

Name: \_\_\_\_\_

Class: \_\_\_\_\_

**St George Girls High School****Year 10****May 2006**

# Advanced Mathematics

*Time Allowed: 75 minutes***Instructions:**

1. All necessary working must be shown.
2. All questions may be attempted.
3. Calculators may be used.

| <b>Question</b>          | <b>Mark</b>    |
|--------------------------|----------------|
| <b>Part A</b>            | /15            |
| <b>Part B</b>            |                |
| 1. Algebra and Equations | /15            |
| 2. Measurement           | <del>/15</del> |
| 3. Surds and Indices     | /15            |
| 4. Quadratics            | /15            |
| <b>Total</b>             | /75            |

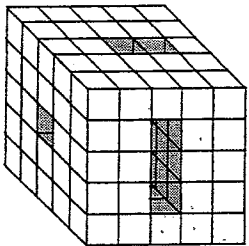
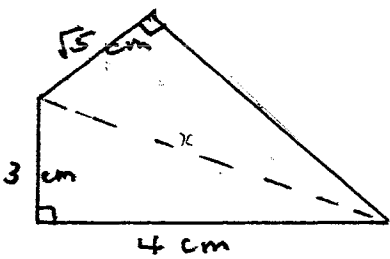
**Part A**

1 mark each

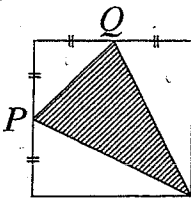
Write your answers in the answer column. You may do your working in the question column.

| Question   | Answer |
|--|--------|
| 1. Simplify $3x + 2x^0$  |        |
| 2. What number must be added to the expression $x^2 + 5x$ in order to make it a perfect square?  |        |
| 3. Evaluate $6y - 3y^2$ when $y = -5$  |        |
| 4. Divide an amount of \$560 in the ratio of 3:7   |        |
| 5. The speed of light is approximately $3 \times 10^5$ km/s. How long would it take light to travel from the Sun to Mars, a distance of $2.3 \times 10^8$ km. (Give your answer in minutes). |        |
| 6. A closed cube has a volume of $125\text{cm}^3$ . Calculate its surface area.  |        |
| 7. Express $4x^{-1}$ with a positive index.  |        |

Part A (cont'd)

| Question  | Answer |
|---|--------|
| <p>8.</p>  <p>A <math>5 \times 5 \times 5</math> cube has a <math>1 \times 1 \times 5</math> hole cut through one side, a <math>2 \times 1 \times 5</math> hole through another and a <math>3 \times 1 \times 5</math> hole through the third as shown in the diagram.</p> <p>The volume remaining, in cubic units, is:</p> <p>(A) 95<br/>             B. 99<br/>             C. 100<br/>             D. 101<br/>             E. 102</p> |        |
| <p>9) If <math>2^x = b</math> find an expression for <math>16^x</math></p>  |        |
| <p>10. Calculate the perimeter, giving your answer in simplified form.</p>   |        |
| <p>11) Write as an equation "Six more than the square root of a number is four less than twice the square of the number". (Let the number be <math>n</math>)</p>  |        |

Part A (cont'd)

| Question  | Answer |
|---|--------|
| 12. Make $y$ the subject of $2y = 1 - xy$   |        |
| 13. Give the exact solution: $(2x + 3)^2 = 5$   |        |
| 14. <div style="text-align: center;">  </div> <p><math>P</math> and <math>Q</math> are mid-points of the sides of the square as shown. What is the ratio of the area of the shaded triangle to the area of the square?</p> <p>A. <math>\frac{1}{4}</math>      B. <math>\frac{3}{8}</math>      C. <math>\frac{1}{2}</math></p> <p>D. <math>\frac{5}{8}</math>      E. <math>\frac{3}{4}</math></p> |        |
| 15. Convert 12L/min to mL/s   |        |

**Part B**

Show all working.

**Question 1** – Algebra and Equations – (15 marks)

Marks

a) Simplify:

(i)  $\frac{x-3}{x^2-9}$

(ii)  $x + \frac{x-1}{3}$

1  
each

b) Expand and simplify:

(i)  $(5x+2)^2$

(ii)  $2y(y-1) - (y-1)^2$

1  
2

c) Solve:

(i)  $5(3y-5) = 4(y+2)$

(ii)  $\frac{x-4}{5} - \frac{2x+1}{6} = 3$

(iii)  $5 - 4z \geq 9$

2  
3  
2

d) Solve simultaneously  $5x + y = 9$   
 $3x - 4y = 10$

3

**Question 2 – Measurement – (15 marks)**

**Marks**

a) (i) If  $\$A1 = \text{€}0.415$  (euro) convert  $\text{€}75$  to  $\$A$ .

3

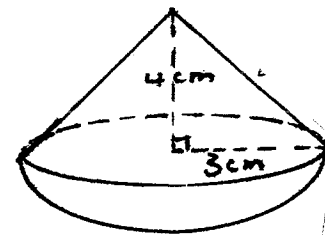
(ii) Convert  $0.35 \text{ cm}^3$  to  $\text{mm}^3$ .

(iii) How many litres of water could be stored in a tank whose volume is  $31.2 \text{ m}^3$ ?

b) Two sheets of paper are similar in shape and their widths are in the ratio  $4:5$ . Find the area of the larger sheet if the smaller one has an area of  $17.6 \text{ cm}^2$

2

c) The composite solid consists of a cone on top of a hemisphere.



Find the:

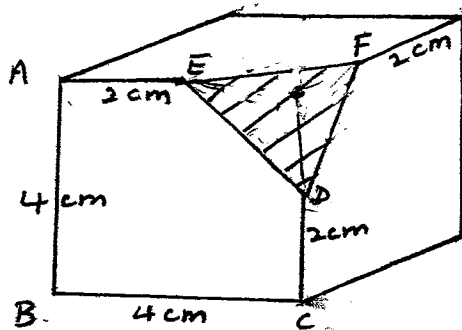
(i) Total volume of the solid.

(ii) Total surface area of the solid.

Question 2 (cont'd)

Marks

d) A solid block in the shape of a cube has a corner cut off as shown.



(i) Find the area of the front face  $ABCDE$ .

2

(ii) Show that the length of the interval  $DE$  is  $2\sqrt{2}$  cm

1

(iii) Find the area of the cut surface  $DEF$ .

2

**Question 3 – Surds and Indices – (15 marks)**

**Marks**

a) Simplify:

3

(i)  $\left(\frac{t^8}{t^2}\right)^2$

(ii)  $(27x^{27})^{\frac{1}{3}}$

(iii)  $4m^{-2} \div \frac{1}{2}m^{-1}$

b) (i) Simplify  $\sqrt{80} + \sqrt{20}$

(ii) Evaluate (giving your answer in scientific notation correct to 3 significant figures:)

4

$$\frac{(3.7 \times 10^3)^2}{2.4 \times 10^{-3}}$$

c) Expand and simplify  $(\sqrt{2} - \sqrt{3})(7\sqrt{2} - 4\sqrt{3})$

2

d) Rationalise the denominator and simplify where possible.

2  
each

(i)  $\frac{\sqrt{7} - \sqrt{3}}{2\sqrt{2}}$

(ii)  $\frac{\sqrt{6}}{\sqrt{6} - 2}$

e) Find values of  $p$  and  $q$

$$(2\sqrt{3} - 1)^2 = p + q\sqrt{3}$$

2



**Question 4 – Quadratics – (15 marks)**

Marks

a) Factorise each of the following:

3

(i)  $3x^2 - 6x$

(ii)  $x^2 - 2x - 63$

(iii)  $6x^2 + 7x - 3$

b) Solve each of the following for  $y$  (give an exact answer).

2  
each

(i)  $3y^2 - 27 = 0$     |    (ii)  $y^2 + 5y + 6 = 0$     |    (iii)  $2y^2 - 3y - 4 = 0$

c) Use the substitution  $u = x + 1$  to solve  $(x + 1)^2 + 3(x + 1) - 4 = 0$

3

d) The height ( $h$ ) metres of a stone thrown vertically upwards is defined by  $h = 30t - 6t^2$ , where  $t$  is the time in seconds. Find when the stone is at a height of 36 metres.

3

Name: Renee a Lowe Class: COM4

St George Girls High School

Year 10

May 2006



# Advanced Mathematics

*Time Allowed: 75 minutes*

**Instructions:**

1. All necessary working must be shown.
2. All questions may be attempted.
3. Calculators may be used.

| Question                 | Mark       |
|--------------------------|------------|
| Part A                   | 109 /15    |
| Part B                   |            |
| 1. Algebra and Equations | 14 /15     |
| 2. Measurement           | 10 1/2 /15 |
| 3. Surds and Indices     | 13 /15     |
| 4. Quadratics            | 15 /15     |
| Total                    | 61 1/2 /75 |

6  
4  
5  
6  
5

62 1/2

Part A

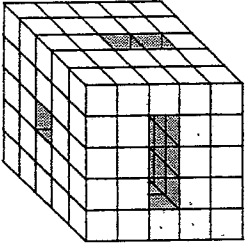
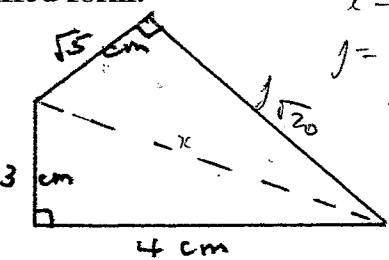
1 mark each

Write your answers in the answer column. You may do your working in the question column.

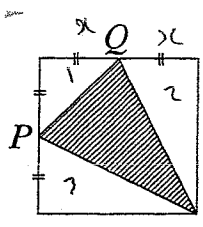
| Question   | Answer   |
|--|--|
| 1. Simplify $3x + 2x^0$<br>$3x + 2$  | $3x + 2$ ✓   |
| 2. What number must be added to the expression $x^2 + 5x$ in order to make it a perfect square?<br>$x^2 + 5x + \frac{25}{4} = (x + 2.5)^2$   | $6.25$ ✓<br>ie. $x^2 + 5x + 6.25 = (x + 2.5)^2$            |
| 3. Evaluate $6y - 3y^2$ when $y = -5$<br>$6(-5) - 3 \times 25 = -30 - 75 = -105$   | $-105$ ✓   |
| 4. Divide an amount of \$560 in the ratio of 3:7<br>$\frac{560}{10} = 56$<br>$56 \times 3 = 168$<br>$56 \times 7 = 392$  | $\$168$ and $\$392$ ✓                                      |
| 5. The speed of light is approximately $3 \times 10^5$ km/s. How long would it take light to travel from the Sun to Mars, a distance of $2.3 \times 10^8$ km. (Give your answer in minutes).<br>$\frac{230000000 \text{ km}}{300000 \text{ km/s}} = 766.67 \text{ s} = 12.77 \text{ mins}$ | $12.7$ minutes ✓<br>$(4.14 \times 10^{15}) \text{ mins}$ ✗ |
| 6. A closed cube has a volume of $125 \text{ cm}^3$ . Calculate its surface area.<br>$\sqrt[3]{125} = 5 \text{ cm edges}$<br>$SA = 5^2 \times 6 = 150 \text{ cm}^2$  | $SA = 150 \text{ cm}^2$ ✓                                  |
| 7. Express $4x^{-1}$ with a positive index.  | $\frac{4}{x}$ ✓  |

(6)

Part A (cont'd)

| Question  | Answer  |
|---|---|
| <p>8.</p>  <p>A <math>5 \times 5 \times 5</math> cube has a <math>1 \times 1 \times 5</math> hole cut through one side, a <math>2 \times 1 \times 5</math> hole through another and a <math>3 \times 1 \times 5</math> hole through the third as shown in the diagram.</p> <p>The volume remaining, in cubic units, is:</p> <p>(A) 95<br/>                 B. 99<br/>                 C. 100<br/>                 D. 101<br/>                 E. 102</p> | <p>A. 95 units<sup>3</sup></p> <p>X</p> <p>= 100 because<br/>                 25 were taken<br/>                 from it.</p> |
| <p>9) If <math>2^x = b</math> find an expression for <math>16^x</math></p> <p><math>2^{4x} = 16^x</math>     <math>2^4 = 16</math>     <math>2^x = b</math>     <math>16^x = (2^4)^x = (2^{4x})^1 = 2^{4x}</math></p>   | <p><math>2^{4x}</math> X</p>  |
| <p>10. Calculate the perimeter, giving your answer in simplified form.</p>  <p><math>x = \sqrt{3^2 + 4^2} = 5</math><br/> <math>y = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}</math><br/> <math>z = \sqrt{5^2 - (\frac{3\sqrt{5}}{5})^2} = \sqrt{20}</math><br/> <math>P = \sqrt{5} + \sqrt{20} + 4 + 3 = \sqrt{5} + 2\sqrt{5} + 7 = 3\sqrt{5} + 7</math></p>   | <p><math>P = (3\sqrt{5} + 7) \text{ cm}</math></p> <p>2 ✓</p>   |
| <p>11) Write as an equation "Six more than the square root of a number is four less than twice the square of the number". (Let the number be <math>n</math>)</p> <p><math>6 + \sqrt{n} = 2n^2 - 4</math><br/> <math>6 = 4n^2 - 16</math><br/> <math>22 = 4n^2</math><br/> <math>n^2 = 5.5</math></p>  | <p><math>n = \sqrt[4]{24}</math> X</p>  |

Part A (cont'd)

| Question   | Answer   |
|--|--|
| <p>12. Make <math>y</math> the subject of <math>2y = 1 - xy</math></p> <p><math>2y = 1 - xy</math><br/><math>y = 1 - x</math></p> <p><math>y = 1 - xy</math><br/><math>y(x+2) = 1</math><br/><math>y = \frac{1}{x+2}</math></p>  | <p><math>y = 1 - x</math></p> <p>X</p>   |
| <p>13. Give the exact solution: <math>(2x+3)^2 = 5</math></p> <p><math>4x^2 + 12x + 9 = 5</math><br/><math>4x^2 + 12x + 4 = 0</math><br/><math>-12 \pm \sqrt{144 - 4 \times 4 \times 4}</math><br/><math>8 = \frac{-12 \pm \sqrt{80}}{8}</math></p>  | <p><math>x = \frac{-3 \pm \sqrt{5}}{2}</math> ✓</p>  |
| <p>14.</p>  <p>A square = <math>4x^2</math><br/><math>A_1 = \frac{1}{2}x^2</math><br/><math>2 = \frac{x^2}{2}</math><br/><math>= x^2</math><br/><math>2 \frac{1}{2}x^2</math></p> <p><math>P</math> and <math>Q</math> are mid-points of the sides of the square as shown. What is the ratio of the area of the shaded triangle to the area of the square?</p> <p>A. <math>\frac{1}{4}</math>      B. <math>\frac{3}{8}</math>      C. <math>\frac{1}{2}</math><br/>D. <math>\frac{5}{8}</math>      E. <math>\frac{3}{4}</math></p> | <p><math>\therefore</math> shaded = <math>1 \frac{1}{2}x^2</math><br/>ratio is 3:8</p> <p>C. <math>\frac{1}{2}</math> X</p> <p>2</p> |
| <p>15. Convert 12L/min to mL/s</p> <p><math>= 12\,000 \text{ mL/min}</math><br/><math>= 200 \text{ mL/s}</math></p>  | <p><math>= 200 \text{ mL/s}</math> ✓</p>   |

**Part B**

Show all working.

**Question 1 – Algebra and Equations – (15 marks)**

Marks

a) Simplify:

(i)  $\frac{x-3}{x^2-9}$

$\frac{x-3}{(x-3)(x+3)} = \frac{1}{x+3}$

(ii)  $x + \frac{x-1}{3}$

$= \frac{3x}{3} + \frac{x-1}{3}$   
 $= \frac{4x-1}{3}$   
 $3x + x - 1 = 4x - 1$

1  
each

you can't get rid of the 3 coz it's not an equation

b) Expand and simplify:

(i)  $(5x+2)^2$

$= 25x^2 + 20x + 4$

(ii)  $2y(y-1) - (y-1)^2$

$= 2y^2 - 2y - (y^2 - 2y + 1)$   
 $= 2y^2 - 2y - y^2 + 2y - 1$   
 $= y^2 - 1$

1  
2

c) Solve:

(i)  $5(3y-5) = 4(y+2)$

$15y - 25 = 4y + 8$   
 $11y = 33$   
 $y = 3$

(ii)  $\frac{x-4}{5} - \frac{2x+1}{6} = 3$

$6(x-4) - 5(2x+1) = 90$   
 $6x - 24 - 10x - 5 = 90$   
 $-4x - 29 = 90$   
 $-4x = 119$   
 $x = -29 \frac{3}{4}$

(iii)  $5 - 4z \geq 9$

$-4z \geq 4$   
 $z \leq -1$

2  
3  
2

d) Solve simultaneously  $5x + y = 9$   
 $3x - 4y = 10$

$\begin{matrix} \times 3 \\ \times 5 \end{matrix} \Rightarrow \begin{matrix} 15x + 3y = 27 \\ 15x - 20y = 50 \end{matrix}$

$\downarrow$   
 $-23y = 23$

$y = -1$

$x = \frac{9 - (-1)}{5} = 2$

3

(14)

**Question 2 – Measurement – (15 marks)**

Marks

- a) (i) If \$A1 = €0.415 (euro) convert €75 to \$A.

3

$$\frac{75}{0.415} = \$A 180.723 \quad (\text{3DP})$$

- (ii) Convert  $0.35 \text{ cm}^3$  to  $\text{mm}^3$ .

$350 \text{ mm}^3$  ✓



2

- (iii) How many litres of water could be stored in a tank whose volume is  $31.2 \text{ m}^3$ ?

31.2 L could be stored.

X  
1L does not equal  $1 \text{ m}^3$

- b) Two sheets of paper are similar in shape and their widths are in the ratio 4:5. Find the area of the larger sheet if the smaller one has an area of  $17.6 \text{ cm}^2$

2

$$4:5$$

$$17.6 : x$$

$$\frac{4}{5} = \frac{17.6}{x}$$

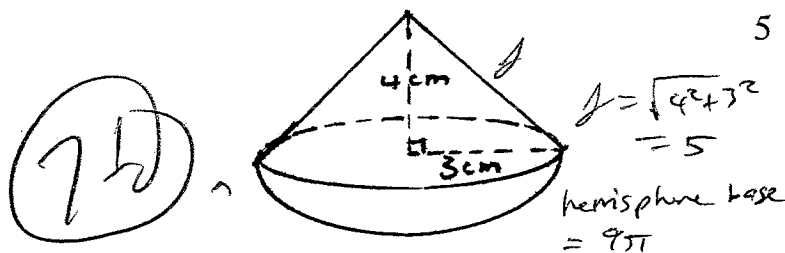
$$x = 22$$

larger sheet is  $22 \text{ cm}^2$

✓  
 $\sqrt{17.6}$  etc -- because its surface area etc.

- c) The composite solid consists of a cone on top of a hemisphere.

5



Find the:

- (i) Total volume of the solid.

- (ii) Total surface area of the solid.

$$V = \frac{1}{3} \times \pi \times 3^2 \times 4 + \frac{2}{3} \times \pi \times 3^3$$

$$= 56.5486 + 37.6991$$

$$\text{total } V = 94.25 \text{ cm}^3 \quad (\text{2DP})$$

$$\text{Curved cone SA} = \pi \times 3 \times 5$$

$$+ \frac{4 \times \pi \times 9}{2}$$

$$= 47.123 + 56.5486$$

$$+ 56.5486$$

$$\text{total SA}$$

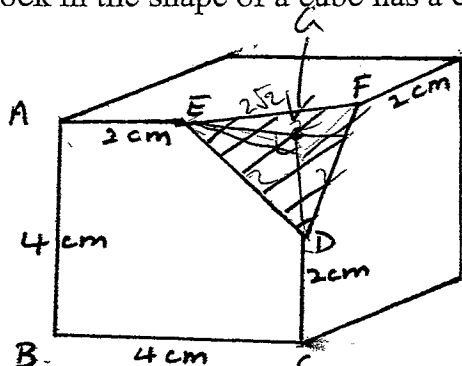
$$= 103.67 \text{ cm}^2 \quad (\text{2DP})$$

~~total V = 150.8 cm<sup>3</sup> (2DP)~~

Question 2 (cont'd)

Marks

d) A solid block in the shape of a cube has a corner cut off as shown.



corner

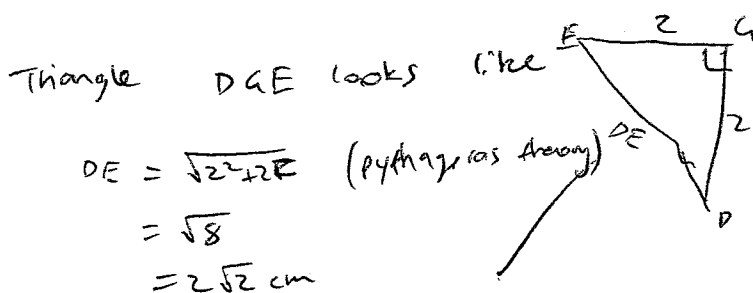


area =  $1 \times 2$  2

(i) Find the area of the front face  $ABCDE$ .

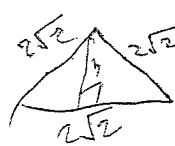
$$\begin{aligned} \text{full front} &= 16 \text{ cm}^2 \\ \text{corner area} &= 2 \text{ cm}^2 \\ \text{total area} &= 16 - 2 \\ &= 14 \text{ cm}^2 \end{aligned}$$

(ii) Show that the length of the interval  $DE$  is  $2\sqrt{2}$  cm



3

(iii) Find the area of the cut surface  $DEF$ .

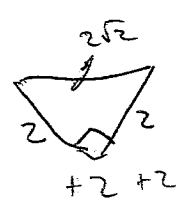


$$\begin{aligned} h^2 + (\sqrt{2})^2 &= (2\sqrt{2})^2 \\ h^2 + 2 &= 8 \\ h^2 &= 6 \\ h &= \sqrt{6} \end{aligned}$$

$$[GE \text{ and } GF = 2\sqrt{2} \text{ cm}]$$

$$\begin{aligned} \therefore A &= \frac{1}{2} \times 2\sqrt{2} \times \sqrt{6} \\ &= \sqrt{12} \\ &= 2\sqrt{3} \text{ cm}^2 \end{aligned}$$

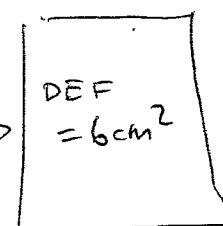
Top view of corner



$$h = \sqrt{2^2 + 2^2}$$

$\therefore EF = 2\sqrt{2}$  cm as well.  
(and  $FD$ )

area = ~~14~~



corner volume =  $\frac{1}{3} \times 2 \times 2 \times 2 = 1\frac{1}{3}$

cube  $V = 4 \times 4 \times 4 = 64$



**Question 3 – Surds and Indices – (15 marks)**

Marks

a) Simplify:

3

(i)  $\left(\frac{t^8}{t^2}\right)^2$

$\frac{t^{16}}{t^4} = t^{12}$

(ii)  $(27x^{27})^{\frac{1}{3}}$

$= 3x^9$

(iii)  $4m^{-2} \div \frac{1}{2}m^{-1}$

$8m^{-1}$

$= \frac{8}{m}$  *actually it's  $\frac{8}{m}$  idiot.*

b) (i) Simplify  $\sqrt{80} + \sqrt{20}$

$= 4\sqrt{5} + 2\sqrt{5}$   
 $= 6\sqrt{5}$

(ii) Evaluate (giving your answer in scientific notation correct to 3 significant figures:)

4

$\frac{(3.7 \times 10^3)^2}{2.4 \times 10^{-3}} = 5.70 \times 10^1$  *(to 3 sig. figs)*

c) Expand and simplify  $(\sqrt{2} - \sqrt{3})(7\sqrt{2} - 4\sqrt{3})$

2

$= 14 - 4\sqrt{6} - 7\sqrt{6} + 12$   
 $= 26 - 11\sqrt{6}$

**(13)**

d) Rationalise the denominator and simplify where possible.

2 each

(i)  $\frac{\sqrt{7} - \sqrt{3}}{2\sqrt{2}}$

$\times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{14} - \sqrt{6}}{4}$   
 $= \frac{2\sqrt{7} - \sqrt{6}}{4}$

(ii)  $\frac{\sqrt{6}}{\sqrt{6} - 2} \times \frac{\sqrt{6} + 2}{\sqrt{6} + 2}$

$= \frac{6 + 2\sqrt{6}}{6 - 4}$   
 $= \frac{6 + 2\sqrt{6}}{2}$   
 $= 3 + \sqrt{6}$

e) Find values of p and q

$(2\sqrt{3} - 1)^2 = p + q\sqrt{3}$  LHS =  $12 - 4\sqrt{3} + 1 = 13 - 4\sqrt{3}$   
 $p = 13$  and  $q = -4$

2

$= (\sqrt{12} - 1)^2$

$p + q\sqrt{3} = 12 - 2\sqrt{12} + 1$

$p + q\sqrt{3} = 11 - 2\sqrt{12} \Rightarrow p + 2\sqrt{3} = 11 - 4\sqrt{3}$

$p + 4q\sqrt{3} = 11$   
 $p = 11 - 4q\sqrt{3}$   
 $4q\sqrt{3} = 11 - p$   
 $q = \frac{11 - p}{4\sqrt{3}}$