St George Girls High School

Trial Higher School Certificate Examination

2006

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Write your student number on every booklet
- Begin each question in a new booklet
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks (120)

- Attempt Questions 1-10
- All questions are of equal value

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

Question 1 (12 marks) – **Start a new booklet** **Marks**

a) Find the value of $\dfrac{16.2}{84.7\times16.8+\sqrt{504.3}}$ correct to 3 significant figures 2

b) Write $\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$ in the form $a + b\sqrt{3}$ 2

c) Simplify $\dfrac{2x}{3}-\dfrac{x+2}{5}$ 2

d) Solve $|5-2x|\geq 3$ 2

e) Factorise $16a^4 - 1$ 2

f) Find the primitive of $x^2\sqrt{x}$ 2

Question 2 (12 marks) – **Start a new booklet** **Marks**

a) The points A(2, -2), B(-2, -3) and C(0, 2) are the vertices of a triangle ABC

(i) Plot these points on the number plane 1

(ii) Find the gradient of AC 1

(iii) Find the angle of inclination of AC to the positive x-axis, to the nearest degree 1

(iv) Show that the equation of AC is $2x + y - 2 = 0$ 1

(v) Calculate the perpendicular distance of B from the side AC 1

(vi) Hence find the area of ΔABC 2

(vii) Find the co-ordinates of D such that ABCD is a parallelogram 1

b) In ΔABC, AB = 2cm, $\angle$ABC = 105° and $\angle$BCA = 30°. Find the length of BC 2

c) For what values of k will x^2+3x+k be positive definite? 2

Question 3 (12 marks) – **Start a new booklet** **Marks**

a) Differentiate with respect to x:

(i) $(e^x + e^{-x})^3$ 2

(ii) $x^2 \sin 3x$ 2

b) (i) Find $\int \frac{3}{5x-3}\,dx$ 2

(ii) Evaluate $\int_0^{\frac{\pi}{4}} \sec^2 x\,dx$ 2

c) A ship sails from a Port A 50 nautical miles due east to a Port B. It then proceeds a distance of 20 nautical miles on a bearing of 020°T to a Port C.

(i) Find the distance of Port C from Port A (correct to 2 decimal places) 2

(ii) Find the bearing of Port C from Port A 2

Question 4 (12 marks) – **Start a new booklet** **Marks**

a) A circular sector AOB, centre O and radius 8cm, contains an angle of $\frac{3\pi}{4}$ radians at the centre. The straight edges OA and OB are joined to form a right circular cone.

Find the:

(i) exact length of the minor arc AB 1

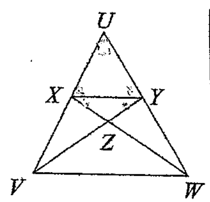
(ii) radius of the base of the cone 2

(iii) curved surface area of the cone 1

b) Explain why the curve $y=\frac{3x+5}{x+1}$ is always decreasing for all real values of x except $x=-1$ 3

c) Evaluate $\sum_{n=3}^{5}\frac{(-1)^n}{2n+1}$ 1

d) In the diagram below $\angle UXY = \angle UYX$ and XZ = YZ



Copy the diagram into your answer booklet.
Prove that $\Delta UVY \equiv \Delta UWX$ 4

Question 5 (12 marks) – **Start a new booklet** **Marks**

a) (i) Copy and complete the table of values for $y=\frac{1}{x+2}$ in your writing booklet 1

x	0	0.5	1	1.5	2
y					

(ii) By using Simpson's rule with 5 function values, estimate the value of the integral $\int_0^2 \frac{1}{x+2}\,dx$ 3

b) The rate at which liquid is flowing into a vessel after t minutes is given by $\frac{dV}{dt}=\frac{1}{t+1}$

(i) If $(\log_e 2)$ m³ of liquid flows into the vessel after 3 minutes, find an expression for $V(t)$, the volume of water in the tank at time t minutes. 2

(ii) Find the volume of the liquid in the vessel after 8 minutes. Give your answer in exact simplest form 2

c) During the first week in January a car dealer sold 15 cars. In the second week he sold 18 cars, and in each succeeding week after that he sold 3 more cars than he sold the previous week.

(i) How many cars did he sell during the last week in December? 2

(ii) Calculate the total number of cars he sold during the year. 2

Question 6 (12 marks) – **Start a new booklet** **Marks**

a) (i) For what values of x will a limiting sum exist for the geometric series $1 - 3x + 9x^2 - \ldots$? 2

(ii) Find the value of x for which the limiting sum is $\frac{4}{5}$ 2

b) The price of one tonne of copper, \$P, was studied over a period of t years.

(i) Throughout the period of study $\frac{dP}{dt} > 0$.
What does this say about the price of copper? 1

(ii) It was also observed that the rate of change of the price of copper decreased over the period of study. What does this statement imply about $\frac{d^2P}{dt^2}$? 1

c) (i) Sketch the parabola which has focus (1, -5) and directrix $y = 1$, showing the co-ordinates of the vertex. 3

(ii) Write down the equation of the parabola 1

d) Find the equation of the tangent to the curve $y = e^{2x-3}$ at the point where $x = \frac{3}{2}$ 2

Question 7 (12 marks) – **Start a new booklet** **Marks**

a) (i) Find the co-ordinates of the points where the curves $y=\sqrt{x}$ and $y=x^2$ intersect 2

(ii) Sketch $y=\sqrt{x}$ and $y=x^2$ on the same axes 2

(iii) Find the volume of the solid generated by rotating the region enclosed between the curves $y=\sqrt{x}$ and $y=x^2$ about the y-axis 3

b) Sue contributes \$1 200 every year, for 20 years, to a superannuation account which pays 9%p.a. compounded monthly.

(i) Show that her total investment at the end of 2 years can be represented by : 2

$$1\,200(1.0075)^{24} + 1\,200(1.0075)^{12}$$

(ii) Hence, find the value of her investment at the end of 20 years. 3

Question 8 (12 marks) – **Start a new booklet** **Mark**

a) The population P of a town at the end of t years is given by $P = Ae^{kt}$, where A and k are constants. After 1 year the population is 1 020.

(i) Find the value of A if the initial population was 1 000 1

(ii) Find the value of k 2

(iii) Calculate the population after 10 years 1

(iv) How many years will it take the population of the town to double? 2

b) (i) Show that $x = \frac{\pi}{8}$ is a solution to $y = 3\sin 2x$ and $y = 3\cos 2x$ 1

(ii) On the same axes sketch $y = 3\sin 2x$ and $y = 3\cos 2x$ for $0 \le x \le \pi$ 2

(iii) Find the area bounded by the curves $y = 3\sin 2x$, $y = 3\cos 2x$ and the x-axis in the domain $0 \le x \le \frac{\pi}{4}$ 3

Question 9 (12 marks) – **Start a new booklet** **Marks**

a) Joe deposited \$20 000 at the beginning of January into an account which paid interest at the rate of $\frac{1}{2}\%$ per month compounded monthly. He withdrew \$50 each month from the account immediately after the interest was paid.

(i) How much money did Joe have in the account after making the first withdrawal? 1

(ii) Show that after making the nth withdrawal, his balance in the account is given by the expression: 3

$$10\,000 \times 1.005^n + 10\,000$$

(iii) Find the minimum number of withdrawals needed for his account balance to show at least \$50 000 2

b) (i) State the condition necessary for a quadratic equation to have rational roots 1

(ii) Prove that the equation $3px^2 = 2px + 3qx - 2q$, where p and q are rational, has rational roots for all values of p and q 4

(iii) What can be concluded about the number of the roots if $p = \frac{3q}{2}$? 1

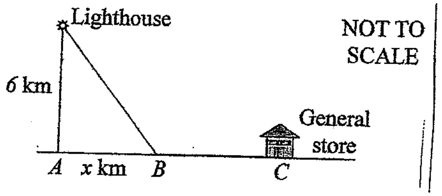
Question 10 (12 marks) – **Start a new booklet** **Marks**

a) (i) Given that the curve $y=x^2e^x$ has turning points at (0, 0) and $(-2, \frac{4}{e^2})$, determine their nature using the first derivative 3

(ii) Discuss the behaviour of the curve for large positive and large negative values of x 1

(iii) Sketch the curve $y=x^2e^x$ 1

b)



The water's edge is a straight line ABC which runs east-west. A lighthouse is 6km due north of A.

10km due east of A is the general store. To get to the general store as quickly as possible, the lighthouse keeper rows to a point B, $x$km from A, and then jogs to the general store. The lighthouse keeper's rowing speed is 6km/h and his jogging speed is 10km/h.

(i) Show that it takes the lighthouse keeper $\frac{\sqrt{36+x^2}}{6}$ hours to row from the lighthouse to B 2

(ii) Show that the total time taken for the lighthouse keeper to reach the general store is given by 1

$$T = \frac{\sqrt{36+x^2}}{6} + \frac{10-x}{10} \text{ hours}$$

(iii) Hence, show that when $x=4\frac{1}{2}$ km the time it takes for the lighthouse keeper to travel from the lighthouse to the general store is a minimum 3

(iv) Hence find the quickest time it takes the lighthouse keeper to go to the general store from the lighthouse. Give your answer correct to the nearest minute. 1

End of Paper

MATHEMATICS TRIAL SOLUTIONS 2006.

1a) $0\cdot011207....$

$= 0\cdot0112$ (to 3 sig. figs)

b) $\frac{\sqrt3+1}{\sqrt3-1} \times \frac{\sqrt3+1}{\sqrt3+1} = \frac{(\sqrt3+1)^2}{3-1}$

$= \frac{3+2\sqrt3+1}{2}$

$= \frac{4+2\sqrt3}{2}$

$= 2+\sqrt3$

c) $\frac{2x}{3} - \frac{x+2}{5} = \frac{10x}{15} - \frac{3(x+2)}{15}$

$= \frac{10x-3x-6}{15}$

$= \frac{7x-6}{15}$

d) $|5-2x| \geq 3$

$5-2x \geq 3$ $\quad$ $5-2x \leq -3$

$2 \geq 2x$ $\quad$ $8 \leq 2x$

$x \leq 1$ $\quad$ $4 \leq x$

$\therefore x \geq 4$

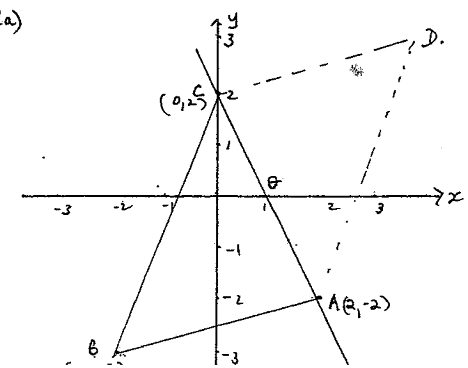
e) $16a^4-1 = (4a^2-1)(4a^2+1)$

$= (2a-1)(2a+1)(4a^2+1)$

f) $\int x^2\sqrt{x}\,dx = \int x^{5/2}\,dx$

$= \frac{2}{7}x^{7/2} + c$

2a)



ii) $m_{AC} = \frac{2--2}{0-2}$

$= -2$

iii) let θ be angle of inclination

$\therefore \tan\theta = -2$

$\therefore \theta = 117^\circ$ (to nrst degree)

iv) $y = mx+b$

$y = -2x+2$

$\therefore 2x+y-2=0$

v) $d = \frac{|-2\times2 - 3\times1 - 2|}{\sqrt{2^2+1^2}}$

$= \frac{9}{\sqrt5}$

vi) Area of $\Delta ABC = \frac12 \times AC \times \frac{10}{\sqrt5}$

length $AC = \sqrt{(2+2)^2+(0-2)^2}$

$= \sqrt{20}$

$\therefore$ area $= \frac12 \times \sqrt{20} \times \frac{9}{\sqrt5}$

$= 9$ sq. units.

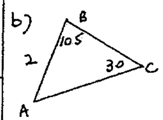
vii) midpt $AC = (1,0)$

$\therefore$ midpt $BD = (1,0)$

$\therefore D = (1+3, 0+3)$

$= (4,3)$

b)



$\frac{BC}{\sin45} = \frac{2}{\sin30}$

$\therefore BC = \frac{2\sin45}{\sin30}$

$= \frac{2\times\frac{1}{\sqrt2}}{1/2}$

$= \frac{4}{\sqrt2} \doteqdot 2.83$ (to 2dp)

c) x^2+3x+k

For positive definite

$a>0$ and $\Delta<0$

$a=1$ $\quad$ $b^2-4ac<0$

$\therefore a>0$ $\quad$ $9-4\times1\times k<0$

$9<4k$

3 a) i) $y = (e^{x} + e^{-x})^3$

$y' = 3(e^{x} + e^{-x})^2 (e^{x} - e^{-x})$

ii) $y = x^2 \sin 3x$

$y' = (\sin 3x)\,2x + x^2\, 3\cos 3x$

$= 2x\sin 3x + 3x^2 \cos 3x$

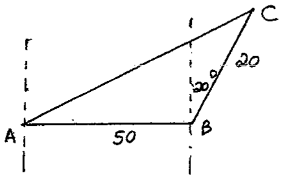
b) i) $\int \frac{3}{5x-3}\,dx = \frac{3}{5}\ln(5x-3) + c$

ii) $\int_0^{\frac{\pi}{4}} \sec^2 x\,dx = [\tan x]_0^{\frac{\pi}{4}}$

$= \tan\frac{\pi}{4} - \tan 0$

$= 1$

c)



i) $AC^2 = 50^2 + 20^2 - 2\times 50\times 20\times \cos 110$

$AC = 59.87$ nm. (to 2dp)

ii) $\frac{20}{\sin C\hat{A}B} = \frac{59.87}{\sin 110}$

$\therefore \sin C\hat{A}B = \frac{20\sin 110}{59.87}$

$C\hat{A}B = 18^0\, 18'$

$\therefore$ Bearing of C from A $= 90 - 18^\circ 18'$

$= 71^\circ 42'$

4. a) i) $l = 8\times \frac{3\pi}{4}$

$\therefore$ arc AB $= 6\pi$

ii) $2\pi r = 6\pi$

$\therefore r = 3$.

iii) C.S.A. $= \pi r l$

$= \pi \times 3\times 8$

b) $y = \frac{3x+5}{x+1}$

$y' = \frac{(x+1)\times 3 - (3x+5)\times 1}{(x+1)^2}$

$= \frac{-2}{(x+1)^2}$

If curve is decreasing $y' < 0$

Since $(x+1)^2 > 0$ for all x except $x=-1$

and $-2 < 0$

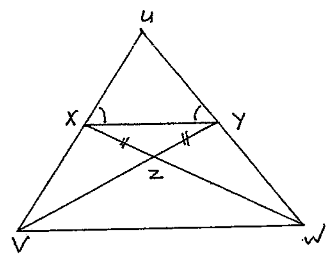
$\therefore \frac{-2}{(x+1)^2} < 0$ for all x except $x=-1$

$\therefore$ Curve is decreasing for all x except $x=-1$.

c) $\sum_{n=3}^{5} \frac{(-1)^n}{2n+1} = \frac{-1}{7} + \frac{1}{9} - \frac{1}{11}$

$= \frac{-85}{693}$

d)



XU = YU (In triangle, sides opposite equal angles are equal)

$Z\hat{X}Y = Z\hat{Y}X$ (In triangle, angles opposite equal sides are equal)

$U\hat{X}Y = U\hat{Y}X$ (given)

$\therefore U\hat{X}Z = U\hat{Y}Z$ Since $U\hat{X}W = U\hat{X}Y + Y\hat{X}Z$
and $U\hat{Y}V = U\hat{Y}X + X\hat{Y}Z$

In ΔUYZ, ΔUWZ

1. $V\hat{U}W$ is common
2. $U\hat{X}W = U\hat{Y}V$ (proven above)
3. XU = YU (proven above)

$\therefore \Delta UVY \equiv \Delta UWX$ (by A.A.S. test)

5 a) i)

x	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2
y	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{2}{7}$	$\frac{1}{4}$

$y = \frac{1}{x+2}$

ii) $\int_0^2 \frac{1}{x+2}\,dx$

$= \frac{1-0}{6}\left[\frac{1}{2} + \frac{2}{5}\times 4 + \frac{1}{3}\right] + \frac{2-1}{6}\left[\frac{1}{3} + 4\times\frac{2}{7} + \frac{1}{4}\right]$

$= \frac{1747}{2520}$

$= 0.693$ (to 3 dp)

b) $\frac{dV}{dt} = \frac{1}{t+1}$

i) $V = \int \frac{1}{t+1}\,dt$

$V = \ln(t+1) + c$

$t=3, V = \ln 2$: $\ln 2 = \ln 4 + c$

$\ln\frac{1}{2} = c$

$\therefore V = \ln(t+1) + \ln\frac{1}{2}$

ii) $t=8$: $V = \ln 9 + \ln\frac{1}{2}$

$= \ln(9\times\frac{1}{2})$

$= \ln 4.5$

c) Sequence 15, 18, 21, ...

Arithmetic where $a = 15$, $d = 3$

(i) $T_{52} = a + 51d$

$= 15 + 51\times 3$

$= 168$

$\therefore$ 168 cars sold during last week of December.

(ii) $S_{52} = \frac{52}{2}(15 + 168)$

$= 4758$

$\therefore$ Total number of cars sold for year is 4758

6. a) i) For S_∞ to exist $-1 < r < 1$

$1 - 3x + 9x^2 \ldots$

$r = -3x$

$\therefore -1 < -3x < 1$

$\frac{1}{3} > x > -\frac{1}{3}$

i.e. $-\frac{1}{3} < x < \frac{1}{3}$

ii) $\frac{4}{5} = \frac{1}{1+3x}$ $\qquad S_\infty = \frac{a}{1-r}$

$4 + 12x = 5$

$12x = 1$

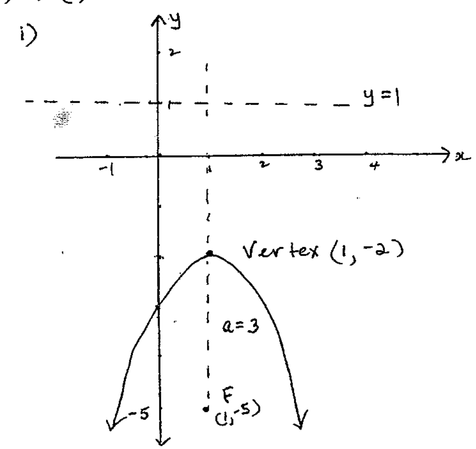
$x = \frac{1}{12}$

b) i) price of copper was rising

ii) $\frac{d^2p}{dt^2} < 0$

c) F(1, −5) $\qquad$ D: $y = 1$

i)



ii) $4a(y+2) = (x-1)^2$

$a = 3$ $\qquad \therefore (x-1)^2 = -12(y+2)$

d) $y = e^{2x-3}$

$y' = 2e^{2x-3}$

m at $x = \frac{3}{2}$: $m = 2e^0$

$= 2$

$x = \frac{3}{2}$: $y = e^0 = 1$

$\therefore$ Equ of tangent: $y - 1 = 2(x - \frac{3}{2})$

$y - 1 = 2x - 3$

$y = 2x - 2$

7. a) i) $\sqrt{x} = x^2$

$x = x^4$

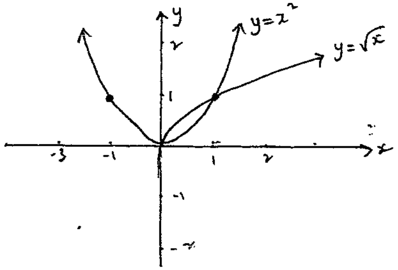
$x^4 - x = 0$

$x^3(x-1) = 0$

$x = 0$ or $x = 1$

$y = 0$ $\quad y = 1$

ii)



iii) $y = \sqrt{x}$ $\qquad y = x^2$

$y^2 = x$

$y^4 = x^2$

$Vol_y = \pi\int_0^1 y - y^4 \, dy$

$= \pi\left[\frac{y^2}{2} - \frac{y^5}{5}\right]_0^1$

$= \pi\left[\left(\frac{1}{2} - \frac{1}{5}\right) - 0\right]$

$= \frac{3\pi}{10}$ cu. units

b) i) 9% pa = 0.0075 per month

Value of first \$1200 is $1200(1.0075)^{24}$

Value of second \$1200 is $1200(1.0075)^{12}$

∴ Total value at end of 2years $= 1200(1.0075)^{24} + 12(1.0075)^{12}$

ii) Total value after 20yrs $= 1200(1.0075)^{12} + \dots + 1200(1.0075)^{240}$

$= 1200\underbrace{(1.0075^{12} + \dots + 1.0075^{240})}$

Sum of geometric series

where $a = 1.0075^{12}$

$r = 1.0075^{12}$

$n = 20$

∴ Value $= 1200 \times \frac{1.0075^{12}(1.0075^{240} - 1)}{1.0075^{12} - 1}$

= \$70 089.23

8a) $P = Ae^{kt}$

i) $t=0$, $P = 1000$

∴ $1000 = Ae^0$

∴ $A = 1000$

ii) $P = 1000e^{kt}$

$t=1$, $P=1020$: $1020 = 1000e^k$

$\frac{1020}{1000} = e^k$

$\ln\frac{1020}{1000} = k$

∴ $k = 0.0198$ (to 4dp)

iii) $t = 10$: $P = 1000e^{10k}$

$= 1000e^{0.198}$

$P = 669.967\dots$

∴ population is 670 (to nrst person)

iv) $2000 = 1000e^{kt}$

$2 = e^{kt}$

$\frac{\ln 2}{k} = t$

∴ $t = 35.007\dots$

∴ town's population doubles in about 35yrs.

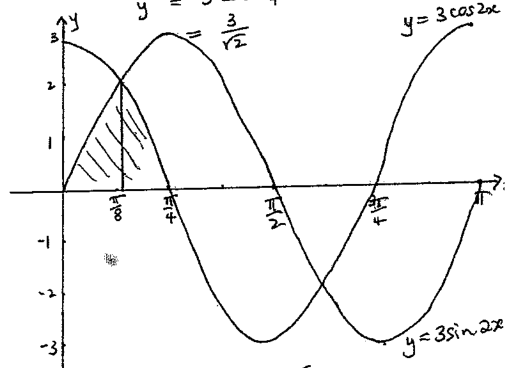
b) i) $x = \frac{\pi}{8}$: $y = 3\sin\frac{\pi}{4}$

$= \frac{3}{\sqrt{2}}$

$y = 3\cos\frac{\pi}{4}$

$= \frac{3}{\sqrt{2}}$

ii)



iii) Area $= \int_0^{\frac{\pi}{8}} 3\sin 2x \, dx + \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} 3\cos 2x \, dx$

$= -\frac{3}{2}\left[\cos 2x\right]_0^{\frac{\pi}{8}} + \frac{3}{2}\left[\sin 2x\right]_{\frac{\pi}{8}}^{\frac{\pi}{4}}$

$= -\frac{3}{2}\left(\frac{1}{\sqrt{2}} - 1\right) + \frac{3}{2}\left[1 - \frac{1}{\sqrt{2}}\right]$

9. i) Amount $= 20000(1.005) - 50$

$= 20050$

ii) Let A_n be the amount in the account after the nth withdrawal

$A_1 = 20000(1.005) - 50$

$A_2 = [20000(1.005) - 50] \times 1.005 - 50$

$= 20000(1.005)^2 - 50 \times 1.005 - 50$

$= 20000(1.005)^2 - 50(1 + 1.005)$

Similarly

$A_n = 20000(1.005)^n - 50(\underbrace{1 + 1.005 + \dots + 1.005^{n-1}})$

$= 20000(1.005)^n - 50 \times$ sum of geom series.

Sum $= \frac{a(r^n - 1)}{r - 1}$

$= \frac{1 \times (1.005^n - 1)}{1.005 - 1}$

$= \frac{1.005^n - 1}{0.005}$

$= 200(1.005^n - 1)$

∴ $A_n = 20000(1.005)^n - 50 \times 200(1.005^n - 1)$

$= 20000(1.005)^n - 10000(1.005^n - 1)$

$= 20000(1.005^n) - 10000 \times 1.005^n + 10000$

$= 10000(1.005^n) + 10000$

iii) If $A_n = 50000$ $n = ?$

$50000 = 10000(1.005^n) + 10000$

$\frac{40000}{10000} = 1.005^n$

$4 = 1.005^n$

$\ln 4 = n \ln 1.005$

$n = \frac{\ln 4}{\ln 1.005}$

$n = 277.95\dots$

∴ min. number of withdrawals is 278.

b) i) $b^2 - 4ac \geq 0$ and $b^2 - 4ac$ is a perfect square.

ii) $3px^2 = 2px + 3qx - 2q$

$3px^2 - (2p + 3q)x + 2q = 0$

$b^2 - 4ac = [-(2p+3q)]^2 - 4 \times 3p \times 2q$

$= 4p^2 + 12pq + 9q^2 - 24pq$

$= 4p^2 - 12pq + 9q^2$

$= (2p - 3q)^2$

Since $b^2 - 4ac \geq 0$ and a perfect square roots will be rational if p and q are rational.

(iii) If $p = \frac{3q}{2}$

then $b^2 - 4ac = 0$

∴ there is only one rational root.

10 a) i) $y = x^2 e^x$

$y' = e^x \times 2x + x^2 \times e^x$

$= xe^x(2 + x)$

For point $(0,0)$

$x = -\frac{1}{2}$: $y' = -\frac{1}{2} \times e^{-1/2}\left(2 - \frac{1}{2}\right)$

$\doteqdot -0.45..$

$x = \frac{1}{2}$: $y' = \frac{1}{2} e^{1/2}\left(2 + \frac{1}{2}\right)$

$\doteqdot 2.06..$

$- \quad 0 \quad +$ ∴ $(0,0)$ is a minvmum turning point

For $\left(-2, \frac{4}{e^2}\right)$

$x = -2\frac{1}{2}$: $y' = -2e^{-2}\left(2 - 2\frac{1}{2}\right)$

$\doteqdot 0.135...$

$x = -1\frac{1}{2}$: $y' = -1\frac{1}{2} e^{-1\frac{1}{2}}\left(2 - 1\frac{1}{2}\right)$

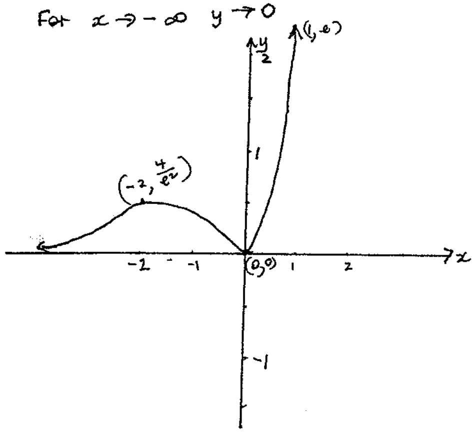
$\doteqdot -0.167$

$+ \quad 0 \quad -$ ∴ $\left(-2, \frac{4}{e^2}\right)$ is a maximum turning point.

ii) $y = x^2 e^x$

For $x \to \infty$ $y \to \infty$

For $x \to -\infty$ $y \to 0$



b) i) Distance from lighthouse to B $= \sqrt{6^2 + x^2}$

$= \sqrt{36 + x^2}$

Time $= \frac{\text{Distance}}{\text{Speed}}$

∴ Time from Lighthouse to B $= \frac{\sqrt{36+x^2}}{6}$

ii) Distance from B to C $= 10 - x$

∴ time $= \frac{10-x}{10}$

∴ Total time for journey: $T = \frac{\sqrt{36+x^2}}{6} + \frac{10-x}{10}$

iii) For minimum $\frac{dT}{dx} = 0$ and $\frac{d^2T}{dx^2} > 0$

$T = \frac{\sqrt{36+x^2}}{6} + \frac{10-x}{10}$

$\frac{dT}{dx} = \frac{\frac{1}{2}(36+x^2)^{-1/2} \times 2x}{6} - \frac{1}{10}$

$= \frac{x}{6\sqrt{36+x^2}} - \frac{1}{10}$

For minimum let $\frac{dT}{dx} = 0$.

$\frac{x}{6\sqrt{36+x^2}} = \frac{1}{10}$.

$10x = 6\sqrt{36+x^2}$

$100x^2 = 36(36 + x^2)$

$100x^2 = 1296 + 36x^2$

$64x^2 = 1296$

$x^2 = 20\frac{1}{4}$

$x = \pm 4.5$

Since x represents distance $x > 0$

∴ $x = 4.5$

To prove a minimum use sign change in $\frac{dT}{dx}$

$x = 4$: $\frac{dT}{dx} = \frac{4}{6\sqrt{52}} - \frac{1}{10}$

$\doteqdot -7.5$

$x = 5$: $\frac{dT}{dx} = \frac{5}{6\sqrt{61}} - \frac{1}{10}$

$\doteqdot 6.7$

Since $\frac{dT}{dx}$: $- \quad 0 \quad +$ ∴ $x = 4.5$ will give minimum time

iv) $x = 4\frac{1}{2}$: $T = \sqrt{36 + 4.5^2} + \frac{10 - 4\frac{1}{2}}{10}$