4}$ in part a) to sketch the graph of $y = \left(\frac{x+3}{x+4}\right)^2$ and state the nature of the point $(-3, 0)$ 3

d) Draw a neat sketch of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ clearly showing the *x* and *y* intercepts, the coordinates of the foci and the equations of the directrices. 3

Marks

Question 7 continued

e) Consider the ellipse, E with equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b)$$

(i) Show that the equation of the tangent to the ellipse, E, at the point $P(a\cos\theta, b\sin\theta)$ is $bx\cos\theta + ay\sin\theta = ab$ 3

(ii) Find the equation of the normal at P 2

(iii) The tangent to E at the point *P* cuts the *y* axis at A and the normal to E at the point *P* cuts the *y* axis at B. Find the coordinates of A and B. 2

(iv) Show that a focus, S lies on the circumference of the circle which has AB as the diameter (for each choice of P) 4

Marks

Question 8 (22 marks) Start a new booklet

a) (i) Express $\frac{x^2-8}{x^2-4}$ in the form $c+\frac{d}{x^2-4}$ where c and d are integers. 1

(ii) Draw a neat sketch of $y=\frac{x^2-8}{x^2-4}$,

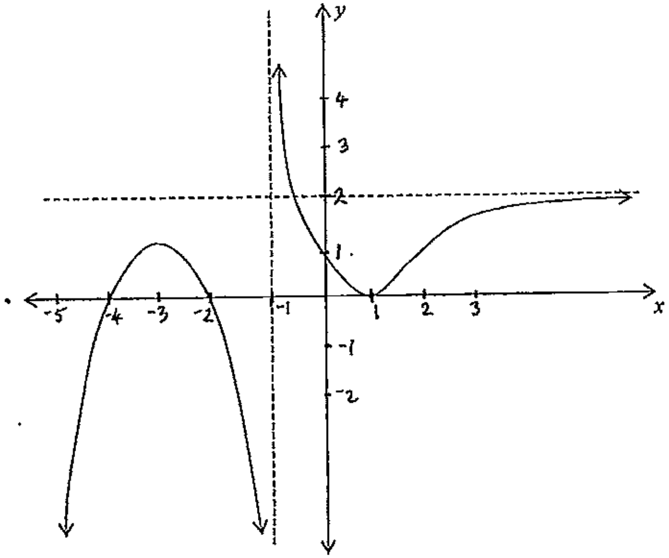
clearly indicate the intercepts with the coordinate axes and the position and equation of all asymptotes. 3

b) Solve the inequality

$$|x+3|+|x-2|\geq 6$$

by first drawing an appropriate sketch. 4

c) The diagram shows the graph of $y=f(x)$



Marks

Question 8 c) continued

Draw separate sketches of the graphs of the following on the templates provided on pages 15 and 16.

(i) $y=f(x+1)$ 1

(ii) $y=[f(x)]^2$ 2

(iii) $y=f(x)+|f(x)|$ 2

(iv) $y=e^{f(x)}$ 2

d) A curve has the equation $x^2+3xy+9y^2=3$

(i) Show that the point $\left(1,\frac{1}{3}\right)$ lies on the curve. 1

(ii) Show that

$$\frac{dy}{dx}=\frac{-(2x+3y)}{3x+18y}$$

1

(iii) Find the equation of the tangent to the curve at $\left(1,\frac{1}{3}\right)$ 2

(iv) Find the coordinates of the point(s) of contact where the tangent(s) to the curve is vertical 3

Marks

Question 9 (22 marks) Start a new booklet

a) Show the curves $x^2 - y^2 = c^2$ and $xy = c^2$ cross at right angles. 4

b) With respect to the x and y axes, the line $x = 1$ is a directrix and the point $(2, 0)$ is a focus of a conic of eccentricity $\sqrt{2}$.

Find the equation of the conic and the sketch the curve indicating its asymptotes, foci and directices. 4

c) (i) Derive the equation of the tangent to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

at the point $P(a\sec\theta,\ b\tan\theta)$ 2

(ii) Show that the tangent intersects the asymptotes of the hyperbola at the points

$$A\left(\frac{a\cos\theta}{1-\sin\theta}, \frac{b\cos\theta}{1-\sin\theta}\right) \text{ and } B\left(\frac{a\cos\theta}{1+\sin\theta}, \frac{-b\cos\theta}{1+\sin\theta}\right)$$ 2

(iii) Prove that the area of the triangle OAB is ab 4

d) (i) Determine the real values of k for which the equation

$$\frac{x^2}{21-k} + \frac{y^2}{9-k} = 1$$

defines (I) an ellipse 1

(II) a hyperbola 1

Marks

Question 9 d) continued

(ii) Sketch the curve corresponding to the value k = 5 2

(iii) Describe how the shape of this curve changes as k increases in value from 5 to 9. 1

(iv) What is the limiting position of the curve? 1

End of Paper

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Student Name: ____________________

Class Teacher: ____________________

Section 1

Multiple-choice Answer Sheet - Questions 1 – 6

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ☒ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ☒ B ☒ C ○ D ○

1.	A	○	B	○	C	○	D	○
2.	A	○	B	○	C	○	D	○
3.	A	○	B	○	C	○	D	○
4.	A	○	B	○	C	○	D	○
5.	A	○	B	○	C	○	D	○
6.	A	○	B	○	C	○	D	○

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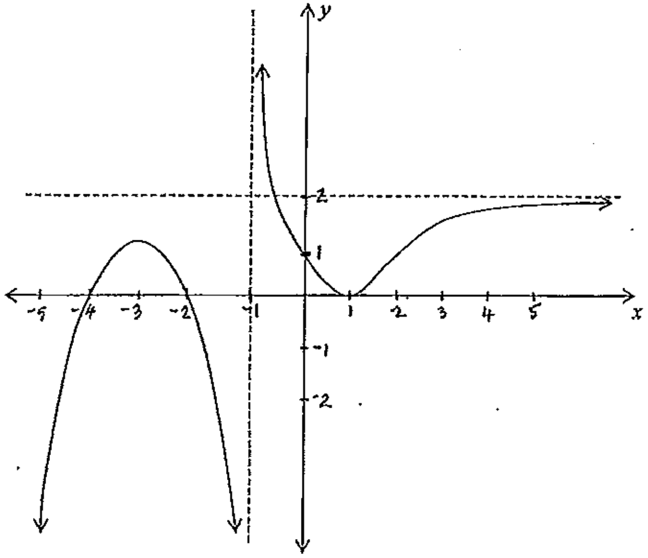
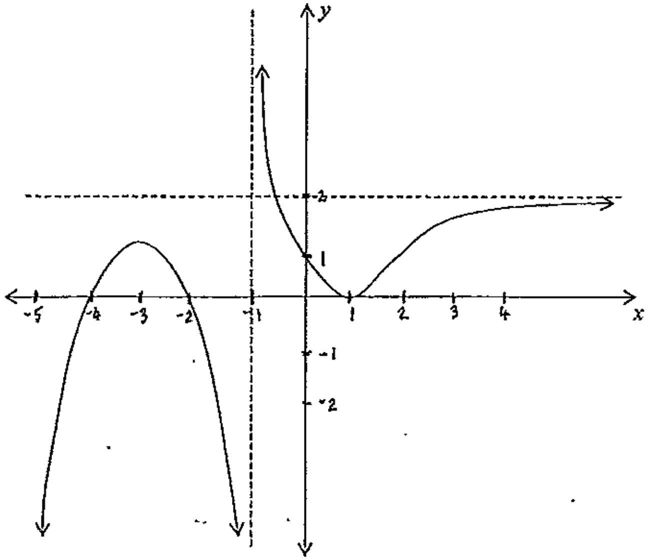
NAME: ______________________ Class Teacher: ______________________

Answers to Question 8 c)

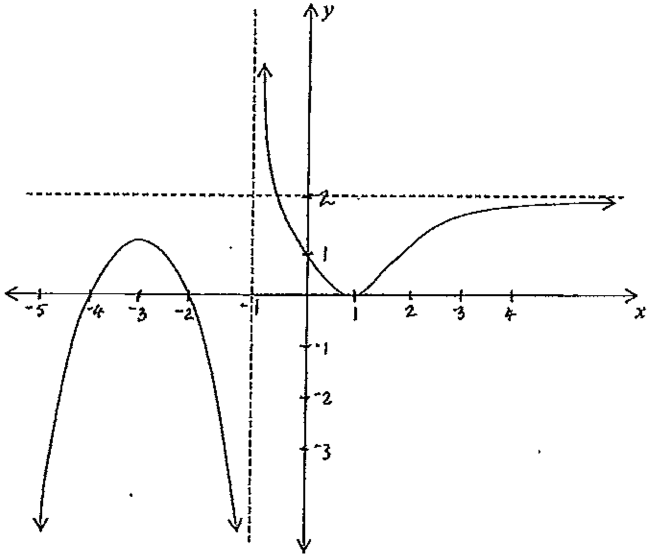
(i)



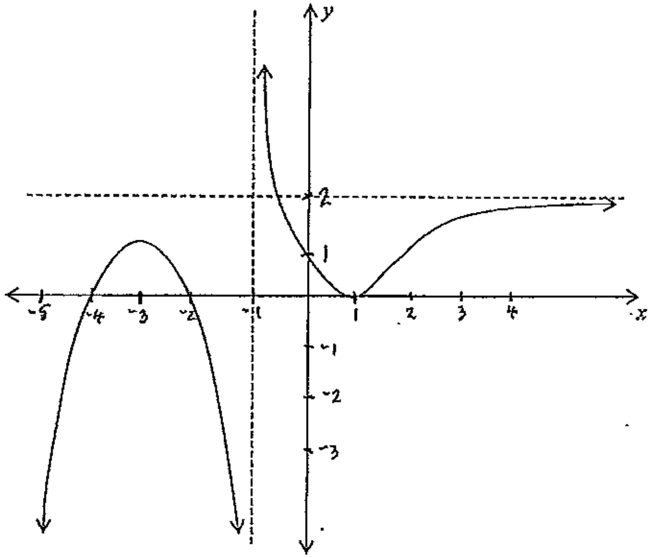
(ii)



(iii)



(iv)



YR 12 MID HSC MATHEMATICS EXT 2 2013

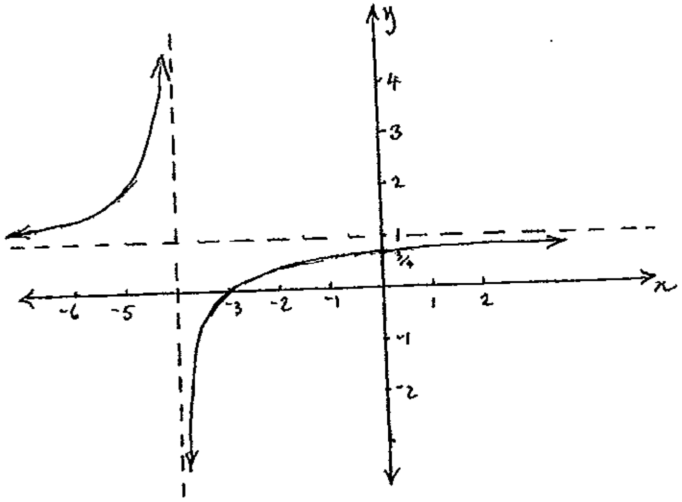
SECTION I

1 A 2 C 3 A 4 B 5 C 6 D

SECTION II

Question 7

a)



b)(i) $\frac{x+3}{x+4} \geqslant 0$ $x < -4$, $x \geqslant -3$

(ii) $y = x - \log_e(x+4)$

$\frac{dy}{dx} = 1 - \frac{1}{x+4}$

$= \frac{x+3}{x+4}$

The function is increasing when $\frac{dy}{dx} > 0$

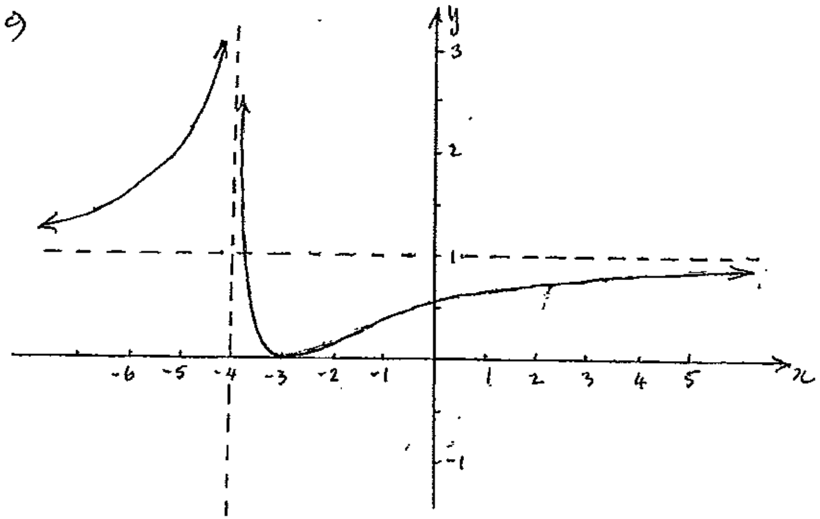
$\therefore x < -4$, $x > -3$

but $\log_e(x+4)$ is only defined for $x > -4$

$\therefore$ the function is increasing when $x > -3$

Question 7

c)



$(-3, 0)$ is a minimum turning point

d) $a^2 = 25$ $b^2 = 4$
$a = \pm 5$ $b = \pm 2$

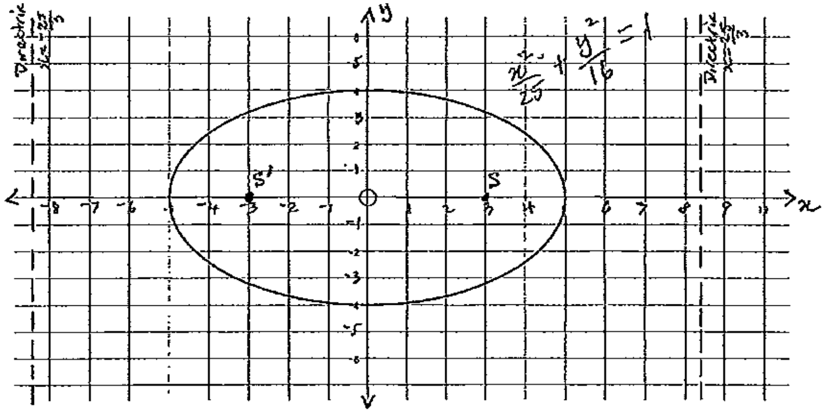
$e^2 = 1 - \frac{b^2}{a^2}$
$= 1 - \frac{16}{25}$
$= \frac{9}{25}$

$e = \frac{3}{5}$

$ae = \pm 5 \times \frac{3}{5}$
$= \pm 3$ (foci)

$\frac{a}{e} = \pm 5 \times \frac{5}{3}$
$= \pm \frac{25}{3}$

directrices $x = \pm \frac{25}{3}$



Question 7

e) (i) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{2x}{a^2} \times \frac{b^2}{2y}$$

$$= \frac{-b^2x}{a^2y}$$

at $P(a\cos\theta, b\sin\theta)$

$$\frac{dy}{dx} = \frac{-b^2 a\cos\theta}{a^2 b\sin\theta}$$

$$= \frac{-b\cos\theta}{a\sin\theta}$$

Equation of the tangent at P

$$y - b\sin\theta = \frac{-b\cos\theta}{a\sin\theta}(x - a\cos\theta)$$

$$ay\sin\theta - ab\sin^2\theta = -bx\cos\theta + ab\cos^2\theta$$

$$bx\cos\theta + ay\sin\theta = ab\sin^2\theta + ab\cos^2\theta$$

$$= ab(\sin^2\theta + \cos^2\theta)$$

$$bx\cos\theta + ay\sin\theta = ab$$

(ii) Gradient of normal at P is $\frac{a\sin\theta}{b\cos\theta}$

Equation of normal at P

$$y - b\sin\theta = \frac{a\sin\theta}{b\cos\theta}(x - a\cos\theta)$$

$$by\cos\theta - b^2\sin\theta\cos\theta = ax\sin\theta - a^2\sin\theta\cos\theta$$

$$ax\sin\theta - by\cos\theta = a^2\sin\theta\cos\theta - b^2\sin\theta\cos\theta$$

$$ax\sin\theta - by\cos\theta = (a^2-b^2)\sin\theta\cos\theta$$

(iii) The tangent to E at P cuts the y axis at A

$$bx\cos\theta + ay\sin\theta = ab$$

when $x = 0$

$$ay\sin\theta = ab$$

$$y\sin\theta = b$$

$$y = \frac{b}{\sin\theta}$$

$\therefore$ A is the point $(0, \frac{b}{\sin\theta})$

Question 7

e) (iii) continued

The normal to E at P cuts the y axis at B

$$ax\sin\theta - by\cos\theta = (a^2-b^2)\sin\theta\cos\theta$$

when $x = 0$

$$-by\cos\theta = (a^2-b^2)\sin x\cos x$$

$$by = (b^2-a^2)\sin x$$

$$y = \frac{b^2-a^2}{b}\sin x$$

$\therefore$ B is the point $(0, \frac{b^2-a^2}{b}\sin x)$

(iv) If AB is the diameter of a circle then the midpoint of AB is the centre

Centre $\left(0, \frac{b}{2\sin\theta} + \frac{(b^2-a^2)\sin\theta}{2b}\right)$

The radius of the circle is half the distance AB

$$r = \frac{b}{2\sin\theta} - \frac{(b^2-a^2)\sin\theta}{2b}$$

$$= \frac{b^2-(b^2-a^2)\sin^2\theta}{2b\sin\theta}$$

The equation of the circle is

$$x^2 + \left(y - \frac{b^2+(b^2-a^2)\sin^2\theta}{2b\sin\theta}\right)^2 = \left(\frac{b^2-(b^2-a^2)\sin^2\theta}{2b\sin\theta}\right)^2$$

If S is the focus, ie. $(ae, 0)$, substitute $y = 0$

$$x^2 + \left(\frac{b^2+(b^2-a^2)\sin^2\theta}{2b\sin\theta}\right)^2 = \left(\frac{b^2-(b^2-a^2)\sin^2\theta}{2b\sin\theta}\right)^2$$

$$x^2 = \left(\frac{b^2-(b^2-a^2)\sin^2\theta}{2b\sin\theta}\right)^2 - \left(\frac{b^2+(b^2-a^2)\sin^2\theta}{2b\sin\theta}\right)^2$$

$$x^2 = \left(\frac{b^2-(b^2-a^2)\sin^2\theta}{2b\sin\theta} - \frac{b^2+(b^2-a^2)\sin^2\theta}{2b\sin\theta}\right)\left(\frac{b^2-(b^2-a^2)\sin^2\theta}{2b\sin\theta} + \frac{b^2+(b^2-a^2)\sin^2\theta}{2b\sin\theta}\right)$$

$$= \left(\frac{-2(b^2-a^2)\sin^2\theta}{2b\sin\theta}\right)\left(\frac{2b^2}{2b\sin\theta}\right)$$

$$= \frac{(a^2-b^2)\sin^2\theta}{b} \times \frac{b}{\sin\theta}$$

$$= a^2 - b^2$$

$$= a^2\left(1 - \frac{b^2}{a^2}\right)$$

$$= (ae)^2$$

$$x = \pm ae$$

$\therefore$ S lies on the circle

Question 8

a) (i) $\frac{x^2-8}{x^2-4}$

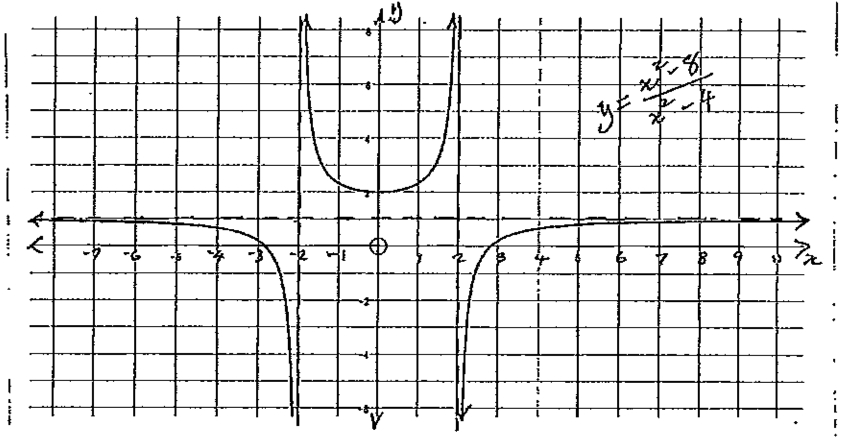
$= \frac{x^2-4-4}{x^2-4}$

$= 1 - \frac{4}{x^2-4}$

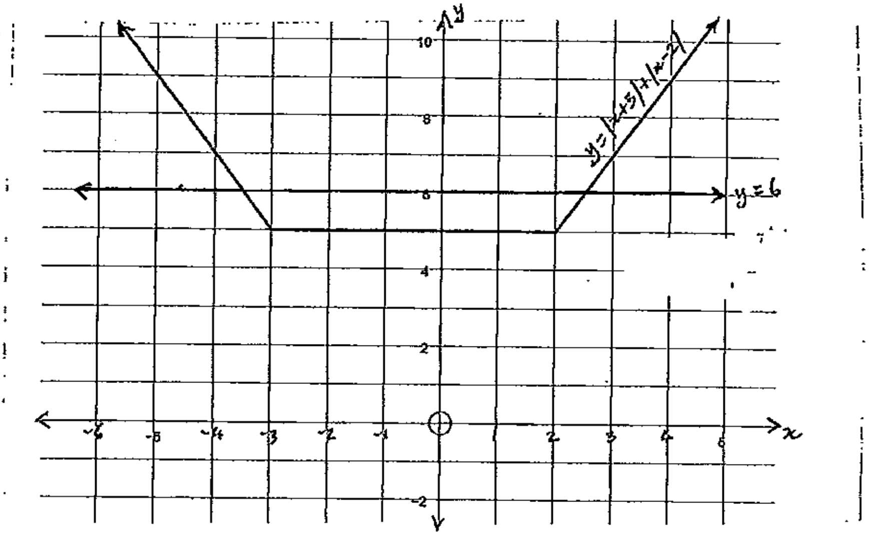
$= 1 + \frac{-4}{x^2-4}$

$\therefore c = 1$ and $d = -4$

(ii)



b)



Question 8

b) continued

$x \leq -3\frac{1}{2}$, $x \geq 2\frac{1}{2}$

c) (i)(ii)(iii)(iv) see separate sheets

d) (i) $LHS = 1^2 + 3\times 1\times\frac{1}{3} + 9\times\left(\frac{1}{3}\right)^2$

$= 1 + 1 + 1$

$= 3$

$= RHS$

$\therefore (1, \frac{1}{3})$ lies on the curve $x^2 + 3xy + 9y^2 = 3$

(ii) $2x + 3x\frac{dy}{dx} + 3y + 18y\frac{dy}{dx} = 0$

$2x + 3y + 3x\frac{dy}{dx} + 18y\frac{dy}{dx} = 0$

$(3x + 18y)\frac{dy}{dx} = -(2x + 3y)$

$\frac{dy}{dx} = \frac{-(2x+3y)}{3x+18y}$

(iii) Gradient of tangent at $(1, \frac{1}{3})$ is

$m = \frac{-(2+1)}{3+6}$

$= -\frac{3}{9}$

$= -\frac{1}{3}$

Equation of tangent at $(1, \frac{1}{3})$ is

$y - \frac{1}{3} = -\frac{1}{3}(x - 1)$

$3y - 1 = -x + 1$

$x + 3y - 2 = 0$

(iv) tangent is vertical when $\frac{dy}{dx}$ is undefined

ie. $3x + 18y = 0$

$y = -\frac{3x}{18}$

$= -\frac{x}{6}$

Question 8

d) (iv) continued

$$x^2+3x\left(-\frac{x}{6}\right)+9\left(\frac{x^2}{36}\right)=3$$
$$x^2-\frac{x^2}{2}+\frac{x^2}{4}=3$$
$$\frac{3x^2}{4}=3$$
$$x^2=4$$
$$x=\pm 2$$

for $x=2$
$$4+6y+9y^2=3$$
$$9y^2+6y+1=0$$
$$(3y+1)^2=0$$
$$y=\frac{-1}{3}$$

for $x=-2$
$$4-6y+9y^2=3$$
$$9y^2-6y+1=0$$
$$(3y-1)^2=0$$
$$y=\frac{1}{3}$$

∴ the points of contact where the tangents to the curve are vertical are
$$\left(2,-\frac{1}{3}\right),\ \left(-2,\frac{1}{3}\right)$$

Question 9

a) $x^2-y^2=c^2$
$$2x-2y\frac{dy}{dx}=0$$
$$2y\frac{dy}{dx}=2x$$
$$\frac{dy}{dx}=\frac{x}{y}$$

$xy=c^2$
$$x\frac{dy}{dx}+y=0$$
$$x\frac{dy}{dx}=-y$$
$$\frac{dy}{dx}=\frac{-y}{x}$$

At any point of intersection (x_1,y_1)
$$\frac{dy}{dx}=\frac{x_1}{y_1} \qquad \frac{dy}{dx}=\frac{-y_1}{x_1}$$

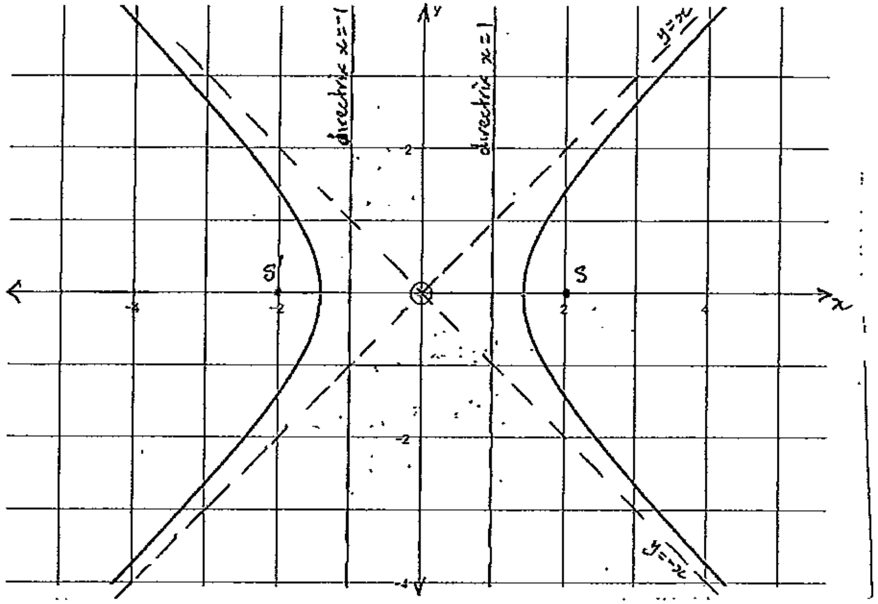
Since $\frac{x_1}{y_1}\times\frac{-y_1}{x_1}=-1$

the tangents are perpendicular at any point of intersection (x_1,y_1)

b) Let $P(x,y)$ be a point on the conic with a focus at $S(2,0)$. M is the point on the directrix $(x=1)$ such that $PM \perp$ the directrix.

Then $PS^2=e^2PM^2$
$$(x-2)^2+y^2=(\sqrt{2})^2(x-1)^2$$
$$x^2-4x+4+y^2=2(x^2-2x+1)$$
$$=2x^2-4x+2$$
$$2=x^2-y^2$$

Question 9 b) continued



c) (i) $x = a\sec\theta$ $\qquad$ $y = b\tan\theta$

$\frac{dx}{d\theta} = a\sec\theta\tan\theta$ $\qquad$ $\frac{dy}{d\theta} = b\sec^2\theta$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$$

$$= \frac{b\sec^2\theta}{a\sec\theta\tan\theta}$$

$$= \frac{b\sec\theta}{a\tan\theta}$$

Equation of the tangent at P is

$$y - b\tan\theta = \frac{b\sec\theta}{a\tan\theta}(x - a\sec\theta)$$

$$ay\tan\theta - ab\tan^2\theta = bx\sec\theta - ab\sec^2\theta$$

$$bx\sec\theta - ay\tan\theta = ab\sec^2\theta - ab\tan^2\theta$$

$$\frac{bx\sec\theta}{ab} - \frac{ay\tan\theta}{ab} = \sec^2\theta - \tan^2\theta$$

$$\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$$

Question 9

c) (ii) Asymptotes of the hyperbola are $y = \pm\frac{b}{a}x$

For $y = \frac{b}{a}x$ $\qquad$ $\frac{x\sec\theta}{a} - \frac{x\tan\theta}{a} = 1$

$$x\sec\theta - x\tan\theta = a$$

$$x = \frac{a}{\sec\theta - \tan\theta}$$

$$= \frac{a\cos\theta}{1-\sin\theta}$$

$$y = \frac{b}{a}\left(\frac{a\cos\theta}{1-\sin\theta}\right)$$

$$= \frac{b\cos\theta}{1-\sin\theta}$$

∴ the tangent meets $y = \frac{b}{a}x$ at $A\left(\frac{a\cos\theta}{1-\sin\theta}, \frac{b\cos\theta}{1-\sin\theta}\right)$

For $y = -\frac{b}{a}x$ $\qquad$ $\frac{x\sec\theta}{a} + \frac{x\tan\theta}{a} = 1$

$$x\sec\theta + x\tan\theta = a$$

$$x = \frac{a}{\sec\theta + \tan\theta}$$

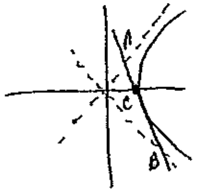
$$= \frac{a\cos\theta}{1+\sin\theta}$$

$$y = -\frac{b}{a}\left(\frac{a\cos\theta}{1+\sin\theta}\right)$$

$$= \frac{-b\cos\theta}{1+\sin\theta}$$

∴ the tangent meets $y = -\frac{b}{a}x$ at $B\left(\frac{a\cos\theta}{1+\sin\theta}, \frac{b\cos\theta}{1+\sin\theta}\right)$

(iii)



Let the point where the tangent meets the x axis be C $(y=0)$

$$\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$$

$$\frac{x\sec\theta}{a} = 1$$

$$x = \frac{a}{\sec\theta}$$

$$= a\cos\theta$$

∴ the point C is $(a\cos\theta, 0)$

Question 9

c)(iii) continued

$\triangle OAB = \triangle OAC + \triangle OBC$

For $\triangle OAC$ base $= a\cos\theta$

height $= \dfrac{b\cos\theta}{1-\sin\theta}$

$$\text{Area} = \tfrac{1}{2}\times a\cos\theta \times \frac{b\cos\theta}{1-\sin\theta}$$

$$= \frac{ab\cos^2\theta}{2(1-\sin\theta)}$$

For $\triangle OBC$ base $= a\cos\theta$

height $= \dfrac{b\cos\theta}{1+\sin\theta}$

$$\text{Area} = \tfrac{1}{2}\times a\cos\theta \times \frac{b\cos\theta}{1+\sin\theta}$$

$$= \frac{ab\cos^2\theta}{2(1+\sin\theta)}$$

$$\text{Area of } \triangle OAB = \frac{ab\cos^2\theta}{2(1-\sin\theta)} + \frac{ab\cos^2\theta}{2(1+\sin\theta)}$$

$$= \frac{ab\cos^2\theta}{2}\left(\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta}\right)$$

$$= \frac{ab\cos^2\theta}{2}\left(\frac{1+\sin\theta+1-\sin\theta}{1-\sin^2\theta}\right)$$

$$= \frac{ab\cos^2\theta}{2}\times\frac{2}{\cos^2\theta}$$

$$= ab$$

d)·(I) For an ellipse

(i) (I) For an ellipse $21-k>0$ and $9-k>0$

$k<21$ $k<9$

$\therefore\ k<9$

II For a hyperbola $21-k>0$ and $9-k<0$

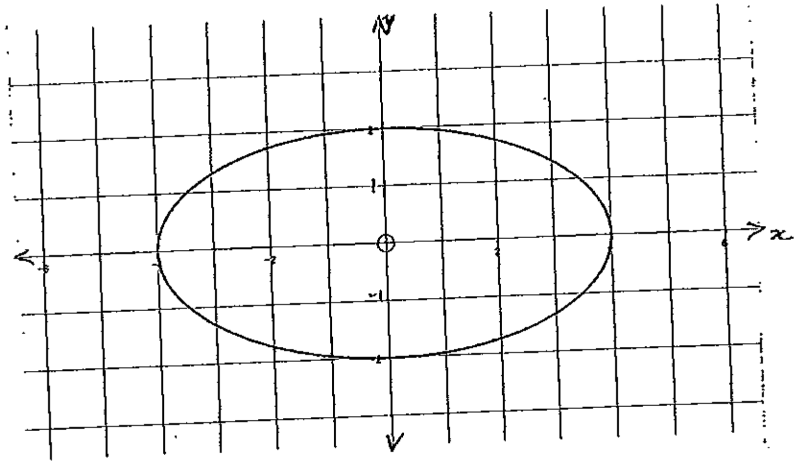
$k<21$ $k>9$

$\therefore\ 9<k<21$

Question 9

d) (ii) If $k=5$ the equation becomes

$$\frac{x^2}{16}+\frac{y^2}{4}=1$$



(iii) As $k\to 9$ $a^2\to 21-9$ ie. $a\to\pm2\sqrt{3}$

$b^2\to 9-9$ ie. $b\to 0$

$\therefore$ the curve approaches the line interval from $x=-2\sqrt{3}$ to $x=2\sqrt{3}$.

(iv) the interval from $x=-2\sqrt{3}$ to $x=2\sqrt{3}$ on the x axis

Alternative solution to Q 7 e) (iv)

If S lies on the circumference of a circle with diameter AB then $\angle ASB = 90°$ $\quad [S\,(ae, 0)]$

$\therefore$ gradient of AS × gradient of BS = −1

Gradient of AS $= \dfrac{\frac{b}{\sin\theta} - 0}{0 - ae}$ $= \dfrac{-b}{ae\sin\theta}$

Gradient of BS $= \dfrac{\frac{b^2-a^2}{b}\sin\theta - 0}{0 - ae}$ $= \dfrac{(a^2-b^2)\sin\theta}{abe}$

Gradient of AS × gradient of BS $= \dfrac{-b}{ae\sin\theta} \times \dfrac{(a^2-b^2)\sin\theta}{abe}$

$$= -\frac{(a^2-b^2)}{a^2e^2}$$

$b^2 = a^2(1-e^2)$

$$= -\frac{(a^2 - a^2(1-e^2))}{a^2e^2}$$

$$= -\frac{(a^2 - a^2 + a^2e^2)}{a^2e^2}$$

$$= \frac{-a^2e^2}{a^2e^2}$$

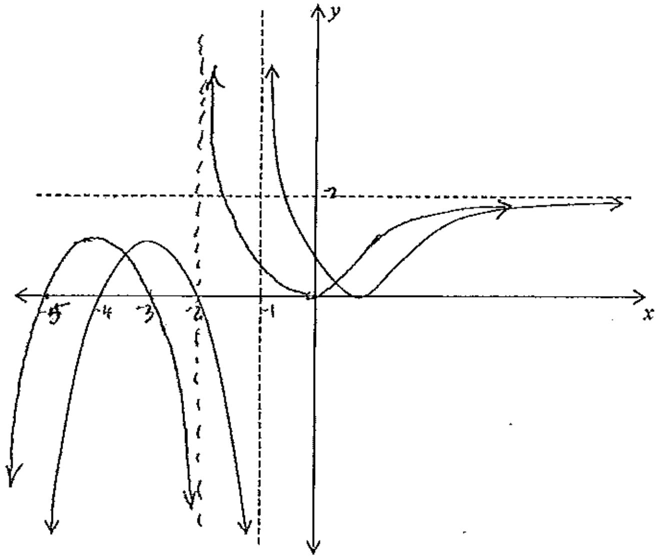
$$= -1$$

$\therefore$ S lies on the circumference of a circle with diameter AB

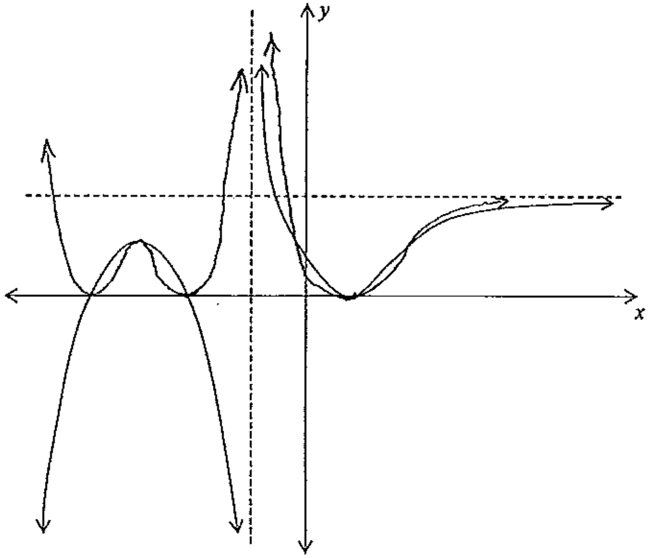
NAME: ____________________ **Class Teacher:** ____________________

Answers to Question 8 c)

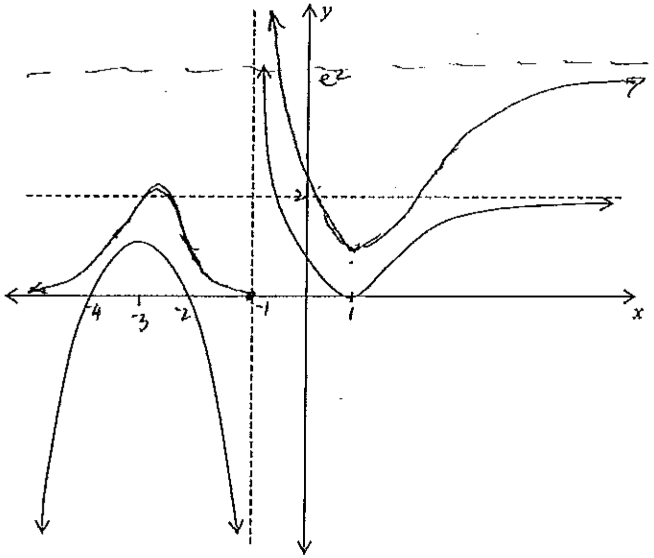
(i)



(ii)



(iv)



(iii)