St George Girls High School

Trial Higher School Certificate Examination

2005

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Write your student number on every booklet
- Begin each question in a new booklet
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks (120)

- Attempt Questions 1-10
- All questions are of equal value

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

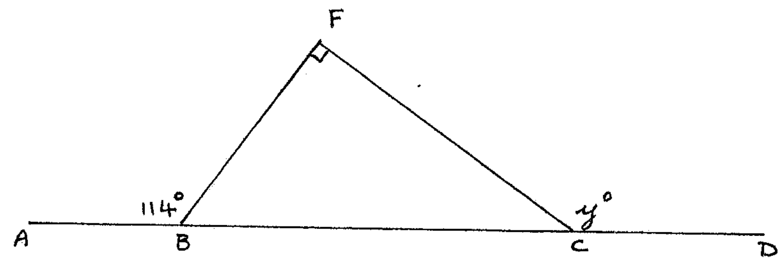
Question 1 (12 marks) – **Start a new booklet** **Marks**

a) Expand and simplify $\left(\sqrt{3}-2\right)^2$ 2

b) Find the value of $\dfrac{25.3}{56.1\times\sqrt{29.02}}$ correct to 3 significant figures 2

c) Solve the equation $9x^2 = x$ 2

d)



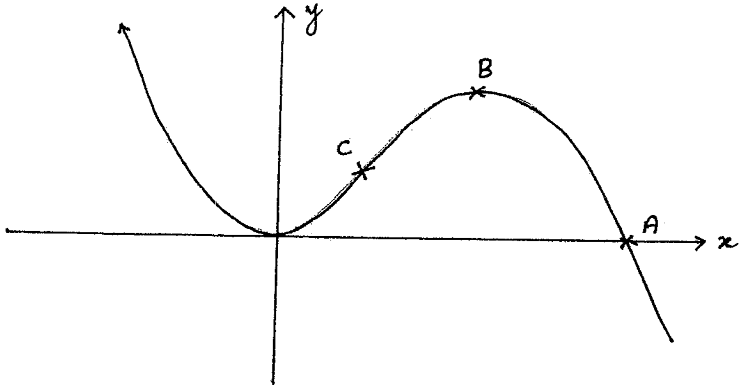
In the diagram $\angle ABF = 114^\circ$ and $\angle BFC = 90^\circ$. Find the value of y. Give all reasons. 2

e) Differentiate $\tan\dfrac{x}{2}$ with respect to x 1

f) Sketch the parabola $x^2 = -4y + 8$ showing its focus and directrix. 3

Question 2 (12 marks) – **Start a new booklet** **Marks**

a)



The graph represents the function $y = 3x^2 - x^3$

The point A is the x-intercept.

The point B is a maximum turning point.

The point C is a point of inflexion.

Find:

(i) the coordinates of A 1

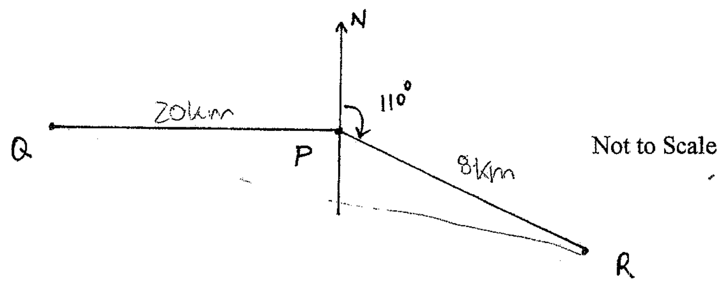
(ii) the coordinates of B 2

(iii) the coordinates of C 2

Question 2b) and c) on next page

Question 2 (cont'd) **Marks**

b)



The diagram shows a point P which is 20km due east of the point Q. The point R is 8km from P and has a bearing from P of 110°.

(i) Find the distance of R from Q. 2

(ii) Find the bearing of R from Q. 2

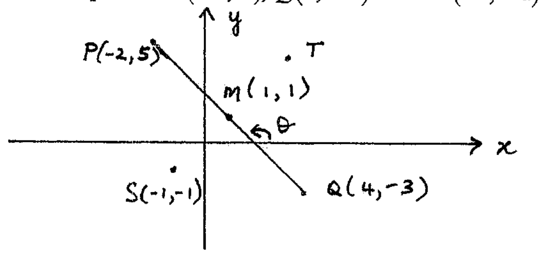
c) The decimal $0 \cdot \dot{5}\dot{4}$ (i.e. $0 \cdot 545454\ldots$) can be considered as a geometric series.

(i) What is the value of the first term, a, and the common ratio, r. 2

(ii) Hence or otherwise express $0 \cdot \dot{5}\dot{4}$ as a fraction in simplest form. 1

Question 3 (12 marks) – **Start a new booklet** **Marks**

The diagram shows the points $P(-2, 5)$, $Q(4, -3)$ and $S(-1, -1)$



You are given that $M(1, 1)$ is the midpoint of PQ.

a) (i) Find the coordinates of T so that M is the midpoint of ST. 1

(ii) Without any further calculations explain why $PSQT$ is a parallelogram. 1

(iii) Find the size of θ to the nearest degree. 1

(iv) Show that the equation of PQ is $4x+3y-7=0$ 1

(v) Find the perpendicular distance from S to the line PQ. 2

(vi) Deduce the area of $PSQT$. 2

(vii) Shade the region inside the quadrilateral for which $4x+3y-7>0$ 1

b) A particle moves in a straight line so that its displacement x in metres, at time t seconds is given by $x = t^3 - 6t^2$ 3

(i) At which times is the particle at rest?

(ii) How far does the particle travel between these times?

Question 4 (12 marks) – **Start a new booklet** **Marks**

a) Differentiate with respect to x

(i) $(1+\log_e x)^3$ 2

(ii) xe^{x^2} 2

b) (i) Evaluate $\int_0^1 e^{2x}\,dx$ 2

(ii) Find $\int \frac{x^2}{x^3+1}\,dx$ 2

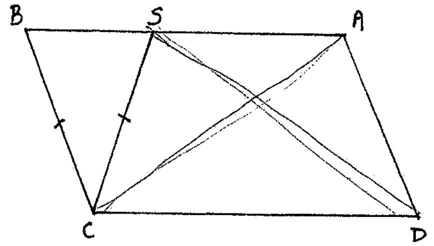
c) Simona wishes to invest $\$A$ at the beginning of each month at a compound interest rate of 0.5% per month for 3 years. This means she makes 36 monthly investments.

(i) Find an expression for the value of the first $\$A$ invested, at the end of the 3 years. 1

(ii) If Simona wishes to save \$30 000 as a house deposit in this time, find how much each monthly investment should be in order to save this amount. 3

Question 5 (12 marks) – **Start a new booklet** **Marks**

a)



ABCD is a parallelogram
$CS = CB$

Copy the diagram into your writing booklet.

(i) Explain why $\angle CSB = \angle CBS$ 1

(ii) Prove that $\angle SCD = \angle ADC$ 2

(iii) Prove that $\Delta SCD \equiv \Delta ADC$ 3

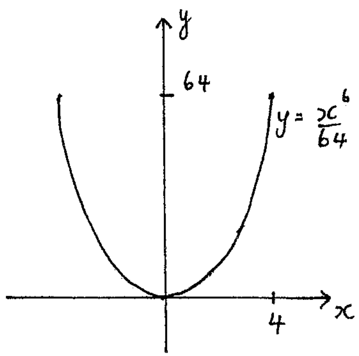
b)

x	1	3	5
$x\ln x$	0	3.296	8.047

The table shows the values of $x\ln x$ for 3 values of x.

Find an approximate value for $\int_1^5 x\ln x\,dx$ using Simpson's rule with the three function values in the table. Express your answer correct to 2 decimal places. 2

c)



A bowl is formed by rotating the part of the curve $y = \dfrac{x^6}{64}$ between $x = 0$ and $x = 4$ about the y-axis.

(i) Show that $x^2 = 4y^{\frac{1}{3}}$ 1

(ii) Find the volume of the bowl. 3

Question 6 (12 marks) – **Start a new booklet** **Marks**

a) (i) On the same axes sketch $y = x^2 + 6$ and $y = 12 - x$ 2

(ii) Find the area in the first quadrant bounded by the y-axis, $y = x^2 + 6$ and $y = 12 - x$ 3

b) A jet engine uses fuel at a rate of R litres per minute.

The rate of fuel used t minutes after the engine starts is given by $R = 10 + \dfrac{15}{1+t}$

(i) What is R when $t = 0$? 1

(ii) What is R when $t = 14$? 1

(iii) What value does R approach as t becomes very large? 1

(iv) Draw a sketch of R as a function of t. 2

(v) Calculate the total amount of fuel burned during the first 14 minutes. Give your answer to the nearest litre. 2

Question 7 (12 marks) – **Start a new booklet** **Marks**

a) The quadratic equation $x^2+(k+3)x-k=0$ has real roots. Find all the possible values k can take. 3

b) A census taken on 1st July 1949 showed the population of a major city to be 2 million. Since then, the Australian population has been increasing at a rate proportional to the population, that is, $\frac{dP}{dt}=kP$ where k is a constant.

(i) Show that the function $P=Ae^{kt}$ satisfies $\frac{dP}{dt}=kP$ 1

(ii) What is the value of A? 1

(iii) The 1st July 1991 census shows the population to be 3.2 million. Find the value of k, correct to 2 decimal places. 2

(iv) Calculate in what year the population will reach 5 million. 2

c) The volume, V, of water in a dam at time t was monitored over a period. When monitoring began, the dam was 75% full. During the monitoring period, the volume decreased at an increasing rate due to a long period of drought. 3

(i) What does this tell us about $\frac{dV}{dt}$ and $\frac{d^2V}{dt^2}$

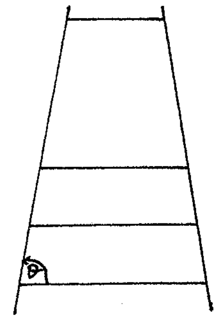
(ii) Sketch the graph of V against t.

Question 8 (12 marks) – **Start a new booklet** **Marks**

a) A ladder tapers from bottom to top, as shown in the diagram. The ladder has two side rails and twenty steps.

The bottom step is 600mm long. Each subsequent step is 15mm shorter than the one below.

The perpendicular distance between each step is 250mm.

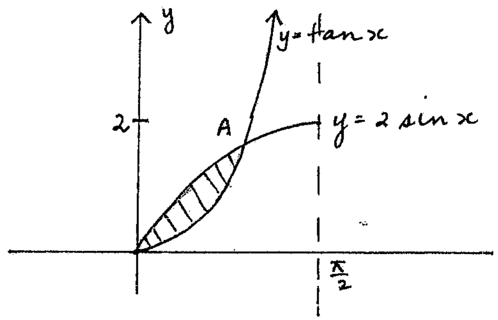


(i) Calculate the length of the top step. 2

(ii) Calculate the total length of all 20 steps. 2

(iii) The angle formed between the side rail and each step has been labelled θ. Calculate the size of this angle to the nearest whole degree. 2

b)



The diagram shows the curves $y=\tan x$ and $y=2\sin x$ for $0\le x\le\frac{\pi}{2}$.

(i) Show that the coordinates of A are $\left(\frac{\pi}{3},\sqrt{3}\right)$ 2

(ii) Show that $\frac{d}{dx}(\ln\cos x)=-\tan x$ 1

(iii) Hence find the shaded area in the diagram. 3

Question 9 (12 marks) – **Start a new booklet** **Marks**

a) Solve $2^{2x} - 6\left(2^x\right) + 5 = 0$ 3

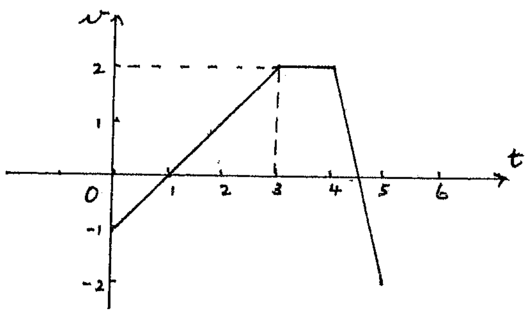
b) Consider the function $f(x) = \dfrac{\log_e x}{x}$, $x > 0$

(i) Show that the graph has a stationary point at $x = e$ 2

(ii) By examining the first derivative each side of e show that this is a maximum turning point. 2

(iii) Given that $x = e^{\frac{3}{2}}$ will give a point of inflexion, sketch the curve noting it is undefined at $x = 0$ 2

c) A particle moves along the x-axis. Initially it is at the origin. The graph shows the velocity, v, of the particle as a function of time for $0 \le t \le 5$.

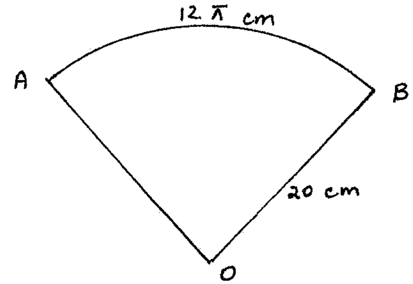


(i) Write down the time(s) when the particle is stationary. 1

(ii) At what time during the interval $0 \le t \le 5$ is the particle furthest from the origin? Give a reason for your answer. 2

Question 10 (12 marks) – **Start a new booklet** **Marks**

a)

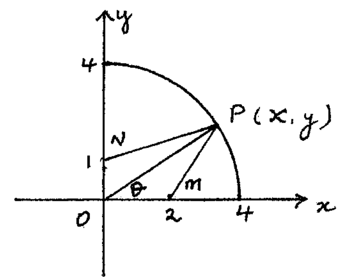


AOB is a sector of a circle, centre O and radius 20cm. The length of the arc is 12π cm. 3

Calculate the exact area of the sector AOB.

b) State the amplitude and period of the function $y = -\frac{1}{2}\cos\left(3x + \frac{\pi}{2}\right)$ 2

c)



The diagram shows the part of the circle $x^2 + y^2 = 16$ that lies in the first quadrant. The point $P(x, y)$ is on the circle, O is the origin, M is on the x-axis at $x = 2$ and N is on the y-axis at $y = 1$. The size of $\angle MOP$ is θ radians.

(i) Show that the area, A, of quadrilateral $OMPN$ is given by $A = 4\sin\theta + 2\cos\theta$ 2

(ii) Find the value of $\tan\theta$ for which A is a maximum. 3

(iii) Hence determine in surd form the coordinates of P for this maximum area. 2

End of Paper

SOLUTIONS.

TRIAL HSC
MATHEMATICS
2005

QUESTION 1:

(a) $(\sqrt{3}-2)^2 = 3 - 4\sqrt{3} + 4$
$= 7 - 4\sqrt{3}$

(b) $0.083716\ldots$
$= 0.0837$ (correct to 3 sig. figs)

(c)
$$9x^2 = x$$
$$9x^2 - x = 0$$
$$x(9x-1) = 0$$
$$\therefore x = 0, \tfrac{1}{9}$$

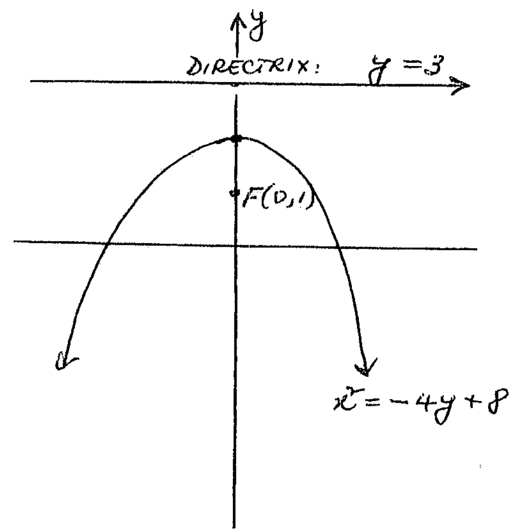
(d) $F\hat{B}C = 66^\circ$ (straight $A\hat{B}C$ is 180°)
$y = 66 + 90$ (exterior angle of triangle equals sum of two interior opposite angles)

$\therefore y = 156$

(e)
$$y = \tan\left(\frac{x}{2}\right)$$
$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)$$

(f)
$$x^2 = -4y + 8$$
$$= -4(y-2)$$
$$= -4(1)(y-2)$$

Vertex $V(0,2)$
Focal length $= 1$
Focus $F(0,1)$



UESTION 2 :

a) (i) at A, $y = 0$ $\therefore$ $3x^2 - x^3 = 0$

$x^2(3-x) = 0$

$x = 0, 3$

$\therefore A \equiv (3,0)$

(ii) at B, $\frac{dy}{dx} = 0$: $y = 3x^2 - x^3$ —— (1)

$\frac{dy}{dx} = 6x - 3x^2$

$\therefore$ $6x - 3x^2 = 0$

$3x(2-x) = 0$

$x = 0, 2$

sub $x = 2$ in (1) $\therefore y = 4$

$\therefore$ $B \equiv (2,4)$

(iii) at C $\frac{d^2y}{dx^2} = 0$: $\frac{d^2y}{dx^2} = 6 - 6x$

$\therefore$ $6 - 6x = 0$

$x = 1$ sub in (1)

$y = 2$

$\therefore$ $C \equiv (1,2)$

b) (i) In ΔPQR by the cosine rule

$QR^2 = 20^2 + 8^2 - 2 \times 20 \times 8 \cos 160^\circ$

$= 764.7\ldots$

$\therefore QR = 27.65\ldots$

$\therefore$ Distance is 27.7 km (correct to 1 decpl)

(ii) By the sine rule

$\frac{\sin P\hat{Q}R}{8} = \frac{\sin 160^\circ}{QR}$

$\therefore \sin P\hat{Q}R = \frac{8 \sin 160^\circ}{QR}$

$= 0.0989\ldots$

$\therefore P\hat{Q}R = 5^\circ 41'$

$\therefore$ Bearing of R from Q is $095^\circ 41'$

(c) $0.\dot{5}\dot{4} = 0.545454\ldots$

$= \frac{54}{100} + \frac{54}{10\,000} + \frac{54}{1\,000\,000} + \ldots$

(i) $a = \frac{54}{100}$ $r = \frac{1}{100}$

(ii) $S = \frac{a}{1-r}$

$= \frac{\frac{54}{100}}{1 - \frac{1}{100}}$

$= \frac{54}{99}$

$= \frac{6}{11}$

$\therefore 0.\dot{5}\dot{4} = \frac{6}{11}$

QUESTION 3 :

(a) (i) T is (a, b) where $\frac{a+(-1)}{2} = 1$ $\quad \frac{b+(-1)}{2} = 1$

$a - 1 = 2 \qquad b - 1 = 2$

$a = 3 \qquad b = 3$

$\therefore \underline{T \equiv (3, 3)}$

(ii) M is mid-point of each diagonal and hence the diagonals bisect each other, indicating a parallelogram.

(iii) $\tan\theta = $ gradient of PQ

$= \frac{8}{-6}$

$\therefore \underline{\theta = 126^\circ 52'}$ (θ is obtuse)

(iv) Equation of PQ is

$y - 5 = -\frac{4}{3}(x + 2)$

$3y - 15 = -4x - 8$

ie $\underline{4x + 3y - 7 = 0}$

(v) $S(-1,-1)$ to $4x + 3y - 7 = 0$

$d = \frac{|4(-1) + 3(-1) - 7|}{\sqrt{4^2 + 3^2}}$

$= \frac{14}{5}$

(vi) Distance $PQ = \sqrt{(5+3)^2 + (-2-4)^2}$

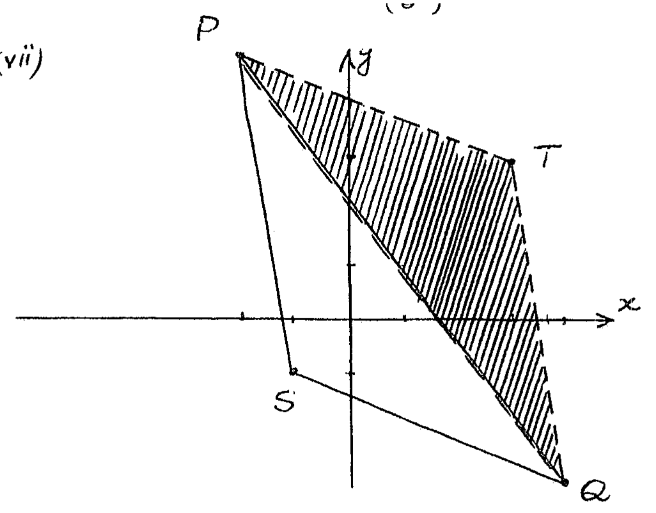
$= 10$

Area $PQST = 2 \times$ area ΔPQS

$= 2 \times \frac{1}{2} \times 10 \times \frac{14}{5}$

$= 28$ units^2

(vii)



(b) $x = t^3 - 6t^2$

(i) $v = 3t^2 - 12t$

at rest when $v = 0$

$\therefore 3t^2 - 12t = 0$

$3t(t - 4) = 0$

$t = 0, 4$

ie when $t = 0, 4$ s.

(ii) at $t = 0$, $x = 0$

at $t = 4$, $x = 64 - 96$

$= -32$

$\therefore$ Distance is 32 m

QUESTION 4:

(a) (i) $y = (1+\log x)^3$

$\frac{dy}{dx} = 3(1+\log x)^2 \cdot \frac{1}{x}$

$= \frac{3}{x} \cdot (1+\log x)^2$

(ii) $y = \underbrace{x}_{u}\underbrace{e^{x^2}}_{v}$

$\frac{dy}{dx} = vu' + uv'$
$= e^{x^2} \cdot 1 + x \cdot 2xe^{x^2}$
$= e^{x^2}(1+2x^2)$

(b) (i) $\int_0^1 e^{2x}\,dx = \left[\frac{e^{2x}}{2}\right]_0^1$

$= \frac{e^2}{2} - \frac{1}{2}$

$= \frac{1}{2}(e^2-1)$

(ii) $\int \frac{x^2}{x^3+1}\,dx = \frac{1}{3}\log|x^3+1| + c$

(c) (i) First \$A grows to \$A$(1.005)^{36}$
(ii) 2^{nd} \$A " " \$A $(1.005)^{35}$

⋮

last \$A " " \$A (1.005)

Lump sum $= [\underbrace{A(1.005) + A(1.005)^2 + \ldots + A(1.005)^{36}}]$ dollars

Geometric series $a = A(1.005)$
$r = 1.005$
$n = 36$

$\therefore 30\,000 = A(1.005)\frac{[1.005^{36}-1]}{1.005-1}$

$\therefore$ \$A = \$758.86 (correct to nearest cent)

QUESTION 5:

(a) (i) $C\hat{S}B = C\hat{B}S$ since base angles of an isosceles triangle are equal.

(ii) $C\hat{B}S = A\hat{D}C$ (opposite angles of parallelogram are equal)

$S\hat{C}D = C\hat{S}B$ (alternate angles equal, $BA \parallel CD$)
$= C\hat{B}S$ (proven above)

$\therefore S\hat{C}D = A\hat{D}C = C\hat{B}S$

(iii) In ΔSCD, ΔADC

(a) CD is common
(b) $SC = AD$ ($BC = SC$ (data)
$BC = AD$ (opposite sides of llogram)
(c) $S\hat{C}D = A\hat{D}C$ (proven in (ii) above)

$\therefore \Delta SCD \equiv \Delta ADC$ (SAS)

(b) $\int_1^5 x \ln x\,dx \doteq \frac{h}{3}[(y_1+y_3)+4y_2]$

$= \frac{2}{3}[(0+8.047)+4(3.296)]$

$= 14.154\ldots$

$= 14.15$ (correct to 2 dec pl)

(c) (i) $y = \frac{x^6}{64}$

$\therefore x^6 = 64y$
$x^2 = (64y)^{\frac{1}{3}}$
$= 4.y^{\frac{1}{3}}$

(ii) $V = \pi\int_0^{64} x^2\,dy$

$= \pi\int_0^{64} 4.y^{\frac{1}{3}}\,dy$

$= 4\pi\left[\frac{3}{4}y^{\frac{4}{3}}\right]_0^{64}$

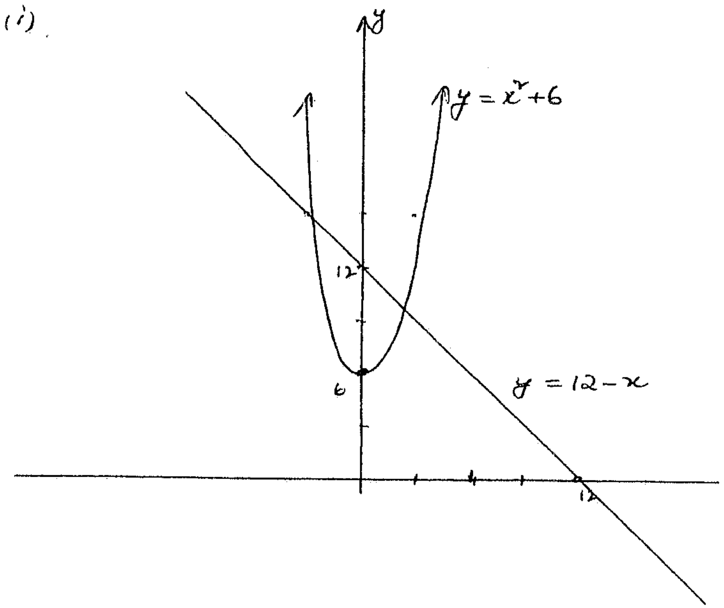
$= 3\pi[256-0]$

$= 768\pi$

$\therefore$ Volume is 768π units^3

QUESTION 6:

(a) (i)



(ii) $y = x^2+6$ —— ①

$y = 12 - x$ —— ②

① − ② $0 = x^2 + x - 6$

$0 = (x+3)(x-2)$

$\therefore x = -3, 2.$

$$A = \int_0^2 \left\{12 - x - (x^2+6)\right\} dx$$

$$= \int_0^2 (6 - x - x^2)\, dx$$

$$= \left[6x - \frac{x^2}{2} - \frac{x^3}{3}\right]_0^2$$

$$= \left[12 - 2 - \frac{8}{3} - 0\right]$$

$$= \frac{22}{3}$$

$\therefore$ Area is $\frac{22}{3}$ units^2

(b) $R = 10 + \frac{15}{1+t}$ litres/min

(i) $t = 0 \Rightarrow R = 10 + 15$

$= \underline{25}$

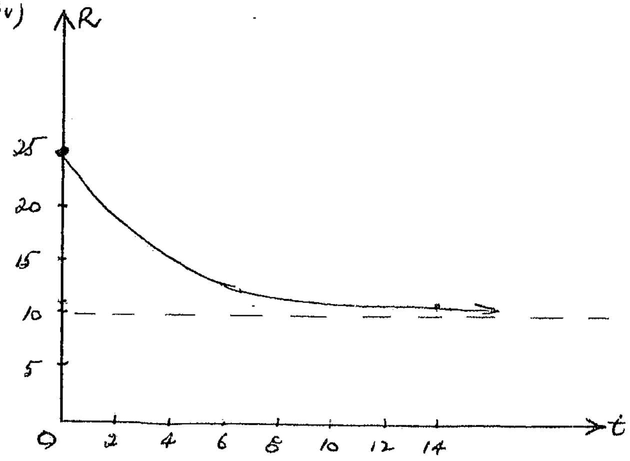
(ii) $t = 14 \Rightarrow R = 10 + \frac{15}{15}$

$= \underline{11}$

(iii) as $t \to \infty$ $R \to 10 + 0$

$= 10$

(iv)



(v) $\frac{dF}{dt} = 10 + \frac{15}{1+t}$

$$\therefore F = \int_0^{14} \left(10 + \frac{15}{1+t}\right) dt$$

$$= \left[10t + 15 \log(1+t)\right]_0^{14}$$

$$= (140 + 15 \log 15) - (0 + 15 \log 1)$$

$$= 140 + 15 \log 15$$

$\therefore$ Fuel used is $(140 + 15 \log 15)$ litres.

QUESTION 7:

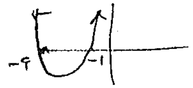
(a) Real roots if $\Delta \geq 0$

ie $(k+3)^2 - 4(1)(-k) \geq 0$

$k^2+6k+9+4k \geq 0$

$k^2+10k+9 \geq 0$

$(k+9)(k+1) \geq 0$



$\therefore k \leq -9$ OR $k \geq -1$

(b) July 1st 1949 ..... 2 million

(i) If $P = Ae^{kt}$ then $\frac{dP}{dt} = A.ke^{kt}$

$= k.Ae^{kt}$

and $kP = k \times Ae^{kt}$

$= \frac{dP}{dt}$

$\therefore P = Ae^{kt}$ satisfies $\frac{dP}{dt} = kP$

(ii) A = 2 million (ie initial value of P)

(iii) July 1st 1949 .... 2 million

" " 1950 ... 1 yr

" " 1951 .... 2 yrs.

" " 1991 .... 42 yrs

$P = 2\,000\,000\,e^{kt}$

$\left.\begin{matrix} t = 42 \\ P = 3\,200\,000 \end{matrix}\right\}$ $3\,200\,000 = 2\,000\,000\,e^{42k}$

$\therefore 42k = \log 1.6$

$k = 0.011..$

$= 0.01$ (correct to 2 dec. pl)

(iv) $P = 2\,000\,000\,e^{kt}$

$P = 5\,000\,000 \Longrightarrow 5\,000\,000 = 2\,000\,000\,e^{kt}$

$e^{kt} = 2.5$

$kt = \ln 2.5$

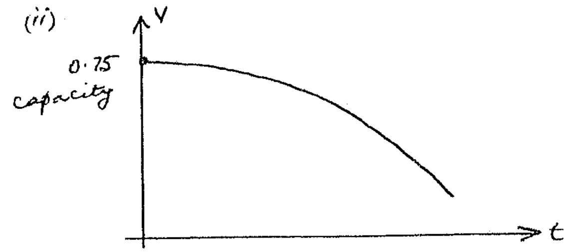
$t = \frac{\ln 2.5}{k}$

$= 81.88...$

$\therefore$ Reaches 5 million during 2031

(c) (i) $\frac{dV}{dt} < 0$ and $\frac{d^2V}{dt^2} < 0$

(ii)



QUESTION 8:

(a) Step lengths are $\underbrace{600, 585, 570, \ldots x}$

arithmetic sequence

$a = 600,\ d = -15,\ n = 20$

(i) $T_n = a + (n-1)d$

$x = 600 - 15(19)$

$= 315$

$\therefore$ Top step is 315 mm

(ii) Consider $\underbrace{600 + 585 + \cdots + 315}$

arithmetic series

$a = 600,\ d = -15,\ n = 20$

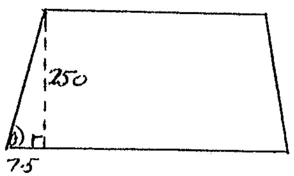
$S_n = \frac{n}{2}(a + l)$

$S_{20} = \frac{20}{2}(600 + 315)$

$= 9150$

$\therefore$ Total length is 9.15 m

(iii)



$\tan\theta = \frac{250}{7.5}$

$\therefore \theta = 88°17'$

(b) (i) Check $(\frac{\pi}{3}, \sqrt{3})$ in $y = \tan x$

$\sqrt{3} = \tan\frac{\pi}{3}$ is true

Check $(\frac{\pi}{3}, \sqrt{3})$ in $y = 2\sin x$

$\sqrt{3} = 2\sin\frac{\pi}{3}$

$= 2 \times \frac{\sqrt{3}}{2}$

$= \sqrt{3}$ true

$\therefore (\frac{\pi}{3}, \sqrt{3})$ satisfies both curves

(ii) $\frac{d}{dx}\left[\ln(\cos x)\right] = -\frac{\sin x}{\cos x}$

$= -\tan x$

(iii) $A = \int_0^{\frac{\pi}{3}} (2\sin x - \tan x)\, dx$

$= \left[-2\cos x + \ln(\cos x)\right]_0^{\pi/3}$

$= \left[-2\cos\frac{\pi}{3} + \ln(\tfrac{1}{2})\right] - \left[-2 + \ln 1\right]$

$= -1 + \ln\frac{1}{2} + 2$

$= 1 + \ln\frac{1}{2}$

$\therefore$ Area is $(1 + \ln\frac{1}{2})$ units^2

QUESTION 9:

(a) $(2^x)^2 - 6(2^x) + 5 = 0$

let $v = 2^x$

$\therefore v^2 - 6v + 5 = 0$

$(v-5)(v-1) = 0$

$v = 1, 5$

$\therefore 2^x = 1, 5$

$2^x = 1 \Rightarrow x = 0$ $\quad$ $2^x = 5 \Rightarrow x = \log_2 5 = \frac{\log 5}{\log 2} = 2.32$ (correct to 2 dec pl)

$\therefore x = 0, 2.32$

(b) $f(x) = \frac{\log_e x}{x}$

(i) $f'(x) = \frac{x \cdot \frac{1}{x} - \log_e x}{x^2}$

$= \frac{1 - \log_e x}{x^2}$

stationary point at $f'(x) = 0$

ie $1 - \log_e x = 0$

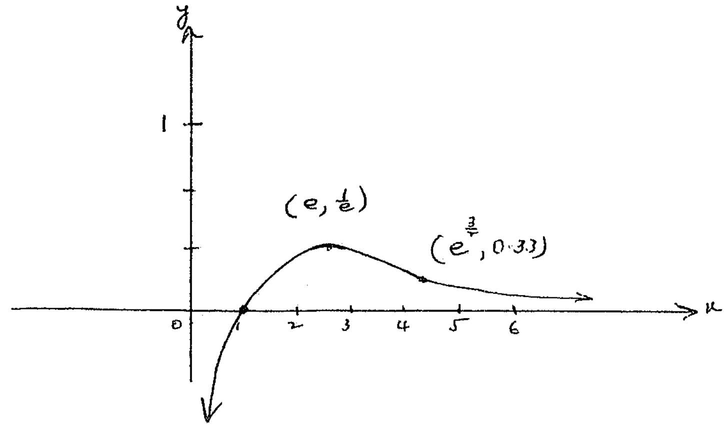
$x = e$

(ii) $f'(2.7) = \frac{1 - \log 2.7}{2.7^2} = 9.25 \times 10^{-4}$

$f'(2.8) = \frac{1 - \log 2.8}{2.8^2} = -3.8 \times 10^{-4}$

$+ \quad 0 \quad -$

$\therefore (e, \frac{1}{e})$ is relative maximum turning point.



(c) (i) stationary when $v = 0$

ie $t = 1, 4.5$ (approx)

(ii) at $t = 4.5$ (area under curve is greatest)

Particle is back at origin at $t = 2$ and then proceeds to the right for $2 < t \leq 4.5$ and in this time its distance travelled is given by the area under the curve.

QUESTION 10:

(a) $l = r\theta$

$\Rightarrow 12\pi = 20\theta$

$\therefore \theta = \frac{12\pi}{20}$

$= \frac{3\pi}{5}$

$A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 20^2 \times \frac{3\pi}{5}$ cm^2

$= 120\pi$ cm^2

(b) amplitude $= \frac{1}{2}$

period $= \frac{2\pi}{3}$

(c)(i) area $\Delta POM = \frac{1}{2} \times 2 \times y$

$= y$ where $\sin\theta = \frac{y}{4}$

$y = 4\sin\theta$

area $\Delta PON = \frac{1}{2} \times 1 \times x$

$= \frac{1}{2}x$ where $\cos\theta = \frac{x}{4}$

$x = 4\cos\theta$

$\therefore$ area $OMPN = 4\sin\theta + 2\cos\theta$

(ii) $A = 4\sin\theta + 2\cos\theta$

$\frac{dA}{d\theta} = 4\cos\theta - 2\sin\theta$

Stat. pt at $\frac{dA}{d\theta} = 0$

ie $4\cos\theta - 2\sin\theta = 0$

ie $\tan\theta = 2$

$\frac{d^2A}{d\theta^2} = -4\sin\theta - 2\cos\theta$

< 0 for $\tan\theta = 2$ and θ_{acute}

$\therefore$ A is maximum when $\tan\theta = 2$

(iii) $\tan\theta = 2 = \frac{y}{x}$

$\therefore y = 2x$ sub $- x^2 + y^2 = 16$

$x^2 + 4x^2 = 16$

$5x^2 = 16$

$x^2 = \frac{16}{5}$

$x = \frac{4}{\sqrt{5}}$

$y = \frac{8}{\sqrt{5}}$

$\therefore P \equiv \left(\frac{4}{\sqrt{5}}, \frac{8}{\sqrt{5}}\right)$