

Trial Higher School Certificate Examination

2004



Mathematics

Extension 1

Total Marks – 84

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Attempt ALL questions.
- Begin each question on a new page
- Write your student number on each page
- All necessary working must be shown.
- Diagrams are not to scale.
- Board-approved calculators may be used.
- The mark allocated for each question is listed at the side of the question.

Question	Mark
Q1	/12
Q2	/12
Q3	/12
Q4	/12
Q5	/12
Q6	/12
Q7	/12
Total	/84

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

Question 1 – (12 marks) – Start a new page**Marks**

a) Differentiate the following:

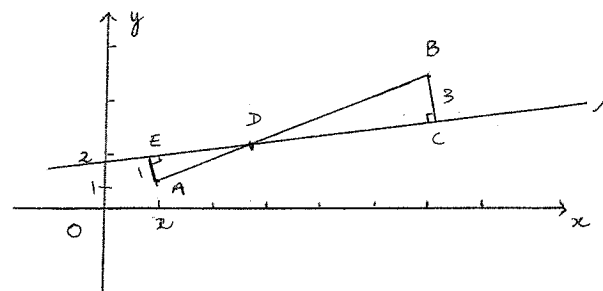
3

(i) $f(x) = \tan^{-1} 3x$

(ii) $y = \log \left(\frac{6-x^2}{1+2x} \right)$

b) Solve $\frac{3}{1-x} \geq 2$

c)



The points $A(2, 1)$ and $B(6, 5)$ are 1 unit and 3 units respectively from the line l and are on opposite sides of l .

Find the coordinates of the point where the interval AB crosses the line l .

2

d) Using the substitution $u = 2x + 1$ evaluate $\int_0^2 \frac{2x}{(2x+1)^2} dx$

4

Question 2 – (12 marks) – Start a new page

Marks

- a) Find the value of $\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos 2x}$ 3
- b) The graphs of $y = \frac{1}{x}$ and $y = x^3$ intersect at $x = 1$. Find the size of the acute angle between these curves at $x = 1$. 4
- c) Find the exact value of $\int_1^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}} dx$ 2
- d) Using the identity $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ solve the equation $\sin 3\theta = 2\sin\theta$ $0 \leq \theta \leq 2\pi$ 3

370

Question 3 – (12 marks) – Start a new page

Marks

- a) Find the term independent of x in the expansion of $\left(2x^2 - \frac{3}{x}\right)^{12}$ 3
- b) Taking $x = 0.5$ as the first approximation use Newton's method to find a second approximation to the root of $x - e^{-x} = 0$ 3
- c) Two points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$. 6
- (i) Show that the equation of the tangent to the parabola at P is $y = px - ap^2$
- (ii) The tangent at P and the line through Q parallel to the y -axis intersect at T . Find the coordinates of T .
- (iii) Write down the coordinates of M , the midpoint of PT .
- (iv) Determine the locus of M if $pq = -1$.

$$x^2 = 4ay$$

$$y = \frac{x^2}{4a}$$

$$\frac{dy}{dx} = \frac{2x}{4a} = \frac{x}{2a}$$

at $P = \frac{2ap}{2a}$

$$\therefore y - ap^2 = p(x - 2ap)$$

Question 4 – (12 marks) – Start a new page

Marks

a) Find the gradient of the tangent to $y = \cos^{-1} \frac{x}{3}$ at the point where $x = 0$. 2

b) The acceleration of a particle moving in a straight line is given by $\ddot{x} = -\frac{900}{x^3}$
where x metres is the displacement from the origin after t seconds.
Initially the particle is 10m to the right of the origin with velocity $3ms^{-1}$. 6

(i) Show that the velocity is given by $\dot{x} = \frac{30}{x}$

(ii) Find an expression for the time (t) as a function of x .

c) Prove by mathematical induction that 4

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

for all positive integral values of n
greater than or equal to 1.

Question 5 – (12 marks) – Start a new page

Marks

a) The polynomial $P(x) = x^3 + ax^2 + bx + c$ has real roots \sqrt{k} , $-\sqrt{k}$ and γ . 4

(i) Explain why $\gamma + a = 0$

(ii) Show that $k\gamma = c$

(iii) Show $ab = c$

b) Consider the function $f(x) = \frac{x}{x-3}$

(i) Show that $f'(x) < 0$ for all x in the domain. 2

(ii) State the equation of the horizontal asymptote. 1

(iii) Without using any further calculus sketch the graph of $y = f(x)$.
(You should show relevant intercepts and asymptotes). 2

(iv) Explain why $f(x)$ has an inverse function $f^{-1}(x)$ 1

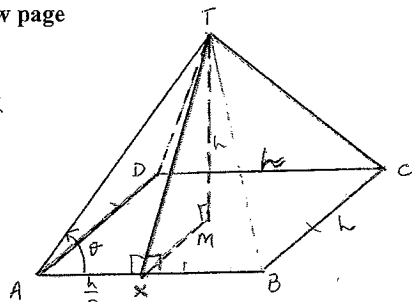
(v) Find an expression for $f^{-1}(x)$ 1

(vi) Write down the domain of $f^{-1}(x)$ 1

Question 6 – (12 marks) – Start a new page

a)

$$TA = \sqrt{\frac{k^2}{4} + \frac{5h^2}{4}}$$



Marks

3

The diagram shows a right square pyramid with base $ABCD$, vertex T and height TM . It is given that $TM = AB = h$ units. X is the midpoint of AB .

(i) Show the length of TX is $\frac{h}{2}\sqrt{5}$

(ii) Hence show that if $\widehat{TAB} = \theta$, then $\cos\theta = \frac{1}{\sqrt{6}}$

b) A saucepan of water at temperature $T^\circ\text{C}$ loses heat when placed in a cooler environment. It cools according to the law $\frac{dT}{dt} = k(T - T_0)$ where t is the time elapsed in minutes and T_0 is the temperature of the environment in degrees Celsius.

It is given that $T = T_0 + Ae^{kt}$.

$$T_0 = -10 \quad t = 8$$

(i) A saucepan of water at 100°C is placed in an environment at -10°C for 8 minutes, and cools to 70°C . Find k .

3

(ii) The saucepan of water is left in this environment for a further 8 minutes. Find its temperature after this time.

2

c) A cone-shaped candle whose height is three times its radius is melting at the constant rate of $0.5\text{cm}^3\text{s}^{-1}$.

4

If the proportion of radius to height is preserved as the candle burns:

(i) Show that the volume of the candle is given by $V = \frac{\pi h^3}{27}$

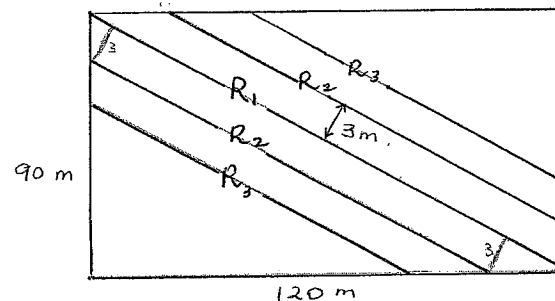
(ii) Find the rate at which the height of the candle is decreasing when the candle height is 12cm.

Question 7 – (12 marks) – Start a new page

Marks

a) A particular paddock in a vineyard measures 90m by 120m. In order to make best use of the sun the grape vines are planted in diagonal rows as shown, with a 3 metre gap between adjacent rows.

6



(i) Find the length of R_1 , the diagonal of the field.

(ii) Show that the length of the equal rows, R_2 is 143.75m.

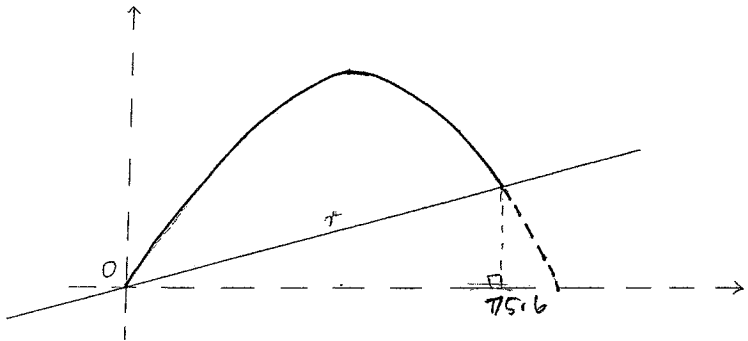
(iii) Given that the rows $R_1 + R_2 + R_3 + \dots$ form an arithmetic series find the total number of rows of vines in the paddock.

Question 7 continued on page 9

Question 7 (continued)

Marks

b) A golf ball is lying at point O on an inclined fairway as shown.



The golf ball is hit with an initial velocity of 30m/s at an angle of elevation of $\tan^{-1} \frac{4}{3}$.
 (You may assume that the acceleration due to gravity is 10m/s^2).

The golf ball's trajectory at time t seconds after being hit may be defined by the equations $x=18t$ and $y=24t-5t^2$, where x and y are the horizontal and vertical displacements, in metres, of the ball from the origin O shown in the diagram.

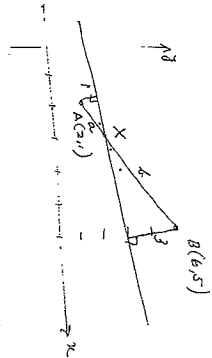
- (i) Find the horizontal range of the ball and its greatest height if it had been hit on a horizontal part of the golf course. 3
- (ii) If the fairway is as shown, inclined at an angle of $\tan^{-1} \frac{1}{6}$, show that the time of flight is 4.2 seconds and calculate the distance (r) the ball has been hit up the fairway (correct to 1 decimal place). 3

End of Paper

2004 30 SOLUTIONS TRIAL

Tony Q3

(c)



By similar triangles $\frac{x}{2} = \frac{y}{3}$ i.e. $x:y = 2:3$

Point X is $(\frac{1 \times 6 + 3 \times 2}{1+3}, \frac{1 \times 5 + 3 \times 1}{1+3})$

$= (3, 2)$

(d) $\int_0^3 \frac{2x}{(2x+1)^2} dx$ $u = 2x+1$
 $du = 2dx$

$= \int_1^{2.5} \frac{u-1}{u^2} \times \frac{du}{2}$

$= \frac{1}{2} \int_1^{2.5} (\frac{1}{u} - \frac{1}{u^2}) du$

$\int u^{-n} du = \frac{u^{-n+1}}{-n+1}$

$= \frac{1}{2} [(ln 5 + \frac{1}{2}) - (ln 1 + 1)]$

$= \frac{1}{2} [ln 5 - \frac{1}{2}]$

1) (i) $y = \tan^{-1} 3x$
 $\frac{dy}{dx} = \frac{1}{1+9x^2} \cdot 3$
 $= \frac{3}{1+9x^2}$ (1)

(ii) $y = \log(6-x^2) - \log(1+2x)$ (1)
 $\frac{dy}{dx} = \frac{-2x}{6-x^2} - \frac{2}{1+2x}$ (1)

$= \frac{-2x-4x^2-12+2x^2}{(6-x^2)(1+2x)}$
 $= \frac{-12-2x-2x^2}{(6-x^2)(1+2x)}$

2) $\frac{3}{1-x} \geq 2$ $x \neq 1$

$\Rightarrow 3(1-x) \geq 2(1-x)^2$

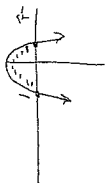
$0 \geq 2(1-x)^2 - 3(1-x)$

$0 \geq (1-x)[2(1-x)-3]$

$0 \geq (1-x)(-1-2x)$

$\Rightarrow -\frac{1}{2} \leq x \leq 1$

but $x \neq 1 \Rightarrow -\frac{1}{2} \leq x < 1$



QUESTION 3:

(a) $(2x^2 - \frac{3}{x})^{12}$

$T_{r+1} = {}^{12}C_r (2x^2)^{12-r} \cdot (-\frac{3}{x})^r$
 $= {}^{12}C_r \cdot 2^{12-r} \cdot (-3)^r \cdot x^{24-2r}$

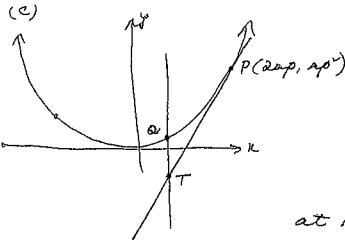
Independent of $x \rightarrow 24-2r=0$
 $\therefore r=12$

Hence term is $T_9 = {}^{12}C_9 \cdot 2^0 \cdot (-3)^8$
 $= 51963120$

(b) Let $f(x) = x - e^{-x} \Rightarrow f'(x) = 1 + e^{-x}$
 If $x_1 = 0.5$ is first approximation then

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$
 $= 0.5 - \frac{0.5 - e^{-0.5}}{1 + e^{-0.5}}$

$= 0.566\dots$
 $= 0.57$ (correct to 2 dec. places)



(i) $x = 4aq$
 $\Rightarrow y = \frac{x^2}{4a}$
 $\frac{dy}{dx} = \frac{2x}{4a}$
 $= \frac{x}{2a}$

at $P(2ap, ap^2) \frac{dy}{dx} = \frac{2ap}{2a}$

QUESTION 2:

(a) $\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos 2x}$
 $= \lim_{x \rightarrow 0} \frac{x^2}{2\sin^2 x}$

$\cos 2x = 1 - 2\sin^2 x$

$= \lim_{x \rightarrow 0} \frac{1}{2} \cdot \left(\frac{x}{\sin x}\right)^2$
 $= \frac{1}{2}$

(b) $y = \frac{1}{x}$
 $\Rightarrow y' = -\frac{1}{x^2}$
 at $x=1, y' = -1 = m_1$

$y = x^3$
 $y' = 3x^2$
 at $x=1, y' = 3 = m_2$

If θ is the acute angle then

$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$
 $= \left| \frac{-1 - 3}{1 + (-1)(3)} \right|$
 $= \left| \frac{-4}{-2} \right|$
 $= 2$

$\therefore \theta = 63^\circ 26'$

\therefore Tangent at P is

$y - ap^2 = p(x - 2ap)$
 $= px - 2ap^2$

is $y = px - ap^2$

(ii) Vertical through O is $x = 2aq$

$y = px - ap^2$
 $x = 2aq$

sub x in ①

$\Rightarrow y = p(2aq) - ap^2$
 $= 2apq - ap^2$

$\therefore T \equiv (2aq, 2apq - ap^2)$

(iii) $M \equiv \left(\frac{2ap + 2aq}{2}, \frac{2apq + ap^2 + ap^2}{2} \right)$

$= (a(p+q), apq)$

(iv) If $pq = -1$ then the locus of M is the line $y = -x$ since M will

then be $(a(p+q), -a)$ Must state locus is

~~hyperbola showing substitute~~

(c) $\int_1^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} = \sin^{-1} \left(\frac{x}{2} \right) \Big|_1^{\sqrt{3}}$
 $= \sin^{-1} \frac{\sqrt{3}}{2} - \sin^{-1} \frac{1}{2}$
 $= \frac{\pi}{3} - \frac{\pi}{6}$
 $= \frac{\pi}{6}$

(d) $\sin 3\theta = 2\sin \theta$
 $\Rightarrow 3\sin \theta - 4\sin^3 \theta = 2\sin \theta$
 $0 = 4\sin^3 \theta - \sin \theta$
 $= \sin \theta (4\sin^2 \theta - 1)$

$\Rightarrow \sin \theta = 0$ OR $4\sin^2 \theta - 1 = 0$
 $\theta = 0, \pi, 2\pi$ $\sin \theta = \pm \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$\therefore \theta = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$

and we aim to then prove it true for $n=k+1$

$$\text{i.e. } \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \dots + \frac{1}{(3k-2)(3k+1)} + \frac{1}{(3k+1)(3k+4)} = \frac{k+1}{3(k+1)+1}$$

$$\text{now } \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \dots + \frac{1}{(3k-2)(3k+1)} + \frac{1}{(3k+1)(3k+4)}$$

$$= \frac{k}{3k+1} + \frac{1}{(3k+1)(3k+4)} \quad \text{using } \textcircled{1}$$

$$= \frac{k(3k+4) + 1}{(3k+1)(3k+4)}$$

$$= \frac{3k^2 + 4k + 1}{(3k+1)(3k+4)}$$

$$= \frac{(k+1)(3k+1)}{(3k+1)(3k+4)}$$

$$= \frac{k+1}{3k+4}$$

$$= \frac{n}{3n+1} \quad \text{where } n=k+1$$

Hence, if the assertion is true for $n=k$ then it is also true for $n=k+1$

But since true for $n=1$, it must be true for $n=2$ and then by the principle of mathematical induction it is true for all integers $n \geq 1$

QUESTION 4:

$$(a) \quad y = \cos^{-1} \frac{x}{3}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1 - \frac{x^2}{9}}} \cdot \frac{1}{3}$$

$$\text{2. at } x=0 \quad \frac{dy}{dx} = \frac{-1}{1} \cdot \frac{1}{3} = -\frac{1}{3}$$

$$(b) \quad \ddot{x} = -\frac{900}{x^3}$$

$$(i) \quad \frac{d}{dx} \left(\frac{1}{2} v^2 \right) = -\frac{900}{x^3}$$

$$\therefore \frac{1}{2} v^2 = \int -\frac{900}{x^3} dx = -\frac{900x^{-2}}{-2} + C$$

$$= \frac{450}{x} + C$$

$$\therefore v^2 = \frac{900}{x} + C$$

$$\text{at } x=10 \quad v=3$$

$$\therefore 9 = 9 + C \Rightarrow C=0$$

$$\therefore v^2 = \frac{900}{x^2}$$

$$\text{i.e. } v = \pm \frac{30}{x}$$

3

but initially $v > 0$ and clearly v cannot be zero

$$\therefore v = \dot{x} = \frac{30}{x}$$

QUESTION 5:

$$(a) \quad p(x) = x^3 + ax^2 + bx + c \quad \text{Roots are } \sqrt{k}, -\sqrt{k}, r$$

$$(i) \quad \text{Sum of roots} \Rightarrow \sqrt{k} + (-\sqrt{k}) + r = -a$$

$$\therefore r + a = 0 \quad \text{--- } \textcircled{1}$$

$$(ii) \quad \text{Product of roots} \Rightarrow \sqrt{k} \times (-\sqrt{k}) \times r = -c$$

$$\therefore kr = c \quad \text{--- } \textcircled{2}$$

$$(iii) \quad \text{sum of roots 2 at time}$$

$$\Rightarrow -k + r\sqrt{k} - r\sqrt{k} = b$$

$$\therefore -k = b$$

$$\text{2. from } \textcircled{1}: \quad r = -a$$

$$\therefore \textcircled{2} \Rightarrow -ka = c$$

$$\text{but } -k = b$$

$$\Rightarrow ab = c$$

$$(b) \quad f(x) = \frac{x}{x-3} \quad x \neq 3$$

$$(i) \quad f'(x) = \frac{(x-3) \cdot 1 - x \cdot 1}{(x-3)^2}$$

$$= \frac{-3}{(x-3)^2}$$

2

$$< 0 \quad \text{for all } x \text{ since } (x-3)^2 > 0 \text{ for all } x \neq 3$$

QUESTION 4:

$$(a) \quad y = \cos^{-1} \frac{x}{3}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1 - \frac{x^2}{9}}} \cdot \frac{1}{3}$$

$$\text{2. at } x=0 \quad \frac{dy}{dx} = \frac{-1}{1} \cdot \frac{1}{3} = -\frac{1}{3}$$

$$(b) \quad \ddot{x} = -\frac{900}{x^3}$$

$$(i) \quad \frac{d}{dx} \left(\frac{1}{2} v^2 \right) = -\frac{900}{x^3}$$

$$\therefore \frac{1}{2} v^2 = \int -\frac{900}{x^3} dx = -\frac{900x^{-2}}{-2} + C$$

$$= \frac{450}{x} + C$$

$$\therefore v^2 = \frac{900}{x} + C$$

$$\text{at } x=10 \quad v=3$$

$$\therefore 9 = 9 + C \Rightarrow C=0$$

$$\therefore v^2 = \frac{900}{x^2}$$

$$\text{i.e. } v = \pm \frac{30}{x}$$

3

but initially $v > 0$ and clearly v cannot be zero

$$\therefore v = \dot{x} = \frac{30}{x}$$

$$(ii) \quad v = \frac{30}{x}$$

$$\Rightarrow \frac{dx}{dt} = \frac{30}{x}$$

$$\therefore \frac{dt}{dx} = \frac{x}{30}$$

$$t = \frac{x^2}{60} + C$$

$$\text{at } t=0, \quad x=10$$

$$\therefore 0 = \frac{100}{60} + C$$

$$\therefore C = -\frac{5}{3}$$

$$\text{3. } \therefore t = \frac{x^2}{60} - \frac{5}{3}$$

$$\text{when } x=100$$

$$t = \frac{100^2}{60} - \frac{5}{3}$$

$$= 165.5$$

$$\therefore \text{at } 165.5 \text{ seconds}$$

$$(c) \quad (i) \quad \text{Test for } n=1$$

$$\text{LHS} = \frac{1}{1 \cdot 4}$$

$$\text{RHS} = \frac{1}{4}$$

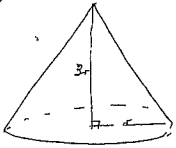
$$= \frac{1}{4}$$

$$\therefore \text{True for } n=1$$

$$(ii) \quad \text{Assume assertion is true for some integer } n=k$$

$$\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \dots + \frac{1}{(3k-2)(3k+1)} = \frac{k}{3k+1} \quad \text{--- } \textcircled{1}$$

(ii) at $t=16$ $T = -10 + 110e^{kt}$
 $= -10 + 110(e^{16k})$
 $= -10 + 110\left(\frac{8}{11}\right)^{16}$
 $= 48.18\dots$
 $= 48.2^\circ\text{C}$ (convert to 1 decpl)



$$\frac{dV}{dt} = 0.5 \text{ cm}^3/\text{s}$$

(i) $V = \frac{1}{3} \cdot \pi r^2 \cdot h$
 $= \frac{1}{3} \cdot \pi \cdot r^2 \cdot 3r$
 $= \pi r^3$ $r = \frac{R}{3}$
 $= \pi \left(\frac{R}{3}\right)^3$
 $= \frac{\pi R^3}{27}$

(ii) $\frac{dV}{dR} = \frac{\pi R^2}{9}$ $\frac{dV}{dt} = 0.5 \text{ cm}^3/\text{s}$

$$\frac{dR}{dt} = \frac{dR}{dV} \cdot \frac{dV}{dt} \text{ cm/s}$$

$$= \frac{9}{\pi R^2} \cdot 0.5 \text{ cm/s}$$

2 at $R=12$

$$\frac{dR}{dt} = \frac{9}{\pi \cdot 144} \times 0.5 \text{ cm/s}$$

$$= \frac{1}{32\pi} \text{ cm/s}$$

QUESTION 7:

(a) (i) $d^2 = 120^2 + 90^2$ where d is diagonal

1 $\therefore d = 150 \text{ m}$

(ii) $\tan \alpha = \frac{3}{4} \Rightarrow x=4$

$$\tan \beta = \frac{4}{3} \Rightarrow \frac{y}{3} = \frac{3}{y}$$

$$\therefore 4y = 9$$

$$y = \frac{9}{4}$$

2

$$\therefore \text{Diagonal} = 150 - 4 - \frac{9}{4}$$

$$= 143\frac{3}{4} \text{ m}$$

(iii) next line = $143\frac{3}{4} - 4 - \frac{9}{4}$
 $= 137.5 \text{ m}$

$$- 150 + 2 \left[143\frac{3}{4} + 137\frac{1}{2} + \dots \right]$$

arithmetic series
 $a = 143\frac{3}{4}, d = -6\frac{1}{4}$

3

$$T_n = 143\frac{3}{4} + (n-1) \cdot (-6\frac{1}{4})$$

$$= 150 - (6\frac{1}{4}) \cdot n$$

want $T_n > 0$
 $\therefore 150 - \frac{25n}{4} > 0$
 $150 > \frac{25n}{4}$
 $600 > 25n$
 $n < 24$
 \therefore There are 23 terms.

QUESTION 6:

a) i) In ΔTMX

$$TX^2 = XM^2 + TM^2$$

$$= \left(\frac{h}{2}\right)^2 + h^2$$

$$= \frac{h^2}{4} + h^2$$

$$= \frac{5h^2}{4}$$

$$\therefore T = \frac{h\sqrt{5}}{2}$$

ii) In ΔTXA

$$AT^2 = TX^2 + AX^2$$

$$= \frac{5h^2}{4} + \left(\frac{h}{2}\right)^2$$

$$= \frac{5h^2}{4} + \frac{h^2}{4}$$

$$= \frac{6h^2}{4}$$

$$AT = \frac{h\sqrt{6}}{2}$$

$$\cos \theta = \frac{AX}{AT}$$

$$= \frac{h}{2} \div \frac{h\sqrt{6}}{2}$$

$$= \frac{h}{2} \times \frac{2}{h\sqrt{6}}$$

$$= \frac{1}{\sqrt{6}}$$

b)

$$\frac{dT}{dt} = k(T - T_0)$$

(i) $\Rightarrow T = T_0 + Ae^{kt}$
 $T = -10 + Ae^{kt}$

at $t=0, T=100$

$$\therefore 100 = -10 + Ae^0$$

$$\therefore A = 110$$

ie $T = -10 + 110e^{kt}$

at $t=8, T=70$

$$\therefore 70 = -10 + 110e^{8k}$$

$$80 = 110e^{8k}$$

$$e^{8k} = \frac{8}{11}$$

$$8k = \ln\left(\frac{8}{11}\right)$$

$$k = \frac{1}{8} \ln\left(\frac{8}{11}\right)$$

(b) $x = 18t$ 15
 $y = 24t - 5t^2$

(i) $\dot{y} = 24 - 10t$
 max ht at $\dot{y} = 0$
 i.e. $24 - 10t = 0$
 $t = 2.4$

$\therefore y_{\max} = 24(2.4) - 5(2.4)^2$
 $= \underline{28.8 \text{ m}}$

Hits ground at $y = 0$

i.e. $24t - 5t^2 = 0$

$t(24 - 5t) = 0$

$t = \frac{24}{5}$

at $t = \frac{24}{5}$ $x = 18 \times \frac{24}{5}$

$= \underline{86.4 \text{ m}}$

(ii) Let ball be at $P(x, y)$

Then $\tan \alpha = \frac{y}{x} \Rightarrow \frac{y}{x} = \frac{1}{6}$

$\frac{24t - 5t^2}{18t} = \frac{1}{6}$

$144t - 30t^2 = 18t$

$0 = 30t^2 - 126t$

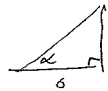
$0 = 3t(10t - 42)$

at P, $t = \frac{42}{10}$
 $= 4.2$

at $t = 4.2$, $x = 18 \times 4.2$
 $= 75.6$

now $\tan \alpha = \frac{1}{6}$

$\Rightarrow \cos \alpha = \frac{6}{\sqrt{37}}$



Then $\frac{6}{\sqrt{37}} = \frac{75.6}{r}$

$\therefore r = \frac{75.6 \times \sqrt{37}}{6}$

$= \underline{76.6 \text{ m}}$