St George Girls High School

Year 12

Mid-HSC Course Examination

2014

Mathematics Extension 2

General Instructions

- Working time – 90 minutes
- Reading time – 5 minutes
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided
- All necessary working should be shown
- Templates are provided for the graphs in question 7and 8

Section I
Total marks (6)
Attempt Questions 1 – 6
Use the answer sheet provided

Section II
Total marks (66)
Attempt Questions 7 – 9
Start each question in a new booklet.

TABLE OF STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n\neq -1;\quad x\neq 0,\ \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

Note $\ln x = \log_e x,\quad x>0$

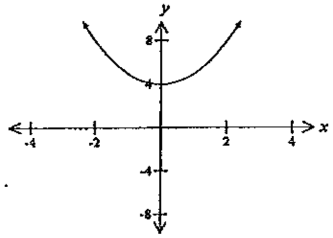
Section I

6 marks
Attempt Questions 1 – 6

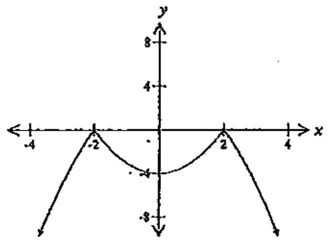
Use the multiple choice answer sheet for Questions 1 – 6

1. What is the graph of $y=|f(x)|$ given that $f(x)=4-x^2$? Change order of answers

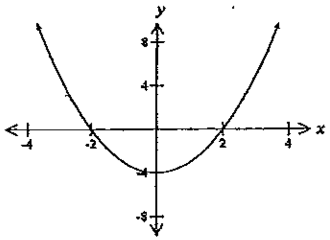
(A)



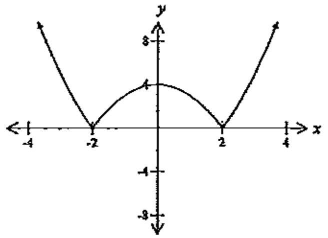
(B)



(C)



(D)



2. The graph of $f(x)=\dfrac{1}{x^2+mx-n}$, where m and n are real constants, has no vertical asymptotes if

(A) $m^2<4n$
(B) $m^2>4n$
(C) $m^2<-4n$
(D) $m^2>-4n$

Section I (cont'd)

3. The graph of the function $f(x)$ is obtained from the graph of the function with equation $y=\sqrt{x}$ by a reflection in the x-axis and also translation 2 units to the left. The rule for f is:

(A) $f(x)=-\sqrt{x+2}$
(B) $f(x)=-\sqrt{x-2}$
(C) $f(x)=-\sqrt{2-x}$
(D) $f(x)=\sqrt{2-x}$

4. The graph of $\left(\frac{x}{2}\right)^2-\left(\frac{y}{3}\right)^2=1$ is a hyperbola.

Which set of equations represents the asymptotes of the hyperbola's graph?

(A) $y=\frac{3}{2}x, y=-\frac{3}{2}x$ (B) $y=\frac{2}{3}x, y=-\frac{2}{3}x$

(C) $y=\frac{1}{2}x, y=-\frac{1}{2}x$ (D) $y=\frac{1}{3}x, y=-\frac{1}{3}x$

5. The distance between the foci of the ellipse with parametric equation

$$x=1+4\cos\theta,\quad y=1+\sin\theta$$

(A) 8 units
(B) 2 units
(C) $2\sqrt{15}$ units
(D) $2(1+\sqrt{15})$

6. The chord of contact from any external point $T(x_0,y_0)$ to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is given by $\frac{xx_0}{a^2}+\frac{yy_0}{b^2}=1$. For the chord of contact on $\frac{x^2}{4}+y^2=1$ from $T(x_0,-2)$ to be a <u>focal</u> chord x_0 has the value.

(A) $\frac{2\sqrt{3}}{3}$ (B) $\frac{4\sqrt{3}}{3}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{\sqrt{3}}{4}$

Section II

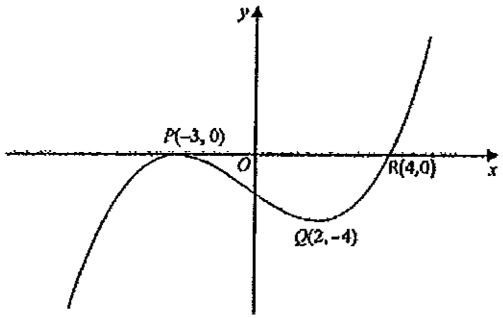
66 marks
Attempt Questions 7 – 9

Answer each question in a separate writing booklet.

Question 7 – (22 marks) – Start a New Booklet

Marks

a) The diagram below shows the graph of $y = f(x)$. The points $P(-3,0)$, $Q(2,-4)$ are stationary points on the graph. $R(4,0)$ is the point where the curve crosses the x axis.



Using the templates provided at the front of the examination paper, draw separate sketches of the graphs of the following showing all essential features.

(i) $y = |f(x)|$ 1

(ii) $y = \sqrt{f(x)}$ 2

(iii) $y = f|(x)|$ 2

b) Sketch the following on the templates provided

(i) $y = x\cos x$ in the domain $0 \le x \le 2\pi$ 2

(ii) $y^2 = \ln(x+1)$ 2

(iii) $|y| = \sin x$ 2

Question 7 (cont'd) Marks

c) For the hyperbola with Cartesian equation $\frac{x^2}{4} - y^2 = 1$

(i) Find the eccentricity of this curve. 1

(ii) Write down the coordinates of

(α) the foci S and S' 1

(β) the equation(s) of the Directrix 1

(iii) Give the equation(s) of the asymptotes of this hyperbola 1

(iv) Sketch [using $\frac{1}{3}$ page] the hyperbola $\frac{x^2}{4} - y^2 = 1$ clearly indicating the features from (i), (ii) and (iii) above. 2

d) The equation $x^2 + 9y^2 + 8x - 18y + 16 = 0$ describes an ellipse.

Determine:

(i) its centre 2

(ii) the eccentricity 'e' 1

(iii) the equation of the tangent at $P(-4,2)$ on the ellipse. 2

Question 8 – (22 marks) – Start a New Booklet Marks

a) Using the template provided and on the same set of axes,

(i) Sketch the graphs of 2

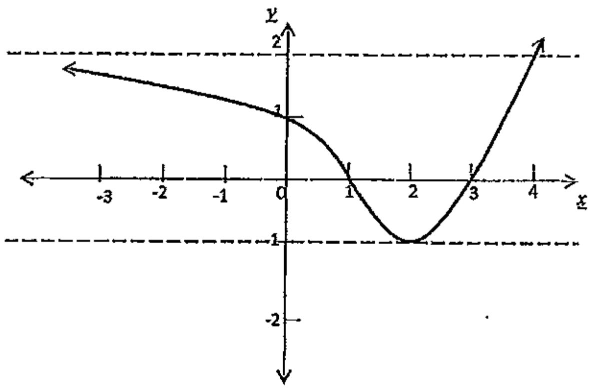
$y = |x+2|$ and $y = |x^2 - 4|$

(ii) On the template from part(i) sketch $f(x) = |x+2| + |x^2 - 4|$ 2

b) Discuss the differentiability of $f(x) = |x+2| + |x^2 - 4|$ when $x = 2$ 2

c) The diagram below shows the graph of $y = f(x)$.

Using the templates provided, on separate diagrams, sketch the following showing essential features



Using the templates provided and on separate diagrams

(i) $y = f(x+2)$ 2

(ii) $y = \dfrac{1}{f(x)}$ 3

Question 8 (cont'd) Marks

d) $P(2\sec\theta,\ \sqrt{3}\tan\theta)$ is an arbitrary point which lies on the hyperbola with Cartesian equation

$$\frac{x^2}{4} - \frac{y^2}{3} = 1$$

(i) Show all necessary steps required to give the equation of the tangent at P as 3

$$\frac{x.\sec\theta}{2} - \frac{y.\tan\theta}{\sqrt{3}} = 1$$

(ii) This tangent cuts the asymptotes in M and N. Prove that $MP = PN$ 5

e) The points $P\left(p, \frac{1}{p}\right)$ and $Q\left(q, \frac{1}{q}\right)$ lie on the rectangular hyperbola $xy = 1$ with $0 < q < p$

The chord PQ has gradient $m = -\dfrac{1}{pq}$

(i) If PQ [extended] passes through $A(2,0)$ then show $p + q = 2$ 2

(ii) If PQ [extended] passes through $A(2,0)$ and M is the mid-point of PQ, completely describe the locus of M as P and Q vary. 1

Question 9 – (22 marks) – Start a New Booklet Marks

a) For the curve given by $x^2 + 2xy + 4y^2 = 1$

(i) Find $\frac{dy}{dx}$ in terms of x and y 2

(ii) Find the coordinates of the critical values and describe their geometrical significance for the curve. 4

b) (i) Using the template provided, graph $y = e^{\sin x}$ in the domain $0 \leq x \leq 2\pi$ 3

(ii) Hence, or otherwise state the number of solutions to $e^{\sin x} = 2$ in the domain $0 \leq x \leq 2\pi$ 1

c) P and Q are the points $\left(ct, \frac{c}{t_1}\right)$ and $\left(ct_2, \frac{c}{t_2}\right)$ on the rectangular hyperbola $xy = c^2$

(i) If $c^2 = 9$, determine

(α) the coordinates of the foci 2

(β) equations of directricies 1

(ii) Given that the gradient of PQ is $\frac{-1}{t_1 t_2}$, prove that if PQ subtends a right angle at a third point R on the hyperbola, THEN the tangent at R is perpendicular to PQ 3

Question 9 (cont'd) Marks

d) (i) Show that the equation of the normal at $P\left(cp, \frac{c}{p}\right)$ on the curve $xy = c^2$ is given by $p^3x - py = c(p^4 - 1)$ 2

(ii) Given the normal at P meets the x-axis at N and the tangent at P meets the y-axis at R. Let M be the midpoint of NR. 4

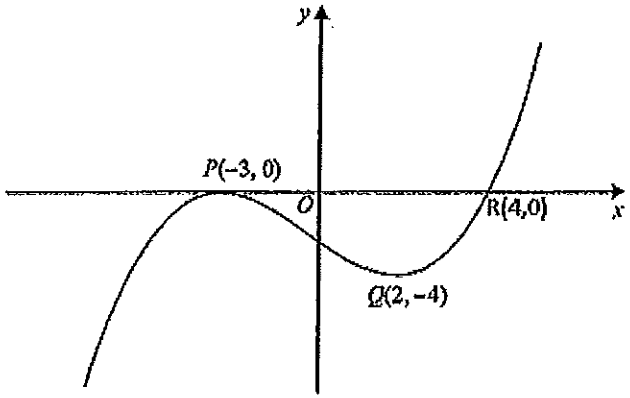
Show that the Cartesian equations of the locus of M is $2c^2xy = c^4 - y^4$

End of Paper

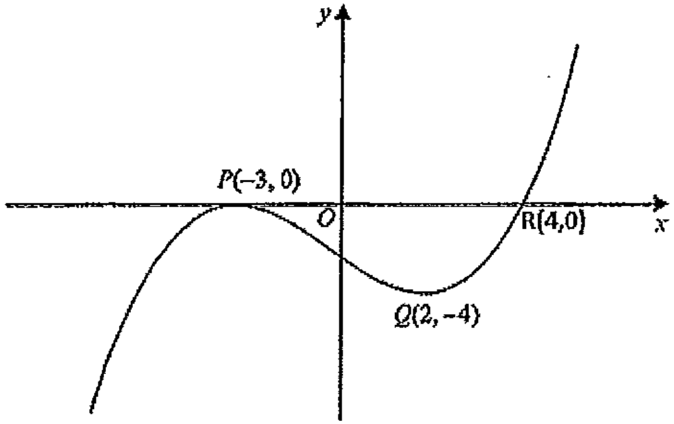
Question 7 Templates Name :

Detach and place completed sheet inside answer booklet for Question 7

a) (i) $y = |f(x)|$

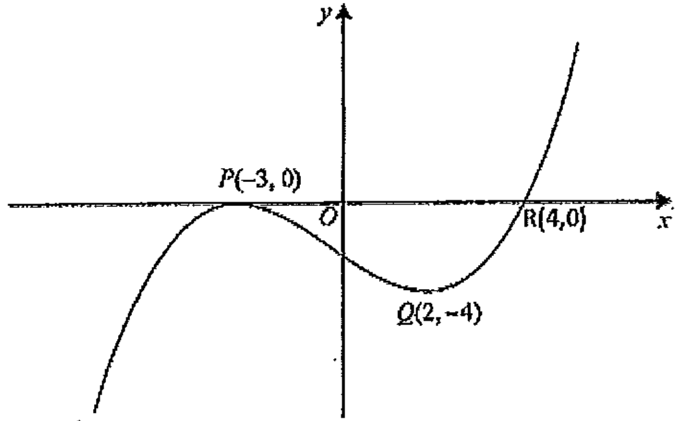


(ii) $y = \sqrt{f(x)}$

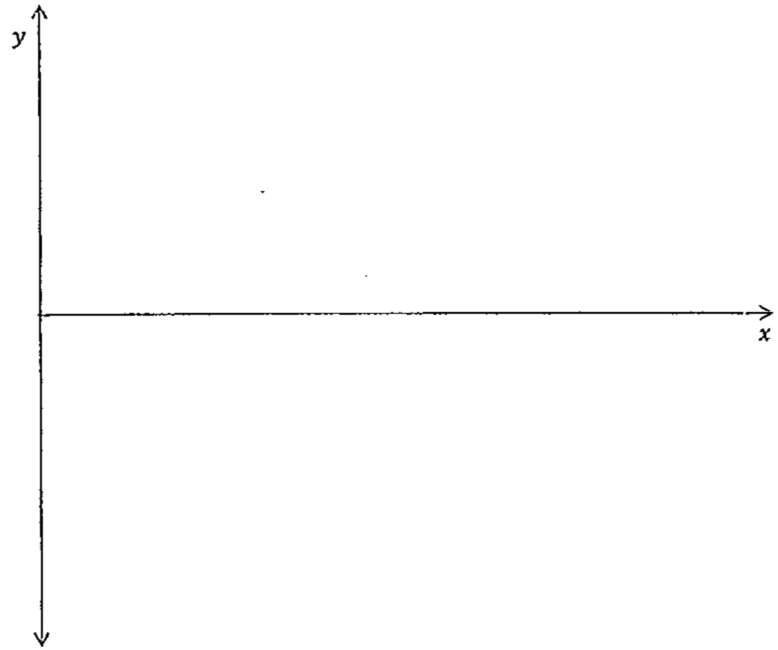


Question 7 Templates Name :

(iii) $y = f|(x)|$

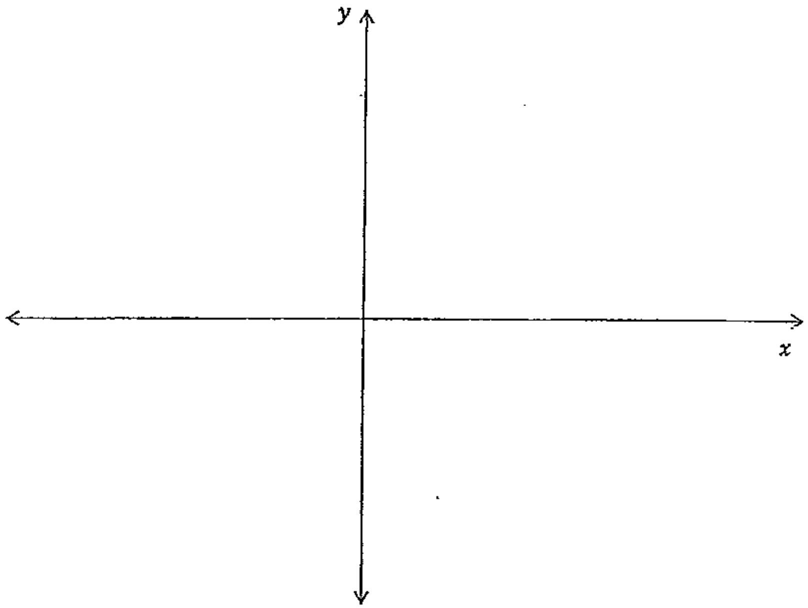


b) (i) $y = x\cos x$ in the domain $0 \le x \le 2\pi$

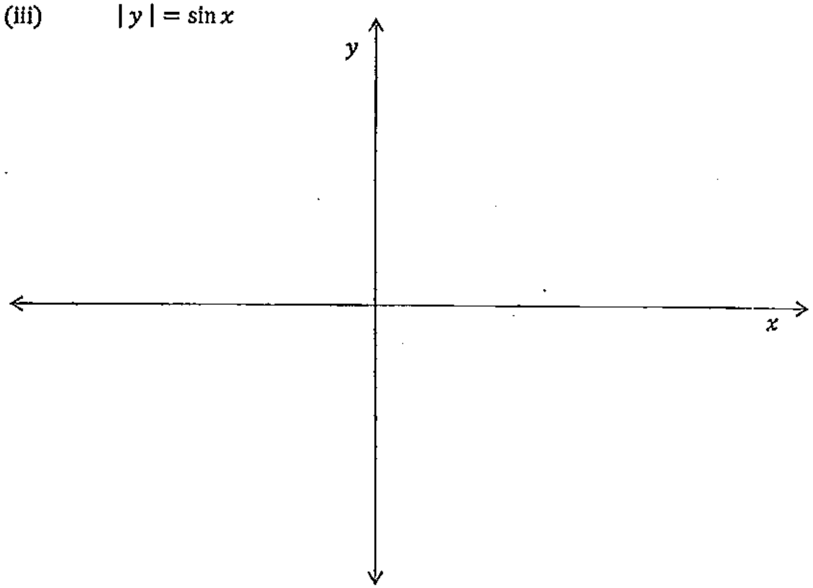


Question 7 Templates **Name :** ____________________

b) (ii) $y^2 = \ln(x+1)$

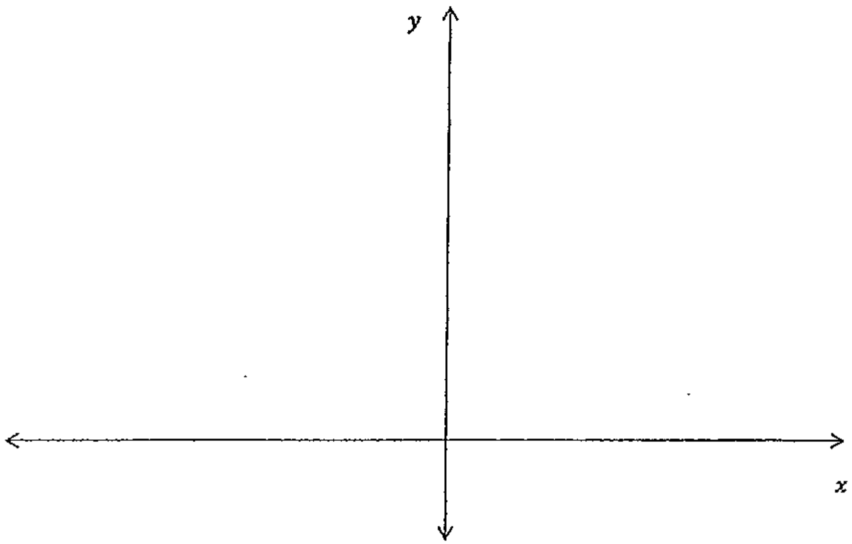


b) (iii) $|y| = \sin x$

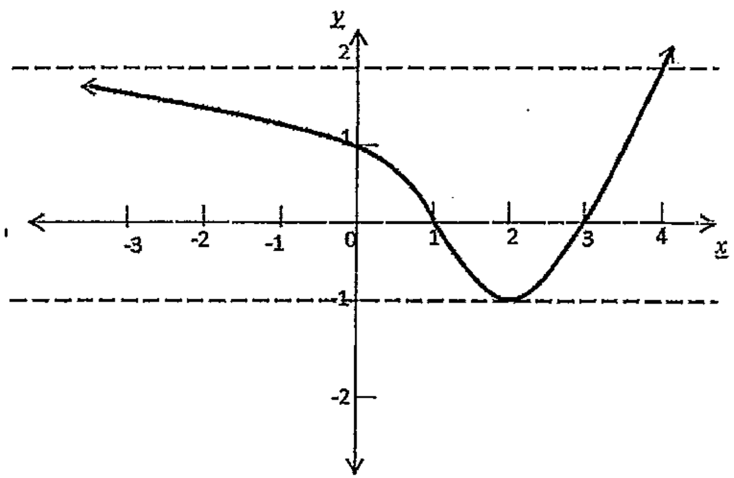


Question 8 Templates **Name :** ____________

a) $y = |x+2|$ and $y = |x^2-4|$ and $f(x) = |x+2| + |x^2-4|$

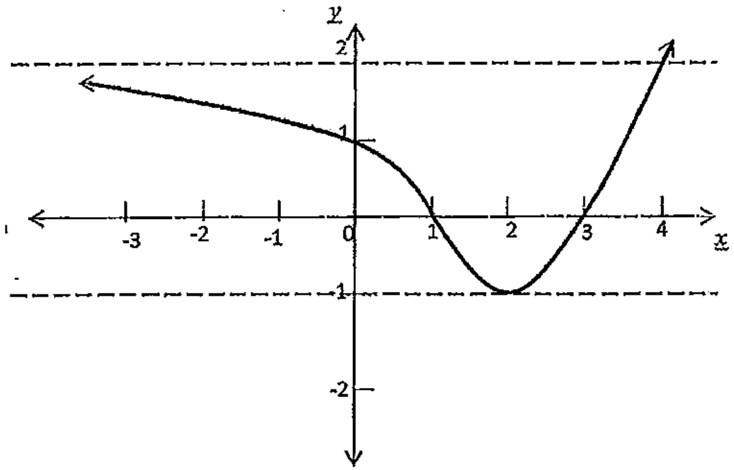


c) (i) $y = f(x+2)$



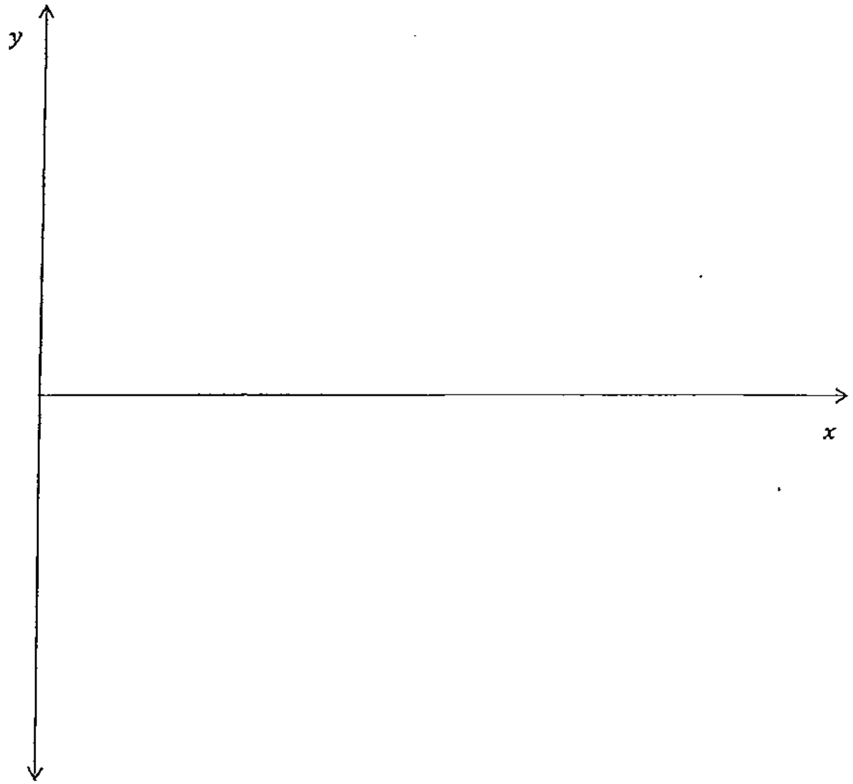
Question 8 Templates **Name :** ____________

b) (ii) $y = \frac{1}{f(x)}$



Question 9b) Templates Name :________________

Using the template provided, graph $y = e^{\sin x}$ in the domain $0 \le x \le 2\pi$



YEAR 12 MID - HSC EXT 2 SOLUTIONS.

SECTION I (Multiple Choice)

① [D]

② No vertical asymptotes if x^2+mx-n is positive definite
$a>0$ and $\Delta<0$ $\therefore$ $m^2+4n<0$
$m^2<-4n$
[C]

③ Reflection : $y=-f(x)$ and translation $y=-f(x+2)$
$\therefore$ $f(x)=-\sqrt{x+2}$
[A]

④ $y=\pm\frac{b}{a}x$ $\therefore$ asymptotes $y=\pm\frac{3}{2}x$
[A]

⑤ Distance is $2ae$ → for $\left(\frac{x-1}{4}\right)^2+(y-1)^2=1$
$a=4$, $e^2=1-\frac{1}{16}$
$e^2=\frac{15}{16}$ ⇒ $e=\frac{\sqrt{15}}{4}$
$\therefore$ distance $2\sqrt{15}$
[C]

⑥ Equation $\frac{xx_0}{4}+\frac{yy_0}{1}=1$, if focal chord
then $\frac{xx_0}{4}-2y=1$ passes through $S(ae,0)$
Here $\frac{x.2e}{4}-0=1$
$x=\frac{2}{e}$
Where $e=\sqrt{1-\frac{1}{4}}$
$=\frac{\sqrt{3}}{2}$ ⟶ $\frac{2}{\frac{\sqrt3}{2}}=\frac{4}{\sqrt3}$
$=\frac{4\sqrt3}{3}$
Hence.. [B]

QUESTION 7

(a) See Template
(b) See Template.

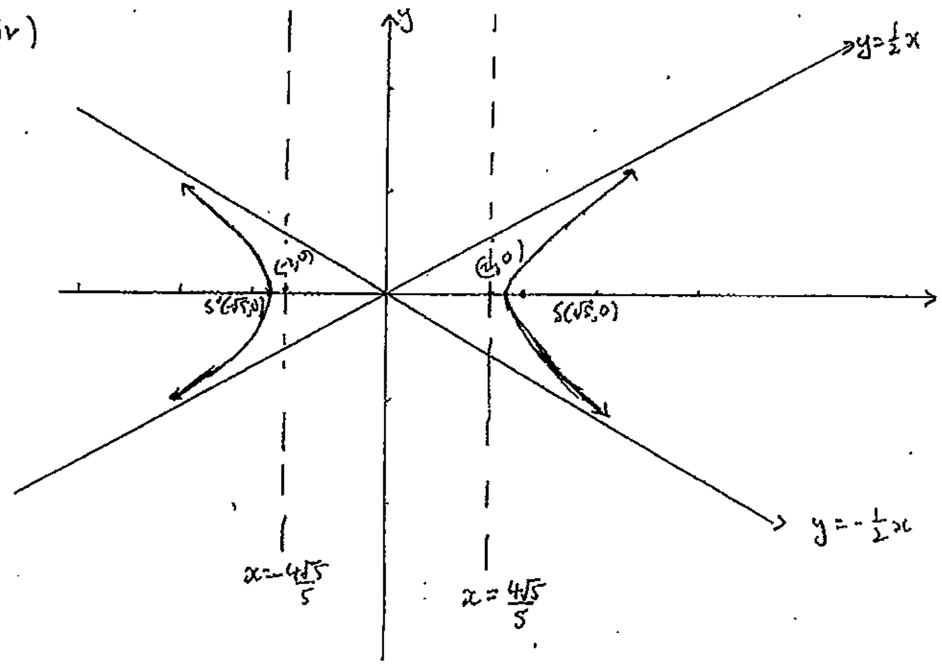
(c) (i) $e^2=\frac{b^2}{a^2}+1$ ⇒ $e^2=\frac{1}{4}+1$
$e=\frac{\sqrt5}{2}$

(ii) (α) $S(ae,0)$ and $S'(-ae,0)$
$\therefore$ $S(\sqrt5,0)$ and $S'(-\sqrt5,0)$

(β) Directrix $y=\pm\frac{a}{e}$
$\therefore$ $x=-\frac{2}{\frac{\sqrt5}{2}}$ and $x=\frac{2}{\frac{\sqrt5}{2}}$
$=-\frac{4\sqrt5}{5}$ $=\frac{4\sqrt5}{5}$

(iii) Equations of asymptotes $y=\pm\frac{b}{a}x$
$\therefore$ $y=-\frac{1}{2}x$ and $y=\frac{1}{2}x$

(iv)



(d) (i) $(x^2+8x+16) + 9y^2-18y = 0$

$(x+4)^2 + 9(y^2-2y+1) - 9 = 0$

$\frac{(x+4)^2}{9} + (y-1)^2 = 1$

Ellipse has centre $(-4, 1)$

(ii) $e^2 = 1 - \frac{b^2}{a^2} \Rightarrow e = \sqrt{1-\frac{1}{9}}$

$= \frac{2\sqrt{2}}{3}$

(iii) By implicit differentiation

$\frac{2}{9}(x+4) + 2(y-1)\cdot\frac{dy}{dx} = 0$

$\therefore \frac{dy}{dx} = -\frac{(x+4)}{9(y-1)}$

at $P(-4, 2)$ gradient is $m = -\frac{(-4+4)}{9(2-1)}$

$m = 0$

Tangent is the horizontal line $y = 2$.

QUESTION 8

(a) See templates.

(b) $\frac{d}{dx}[(x+2) + -(x^2-4)]$ when $x<2$ and $\frac{d}{dx}[x+2 + (x^2-4)]$ when $x>2$

$= (-2x+1)$ when $x<2$ $\qquad = (2x+1)$ at $x<2$

as $x \to 2^-$, $m \to -3$ $\qquad$ as $x \to 2^+$, $m \to 5$

$f(x) = |x+2| + |x^2-4|$ is not differentiable at $x=2$ as different gradients approached from either side.

(c) on template.

(d) (i) Differentiate: $\frac{x}{2} - \frac{2y}{3}\cdot\frac{dy}{dx} = 0$ $\qquad$ or $\frac{dx}{d\theta} = 2\sec\theta\tan\theta$

$\therefore \frac{dy}{dx} = \frac{3x}{4y}$ $\qquad$ $\frac{dy}{d\theta} = \sqrt{3}\sec^2\theta$

$\therefore \frac{dy}{dx} = \frac{\sqrt{3}\sec\theta}{2\tan\theta}$

$P(2\sec\theta, \sqrt{3}\tan\theta) \Rightarrow m = \frac{3\cdot 2\sec\theta}{4\sqrt{3}\tan\theta}$

$= \frac{\sqrt{3}\sec\theta}{2\tan\theta}$.

Then, gradient of tangent at P given by

$y - \sqrt{3}\tan\theta = \frac{\sqrt{3}\sec\theta}{2\tan\theta}(x - 2\sec\theta)$

$y\cdot\frac{\tan\theta}{\sqrt{3}} - \tan^2\theta = x\cdot\frac{\sec\theta}{2} - \sec^2\theta$

$\therefore \frac{x}{2}\cdot\sec\theta - \frac{y}{\sqrt{3}}\cdot\tan\theta = \sec^2\theta - \tan^2\theta$

$= 1$

as required.

(ii) • let M be on $y = \frac{\sqrt{3}}{2}x$.

then by substitution, intersect when

$$\frac{x \cdot \sec\theta}{2} - \frac{\frac{\sqrt{3}}{2}x \cdot \tan\theta}{\sqrt{3}} = 1$$

$$\therefore \quad x(\sec\theta - \tan\theta) = 2$$

$$x = \frac{2}{\sec\theta - \tan\theta}$$

$$\text{then} \quad y = \frac{\sqrt{3}}{\sec\theta - \tan\theta}$$

$$M\left(\frac{2}{\sec\theta - \tan\theta}, \frac{\sqrt{3}}{\sec\theta - \tan\theta}\right)$$

• let N be on $y = -\frac{\sqrt{3}}{2}x$

then by substitution, intersect when

$$\frac{x \cdot \sec\theta}{2} - \frac{-\frac{\sqrt{3}}{2}x\tan\theta}{\sqrt{3}} = 1$$

$$x(\sec\theta + \tan\theta) = 2$$

$$x = \frac{2}{\sec\theta + \tan\theta}$$

$$\therefore \quad y = \frac{-\sqrt{3}}{\sec\theta + \tan\theta}$$

$$N\left(\frac{2}{\sec\theta + \tan\theta}, \frac{-\sqrt{3}}{\sec\theta + \tan\theta}\right)$$

Now $MP = PN$ true if P is mid point of MN.

So, check midpoint

$$\left[\frac{\frac{2}{\sec\theta - \tan\theta} + \frac{2}{\sec\theta + \tan\theta}}{2}, \frac{\frac{\sqrt{3}}{\sec\theta - \tan\theta} + \frac{-\sqrt{3}}{\sec\theta + \tan\theta}}{2}\right]$$

$$= \left[\sec\theta + \tan\theta + \sec\theta - \tan\theta, \frac{\sqrt{3}}{2}\left(\sec\theta + \tan\theta + -(\sec\theta - \tan\theta)\right)\right]$$

$= (2\sec\theta, \sqrt{3}\tan\theta)$. Co-ordinates of P

$\therefore MP = PN$.

(e) (i) Cuts y-axis when $x = 0$

then by $\sec\theta = (a^2 + b^2)\sec\theta \cdot \tan\theta$

$$y = \frac{(a^2+b^2)}{b}\tan\theta$$

and B is $\left(0, \frac{(a^2+b^2)}{b} \cdot \tan\theta\right)$

(ii) If S lies on a circle with diameter AB, then AB subtends a right angle at S.

let m_1 be gradient of AS and m_2 be gradient of BS.

Then $m_1 = \dfrac{0 - \frac{-b}{\tan\theta}}{ae - 0}$; $m_2 = \dfrac{0 - \frac{(a^2+b^2)}{b}\tan\theta}{ae - 0}$

$$m_1 = \frac{b}{ae\tan\theta} \qquad = \frac{-(a^2+b^2)\tan\theta}{abe}$$

Now $m_1 \times m_2 = \dfrac{b}{ae\tan\theta} \times \dfrac{-(a^2+b^2)\tan\theta}{abe}$

$$= -\frac{(a^2+b^2)}{a^2e^2} \qquad \left[* e^2 = 1 + \frac{b^2}{a^2} = \frac{a^2+b^2}{a^2}\right]$$

$$= -\frac{(a^2+b^2)}{a^2\left(\frac{a^2+b^2}{a^2}\right)}$$

$$= -1$$

$\therefore$ $m_1 \perp m_2$ and AB subtends a right angle at S. S lies on a circle with diameter AB.

QUESTION 9:

(a) (i) Differentiate Implicity

$$2x + \left[2y + 2x.\frac{dy}{dx}\right] + 8y.\frac{dy}{dx} = 0$$

then $(2x+8y)\frac{dy}{dx} = -2(x+y)$

$$\frac{dy}{dx} = \frac{-2(x+y)}{2(x+4y)}$$

$$= -\frac{(x+y)}{(x+4y)}$$

(ii) (α) Stationary points when $\frac{dy}{dx} = 0$

then $x+y=0$

$y=-x$

On substitution : $x^2 + 2x.-x + 4(-x)^2 = 1$

$x^2 - 2x^2 + 4x^2 = 1$

$3x^2 - 1 = 0$

$(\sqrt{3}x-1)(\sqrt{3}x+1) = 0$

at $x = -\frac{1}{\sqrt{3}}$ and $x = \frac{1}{\sqrt{3}}$

$= -\frac{\sqrt{3}}{3}$ $\qquad = \frac{\sqrt{3}}{3}$

$y = \frac{\sqrt{3}}{3}$ $\qquad y = -\frac{\sqrt{3}}{3}$

Stationary pts. $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$ $\quad \left(\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}\right)$

x	-1	$-\frac{\sqrt{3}}{3}$	0^-	0^+	$\frac{\sqrt{3}}{3}$	1
$\frac{dy}{dx}$	+ve	0	-ve	+ve	0	+ve
y	≈ 0.5	$\frac{\sqrt{3}}{3}$	0.5	≈ 0.5	$-\frac{\sqrt{3}}{3}$	-0.5

$4y^2 - y - 2 = 0$

$y = \frac{-1 \pm \sqrt{33}}{8}$

Local Max $\qquad$ Local Min

Local min. at $\left(\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}\right)$; Local max a $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

(β) $\frac{dy}{dx}$ Undefined at $x + 4y = 0$

$x = -4y$

On substitution : $(-4y)^2 + 2.-4y.y + 4y^2 = 1$

$16y^2 + -8y^2 + 4y^2 = 1$

$12y^2 - 1 = 0$

$\therefore (2\sqrt{3}y+1)(2\sqrt{3}-1) = 0$

'Vertical' tangents at

$y = -\frac{1}{2\sqrt{3}}$ and $y = \frac{1}{2\sqrt{3}}$

$= -\frac{\sqrt{3}}{6}$ $\qquad = \frac{\sqrt{3}}{6}$

$x = \frac{2\sqrt{3}}{3}$ $\qquad x = -\frac{2\sqrt{3}}{3}$

$\left(\frac{2\sqrt{3}}{3}, -\frac{\sqrt{3}}{6}\right)$ $\qquad \left(-\frac{2\sqrt{3}}{3}, \frac{\sqrt{3}}{6}\right)$

Curve has 'vertical' tangents at $\left(-\frac{2\sqrt{3}}{3}, \frac{\sqrt{3}}{6}\right)$ and $\left(\frac{2\sqrt{3}}{3}, -\frac{\sqrt{3}}{6}\right)$

(b) (i) See template.

(ii) Construct $y = 2$ on same graph.

Two points of intersection, then two solutions in the domain $0 \le x \le 2\pi$.

(c) (i) Gradient function $\frac{2x}{a^2} - \frac{2y}{b^2}.\frac{dy}{dx} = 1$

$$\frac{dy}{dx} = \frac{b^2x}{a^2y}$$

at $P(x_1, y_1)$ gradient of tangent is

$$m = \frac{b^2x_1}{a^2y_1}$$

Then, equation of tangent

$$(y - y_1) = \frac{b^2x_1}{a^2y_1}(x - x_1)$$

$$\frac{yy_1}{b^2} - \frac{y_1}{b^2} = \frac{\quad}{a^2} \quad \frac{\quad}{a^2}$$

gives $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2}$

$= 1$ [Since P lies on the curve.]

$\therefore \boxed{\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1}$

(ii) Let $x = \frac{a}{e}$

then, $\frac{\frac{a}{e} . x_1}{a^2} - \frac{yy_1}{b^2} = 1$

$\frac{x_1}{ae} - \frac{y.y_1}{b^2} = 1$

$\frac{y.y_1}{b^2} = \frac{x_1}{ae} - 1$

$y = \frac{b^2}{y_1}\left(\frac{x_1}{ae} - 1\right)$

* $= \frac{b^2}{y_1}\left(\frac{x_1}{\sqrt{a^2+b^2}} - 1\right)$

$= \frac{ab}{\sqrt{x_1^2-a^2}}\left(\frac{x_1}{\sqrt{a^2+b^2}} - 1\right)$

• $e = \sqrt{1 + \frac{b^2}{a^2}}$

$= \frac{1}{a}\sqrt{a^2+b^2}$

• $y_1 = \frac{b}{a}\sqrt{x_1^2 - a^2}$

(iii) Gradient PS $\rightarrow m_1 = \frac{y_1 - 0}{x_1 - ae}$

$= \frac{y_1}{x_1 - ae}$

Gradient XS $\rightarrow m_2 = \frac{\frac{b^2}{aey_1}(x_1 - ae) - 0}{\frac{a}{e} - ae}$

$= \frac{b^2(x_1 - ae)}{a^2 y_1 (1 - e^2)}$

Now, $m_1 \times m_2 = \frac{y_1}{x_1 - ae} \times \frac{b^2(x_1 - ae)}{a^2 y_1 (1-e^2)}$

$= \frac{b^2}{a^2} \times \frac{1}{1-e^2}$

$= \frac{b^2}{a^2} \times \frac{1}{-\frac{b^2}{a^2}}$

$= -1$

$\left[e^2 - 1 = \frac{b^2}{a^2} \quad \therefore 1 - e^2 = -\frac{b^2}{a^2}\right]$

PS ⊥ XS, $\therefore \angle PSX = 90°$

(d) (i) (α) If elliptical: $4 - \lambda > 0$ and $2 - \lambda > 0$

$4 > \lambda$ and $2 > \lambda$

both true when $\underline{\lambda < 2}$

(β) If hyperbolic $4 - \lambda > 0$ and $2 - \lambda < 0$

$4 > \lambda$ and $2 < \lambda$

true when $\underline{2 < \lambda < 4}$

(ii) As λ moves from 1 to 2 the ellipse ellongates and limiting position is a straight line.

Since, as λ increases from 1 to 2, $4 - \lambda$ decreases from 3 to 2 and $2 - \lambda$ decreases from 1 to 0.

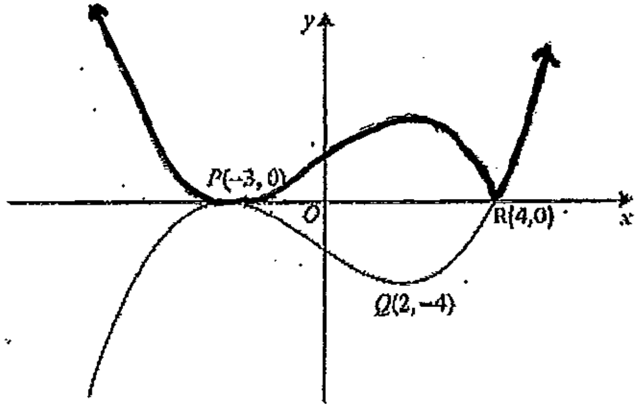
The curve remains an ellipse with 'a' reducing from $\sqrt{3}$ to $\sqrt{2}$ [semi-major axis] and b [semi-minor axis] reducing from 1 to 0.

As $\lambda \rightarrow 2$, the ellipse becomes a line interval from $(-\sqrt{2}, 0)$ to $(\sqrt{2}, 0)$.

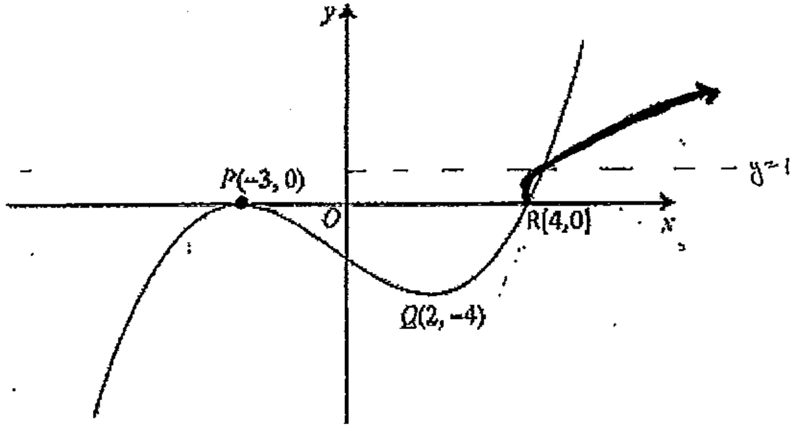
Question 7 Templates Name : ____________

Detach and place completed sheet inside answer booklet for Question 7

a) (i) $y = |f(x)|$

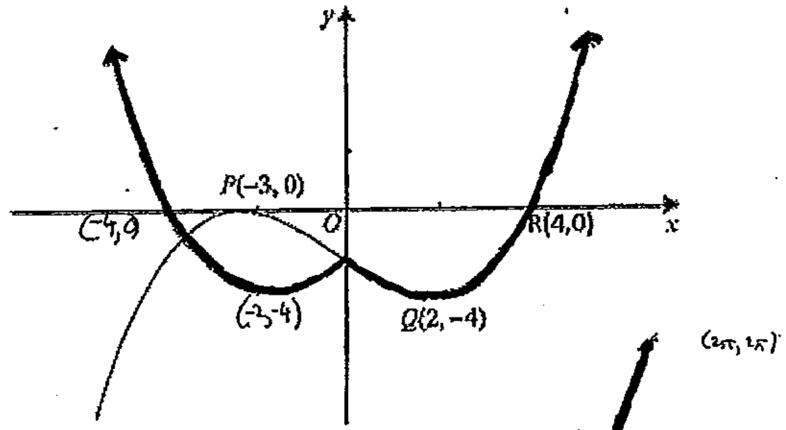


(ii) $y = \sqrt{f(x)}$

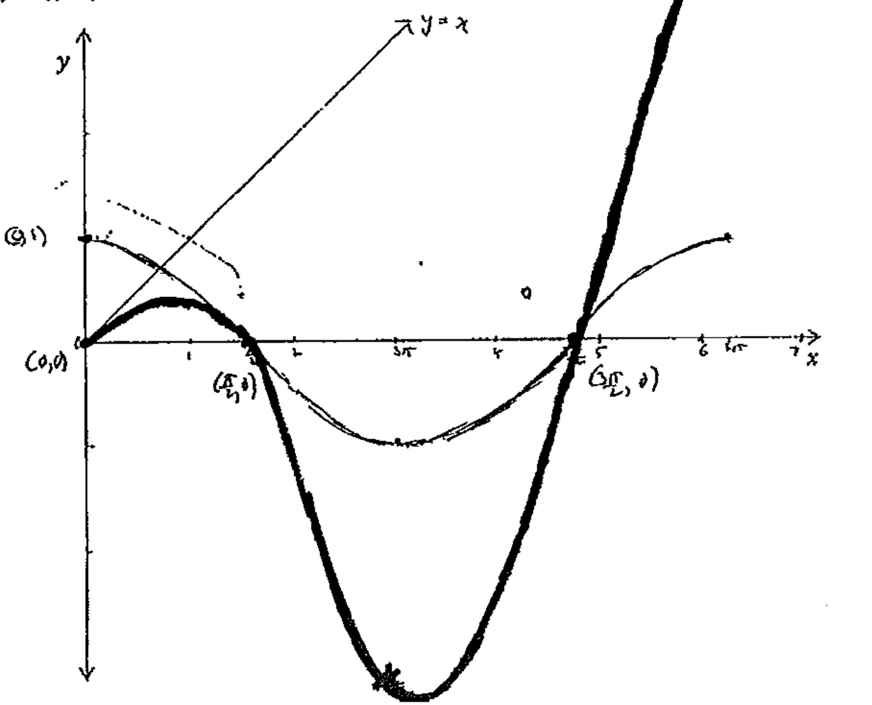


Question 7 Templates Name : ____________

(iii) $y = f|(x)|$

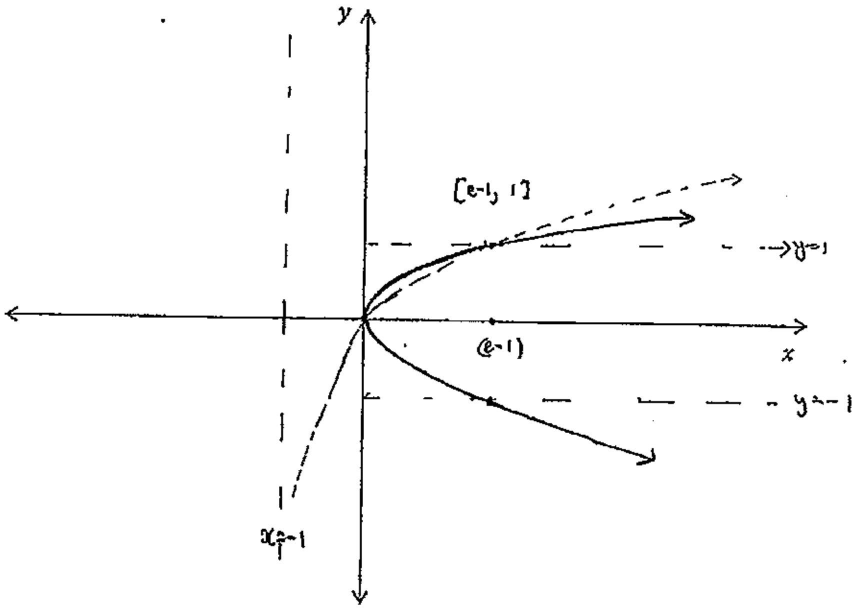


b) (i) $y = x\cos x$ in the domain $0 \le x \le 2\pi$

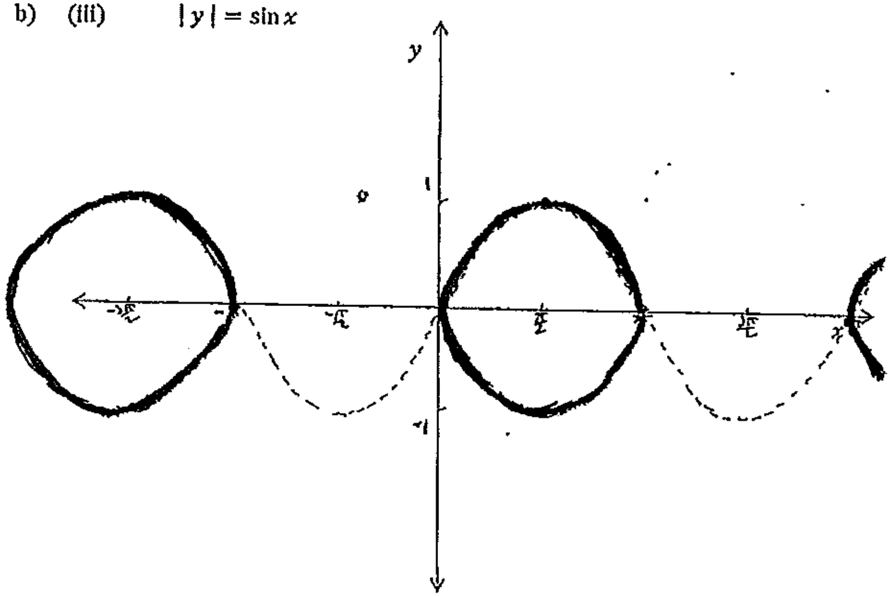


Question 7 Templates **Name :** ______________

b) (ii) $y^2 = \ln(x+1)$

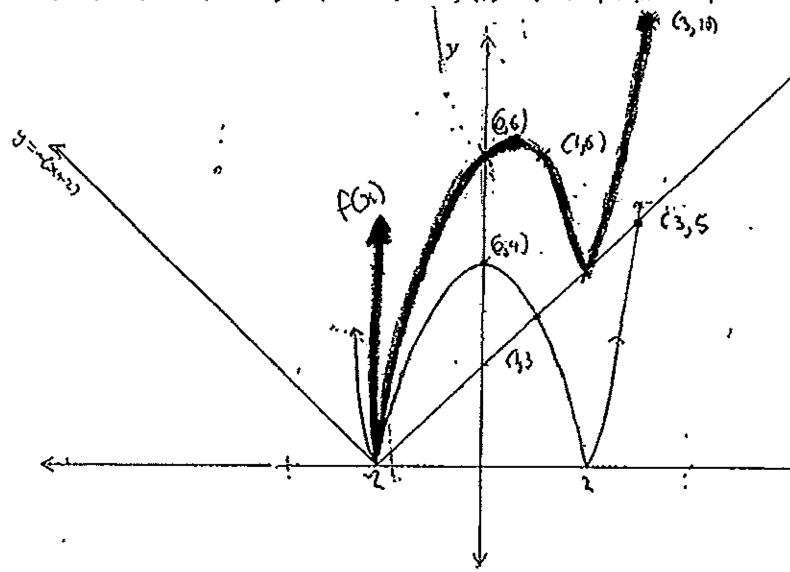


b) (iii) $|y| = \sin x$



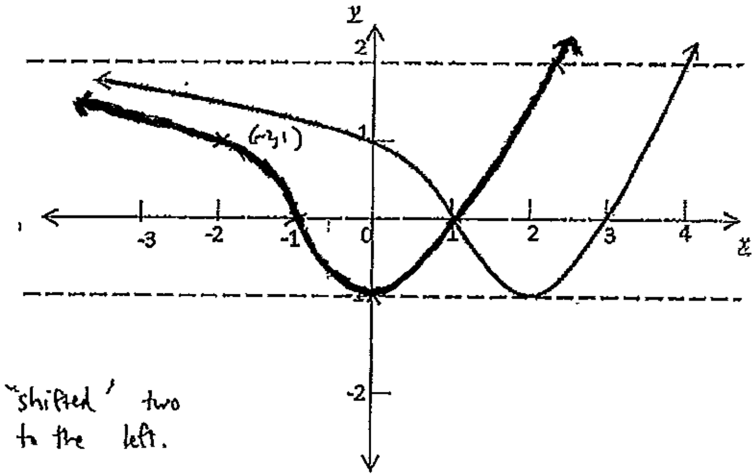
Question 8 Templates **Name :** ______________

a) $y = |x+2|$ and $y = |x^2 - 4|$ and $f(x) = |x+2| + |x^2 - 4|$



$x+2 = x^2-4$
$x^2-x-6=0$
$(x-3)(x+2)=0$
$x=3$
$x+2=-x^2+4$
$x^2+x-2=0$
$(x+2)(x-1)=0$
$x=1$

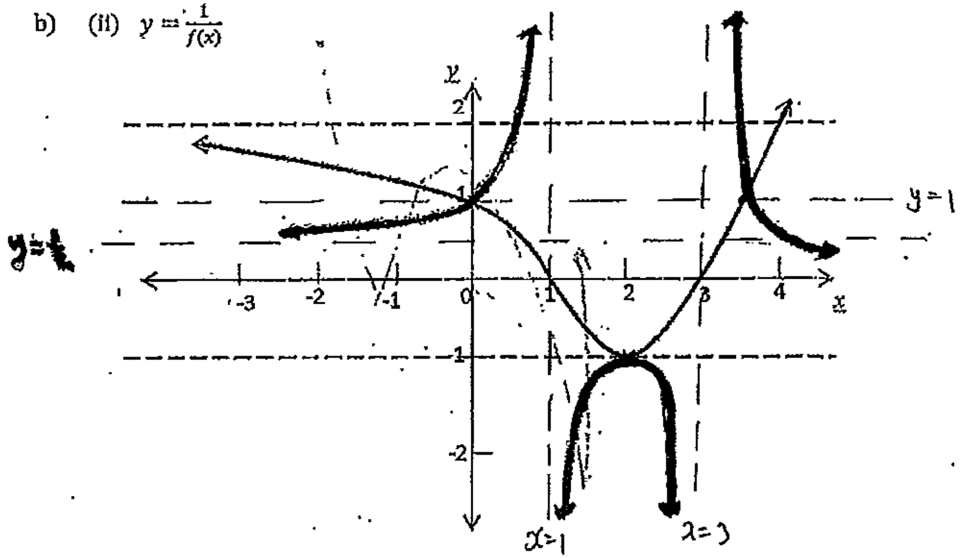
c) (i) $y = f(x+2)$



Curve 'shifted' two units to the left.

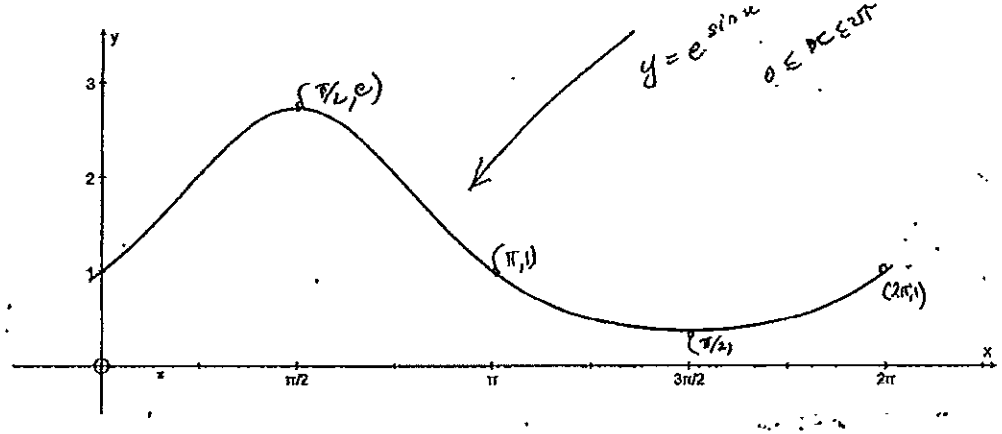
Question 8 Templates **Name :** ____________

b) (ii) $y = \frac{1}{f(x)}$



Question 9b) Templates **Name :** ____________

Using the template provided, graph $y = e^{\sin x}$ in the domain $0 \le x \le 2\pi$



9 a) ii)