St George Girls High School

Year 12

Common Test 3

May 2012

Mathematics

General Instructions

- Time allowed – 70 minutes
- Write using blue or black pen
- Board-approved calculators may be used.
- A table of standard integrals is provided.
- All necessary working should be shown in every question.
- Start each question in a new booklet.

Total marks – 65

- Attempt Questions 1 – 5
- All questions are of equal value

Question 1 – Start a New Booklet – (13 marks) Marks

Questions a) and b) are multiple choice: write the correct letter A, B, C or D in your answer booklet

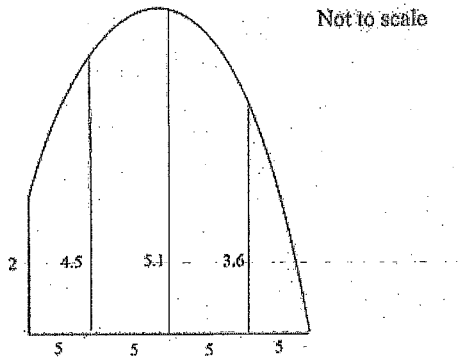
a) The table below shows the values of a function $f(x)$ for five values of x. 1

x	2	2.5	3	3.5	4
$f(x)$	4	1	−2	3	8

What value is an estimate for $\int_2^4 f(x)\,dx$ using Simpson's rule with these five values?

(A) 4
(B) 6
(C) 8
(D) 12

b) The diagram below shows a native garden. All measurements are in metres. 1



What is an approximate value for the area of the native garden using the trapezoidal Rule with 4 intervals?

(A) 31 m²
(B) 62 m²
(C) 71 m²
(D) 74 m²

Question 1 (cont'd) Marks

c) (i) **Copy** and **complete** the table below for the function $f(x) = 3^x$ 2

x	1	2	3	4	5
$f(x)$					

(ii) Use Simpson's Rule with the 5 function values obtained in part (i) to find an approximate value of $\int_1^5 3^x \, dx$ 3

d) Use Simpson's Rule with 3 function values to estimate the volume formed by rotating the curve $y = \frac{1}{1+x^2}$ about the x axis between $x = 0$ and $x = 2$. 3

Answer correct to 4 significant figures.

e) Sketch $y = 3^x + 2$ showing all its essential features and hence state its domain and range. 3

Question 2 – Start a New Booklet – (13 marks) Marks

Questions a) and b) are multiple choice: write the correct letter A, B, C or D in your answer booklet

a) What is the solution to the equation $4^x = 32$? 1

(A) 0.4
(B) 2.5
(C) 3
(D) 8

b) Which of the following is an expression for $\frac{dy}{dx}$ if $y = \ln(x^2 - 4)$? 1

(A) $\frac{dy}{dx} = 2x$ (B) $\frac{dy}{dx} = \frac{1}{x^2-4}$

(C) $\frac{dy}{dx} = \frac{2}{x^2-4}$ (D) $\frac{dy}{dx} = \frac{2x}{x^2-4}$

c) Differentiate the following functions with respect to x

(i) e^{7x} 1

(ii) $4e^{3x^2+1} + 9e^{-x}$ 2

(iii) $\log_e(x^2 - 3x)$ 2

(iv) $x\log_e x$ 2

(v) $\frac{e^x - 1}{x - 1}$ 2

(vi) $\log_e\left(\frac{2x+1}{x+1}\right)$ 2

Question 3 – Start a New Booklet – (13 marks) Marks

Questions a) and b) are multiple choice: write the correct letter A, B, C or D in your answer booklet

a) If $f'(x) = 4e^{2x}$, then $f(x)$ could be equal to 1

(A) $2e^{2x} + 3$

(B) $4e^{2x} + 5$

(C) $8e^{2x} + 2$

(D) $4\log_e(2x) - 4$

b) What is the exact value of $\int_0^1 (e^{2x} + 1)\ dx$? 1

(A) $\frac{1}{2}e^2$ (B) $\frac{1}{2}(e^2 + 1)$

(C) e^2 (D) $e^2 + 1$

c) Find the indefinite integrals

(i) $\int e^{5x}\ dx$ 1

(ii) $\int e^{2-3x}\ dx$ 1

Question 3 (cont'd) Marks

d) Find the area under the curve $y = e^{2x} + 1$ from to $x = 1$ to $x = 3$ 2

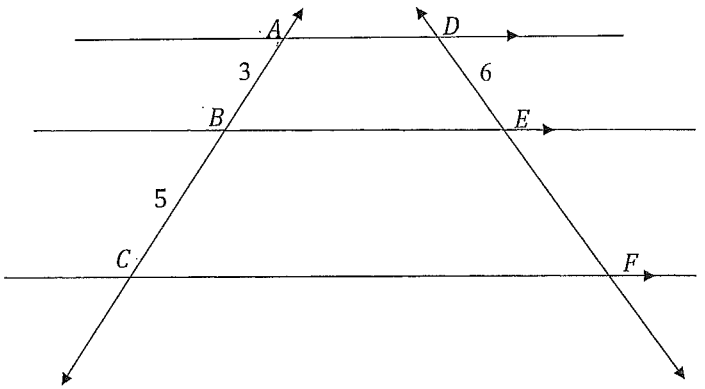
e) Find the volume of the solid of revolution formed when the area between the curve $y = 4e^{-2x}$, the x axis, and the lines $x = 1$ and $x = 2$ is rotated about the x axis. 3

f) Find the equation of the tangent to the curve $y = x^2$ at the point when $x = e$ 2

g) In the diagram below, AD, BE and CF are parallel lines cut by transversals AC and DF. The interval $AB = 3$ cm, $BC = 5$ cm and $DE = 6$ cm. 2

Find the length of DF giving reasons for your answer.

Diagram not to scale



Question 4 – Start a New Booklet – (13 marks) Marks

Questions a) and b) are multiple choice: write the correct letter A, B, C or D in your answer booklet

a) $\log_t r = s$ can also be expressed as 1

(A) $s^t = r$

(B) $r^s = t$

(C) $t^r = s$

(D) $t^s = r$

b) Which of the following expressions is equivalent to $2\log B - \log C + \log D$? 1

(A) $2\log\left(\frac{BD}{C}\right)$ (B) $\log\left(\frac{B^2D}{C}\right)$

(C) $\log\left(\frac{2BD}{C}\right)$ (D) $\log\left(\frac{B^2}{CD}\right)$

c) Evaluate

(i) $\log_2 16$ 1

(ii) $\log_3 \frac{1}{27}$ 1

(iii) $\log_a \sqrt{a}$ 1

d) Solve $\log_4(2x+1) = 3$ 2

e) Solve $\log(3x) + \log 10 = \log 81$ 3

f) Simplify fully $e^{\ln(3x+1)}$ 1

g) Solve $25^{x+2} = 125^{2x}$ 2

Question 5 – Start a New Booklet – (13 marks) Marks

Questions a), b) and c) are multiple choice: write the correct letter A, B, C or D in your answer booklet

a) What is the derivative of $(1+\log_e x)^4$? 1

(A) $4(1+\log_e x)^3$ (B) $\frac{(1+\log_e x)^5}{5}$

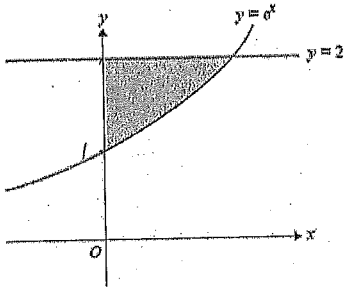
(C) $\frac{4(1+\log_e x)^3}{x}$ (D) $\frac{(1+\log_e x)^5}{5x}$

b) What is the derivative of $\log_e 2x$? 1

(A) $\frac{1}{x}$ (B) $\frac{1}{2x}$

(C) $\ln 2x$ (D) $\frac{1}{x\ln 2}$

c) Parts of the graph of $y = e^x$ and $y = 2$ are shown below. 1



The total area, bounded by the y axis, the line $y = 2$ and the curve with equation $y = e^x$ is given by

(A) $\int_0^{e^2} (e^x - 2)\,dx$ (B) $\int_0^{\ln 2} (e^x - 2)\,dx$

(C) $\int_0^{e^2} (2 - e^x)\,dx$ (D) $\int_0^{\ln 2} (2 - e^x)\,dx$

Question 5 (cont'd) Marks

d) (i) Sketch $y = \log_2(x-1)$ showing its essential features 2

(ii) What is the domain of $y = \log_2(x-1)$? 1

(iii) What is the range of $y = \log_2(x-1)$? 1

e) For the curve $y = \frac{x}{e^x}$

(i) Find the stationary point of the curve and determine its nature. 2

(ii) Find any points of inflexion (Justify your answer) 2

(iii) Sketch the curve. 2

End of Paper

Question 1

A

C

x	1	2	3	4	5
$f(x)$	3	3^2	3^3	3^4	3^5

3 9 27 81 243

$\int_1^5 3^x\,dx \approx \frac{1}{2}\{3+4\cdot 9+2\cdot 27\}+\frac{1}{2}\{27+4\cdot 81+243\}$

$= 220$

$\text{Vol} = \pi\int_a^b [f(x)]^2\,dx$

$= \pi\int_0^2 \left(\frac{1}{1+x^2}\right)^2 dx$

x	0	1	2
$\left(\frac{1}{1+x^2}\right)^2$	1	$\frac{1}{4}$	$\frac{1}{25}$

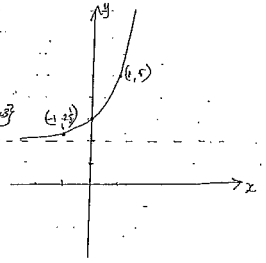
$h=1$

$\text{Vol} \approx \pi\left[\frac{1}{3}\{1+4\cdot\frac{1}{4}+\frac{1}{25}\}\right]$

$\approx \pi\cdot\frac{1}{3}\cdot 2\frac{1}{25}$

≈ 2.136

(a) $y = 3^x + 2$



domain all real x,

range $y > 2$

(a) B

(b) D

(c) (i) $y = e^{7x}$

$y' = 7e^{7x}$

(ii) $y = 4e^{3x^2+1} + 9e^{-x}$

$y' = 4e^{3x^2+1}\cdot 6x + 9e^{-x}\cdot -1$

$= 24xe^{3x^2+1} - 9e^{-x}$

(iii) $y = \ln(x^2-3x)$

$y' = \frac{1}{x^2-3x} \cdot 2x-3$

$= \frac{2x-3}{x^2-3x}$

(iv) $y = x\log x$

$y' = 1\cdot\log x + x\cdot\frac{1}{x}$

$= \log x + 1$

(v) $y = \frac{x}{x-1}$ $u = x$ $v = x-1$ $u' = 1$ $v' = 1$

$y' = \frac{u'v - v'u}{v^2}$

$= \frac{1(x-1) - 1(x)}{(x-1)^2}$

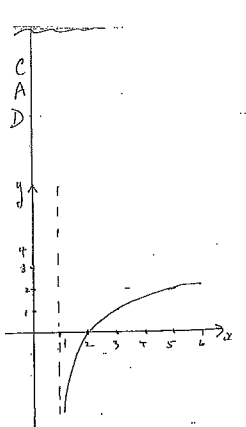
$= \frac{-1}{(x-1)^2}$

(vi) $y = \log\frac{2x+1}{x+1}$

$y = \ln(2x+1) - \ln(x+1)$

$y' = \frac{1}{2x+1}\cdot 2 - \frac{1}{x+1}$

$= \frac{2}{2x+1} - \frac{1}{x+1}$



domain $x > 1$

range all real y.

$y = \frac{x}{e^x}$

$y' = \frac{u'v - v'u}{v^2}$ $u = x$ $v = e^x$ $u' = 1$ $v' = e^x$

$= \frac{e^x - xe^x}{(e^x)^2}$

$= \frac{e^x(1-x)}{(e^x)^2}$

$y' = 0$ for stationary pt's.

$\frac{1-x}{e^x} = 0$

$\therefore 1-x = 0$

$x = 1$

$y'' = \frac{u'v - v'u}{v^2}$ $u = 1-x$ $v = e^x$ $u' = -1$ $v' = e^x$

$= \frac{-e^x - e^x(1-x)}{(e^x)^2}$

$= \frac{e^x(-1-1+x)}{(e^x)^2}$

$= \frac{x-2}{e^x}$ $x = 1$ $y'' = \frac{-1}{e^x}$

$\therefore$ neg. $\therefore$ maximum

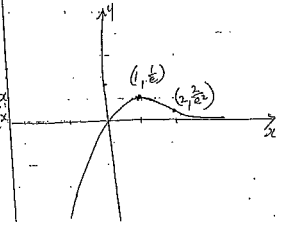
$\therefore \left(1, \frac{1}{e}\right)$ maximum

(ii) $y'' = 0$ for pt. of inflexion.

$\frac{x-2}{e^x} = 0$

$x = 2$

$\therefore$ pt. of inflexion $\left(2, \frac{2}{e^2}\right)$



Question 3.

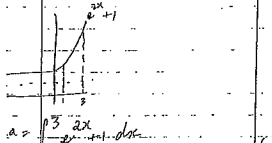
A

B

(i) $\int e^{5x}\,dx = \frac{1}{5}e^{5x} + C$

(ii) $\int e^{2-3x}\,dx = -\frac{1}{3}\int -3e^{2-3x}\,dx$

$= -\frac{1}{3}e^{2-3x} + C$



$a = \int_1^3 e^{2x}+1\,dx$

$= \frac{1}{2}e^{2x} + x\Big]_1^3$

$= \left(\frac{1}{2}e^{2\cdot 3} + 3\right) - \left(\frac{1}{2}e^2 + 1\right)$

$= \frac{1}{2}e^6 + 3 - \frac{1}{2}e^2 - 1$

$= \frac{1}{2}e^6 - \frac{1}{2}e^2 + 2$

$\text{Vol} = \pi\int_1^2 \left[4e^{-2x}\right]^2 dx$

$= \pi\int_1^2 16e^{-4x}\,dx$

$= \pi\left[\frac{16e^{-4x}}{-4}\right]_1^2$

$= \pi\left[-4e^{-4x}\right]_1^2$

$= \pi\left[-4e^{-8} - (-4e^{-4})\right]$

$= \pi\left[4e^{-4} - 4e^{-8}\right]$

$= \pi\left(\frac{4}{e^4} - \frac{4}{e^8}\right)$

$= \frac{4\pi}{e^8}(e^4 - 1)$

(b) $y = x^{2x}$ $x = e$ $y = e^2$ (e, e^2)

$y = 2x$

$x = e$ $y' = 2e$

$\therefore m = 2e$

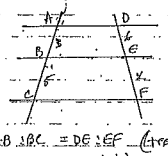
$y - y_1 = m(x - x_1)$

$y - e^2 = 2e(x - e)$

$y - e^2 = 2ex - 2e^2$

$y = 2ex - e^2$

(c)



AB : BC = DE : EF (transversals or parallel lines cut of intercepts in the same ratio)

$\therefore 3:5 = 6:x$

$\frac{3}{5} = \frac{6}{x}$

$\therefore 3x = 30$

$\therefore x = 10$

C Question 4

(a) D

(b) B

(c) (i) $\log_2 16 = \log_2 2^4$

$= 4\log_2 2$

$= 4$

(ii) $\log_3\frac{1}{27} = \log_3 3^{-3}$

$= -3\log_3 3$

$= -3$

(iii) $\log_a\sqrt{a} = \log_a a^{\frac{1}{2}}$

$= \frac{1}{2}\log_a a$

$= \frac{1}{2}$

(d) $\log_4(2x+1) = 3$

$2x+1 = 4^3$

$2x+1 = 64$

$2x = 63$

$x = \frac{63}{2}$

(e) $\log 3x + \log 10 = \log 81$

$\log(3x\times 10) = \log 81$

$\log 30x = \log 81$

$\therefore 30x = 81$

$x = \frac{81}{30}$

(f) let $\ln(3x+1) = y$

$\therefore e^{\ln(3x+1)} = e^y$

but $\ln(3x+1) = y$

$(3x+1) = e^y$

$\therefore e^{(\ln(3x+1))} = 3x+1$

(g) $25^{x+2} = 125^{2x}$

$(5^2)^{x+2} = (5^3)^{2x}$

$5^{2x+4} = 5^{6x}$

$\therefore 2x+4 = 6x$

$4 = 4x$

$\therefore x = 1$