Name: _____   Class: /_____

Advanced Mathematics

Time Allowed: 2 hours
(plus 5 minutes reading time)

Instructions

Section A: Multiple Choice.
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely on the answer sheet.

Section B: Answers only required.

Section C: Free response – all necessary working must be shown or full marks might not be awarded.

	Section A		/20
	Section B		/20
Section C	Question 1		/12
	Question 2		/12
	Question 3		/12
	Question 4		/12
	Question 5		/12
	Total		/100

Section A

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely on the answer sheet.
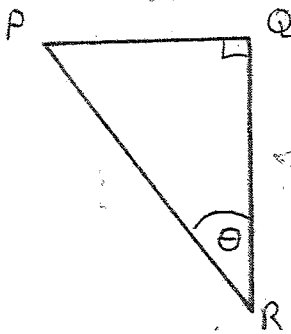
1. Simplify the expression $\dfrac{3-6a}{3}$

    A. $3-2a$ B. $1-2a$ C. $1-6a$ D. $-a$

2. Successive discounts of 10% followed by 10% is equivalent to a single discount of:

    A. 19%   B. 20%   C. 81%   D. 80%

3.

    The ratio of PQ to PR is $2:3$

    The size of $\angle PRQ$ is closest to:

    (not to scale)

    A. $48°11'$ B. $33°41'$ C. $56°19'$ D. $41°49'$

4. The gradient of the line $4x - by + 12 = 0$ is $\frac{1}{3}$

    The value of b is:

    A. 3   B. 12   C. -3 D. -12

5. Solve for x: $x(3x-2)=0$

   A. $x=0,-2$ B. $x=0,\dfrac{3}{2}$ C. $x=0,\dfrac{2}{3}$ D. $x=\dfrac{2}{3}$

6. The expression for the next consecutive odd number following $2n-1$ is:

   A. $2n$ B. $2n-3$ C. $2n+1$ D. $n-1$

7. If $\left(2\sqrt{5}-1\right)^2 = 21-\sqrt{b}$, the value of b is:

   A. 5 B. 20 C. 80 D. 40

8. Expand and simplify: $(1-p)(2p-1)$

   A. p B. $p-1-2p^2$ C. $3p-2p^2-1$ D. $3p-2p^2+1$
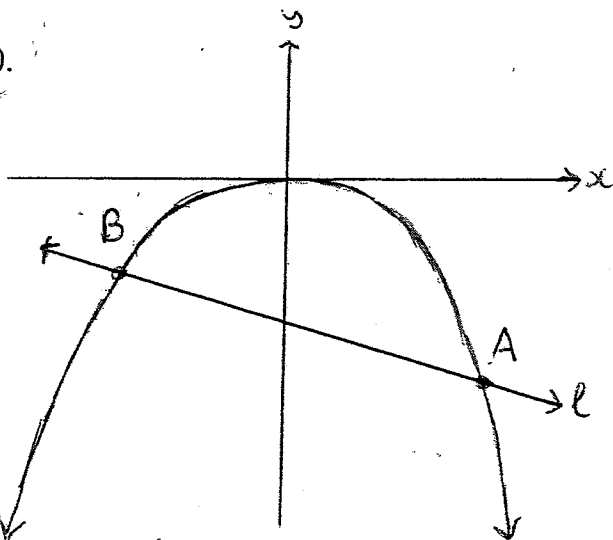
9. The solutions of $2x^2-9x-1=0$ are:

   A. $x=\dfrac{9\pm\sqrt{73}}{4}$ B. $x=\dfrac{9\pm\sqrt{89}}{4}$

   C. $x=\dfrac{9\pm\sqrt{89}}{2}$ D. $x=\dfrac{-9\pm\sqrt{73}}{4}$

10.

    The equation $x^2-2x-3=0$ is solved to find the x-coordinates of the points A and B.
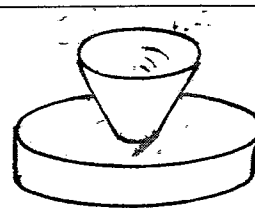
    The equation of the line l could be:
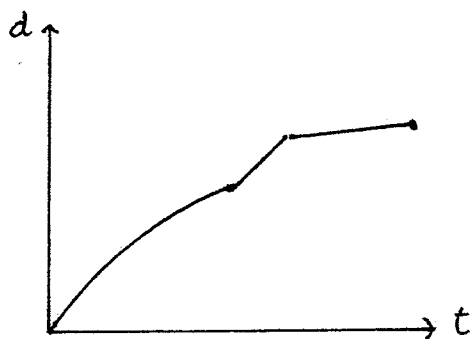
    A. $-2x-3$

    B. $2x+3$
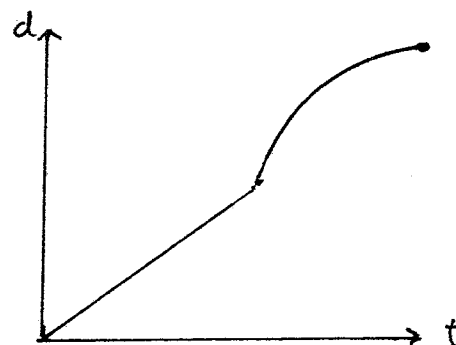
    C. $-2x+3$

    D. $2x-1$

11. This container is filled from the tap at a constant rate. Which of the following graphs would most accurately show the depth of water, d cm, at time t secs?
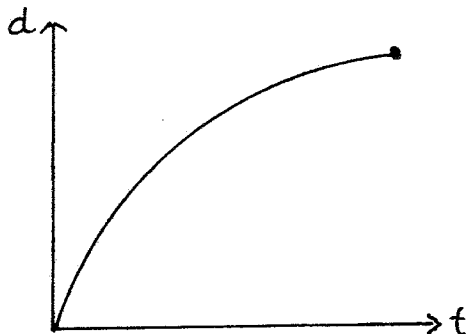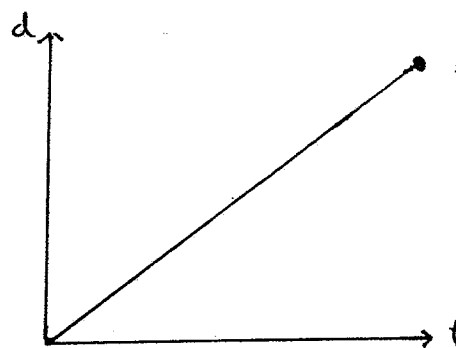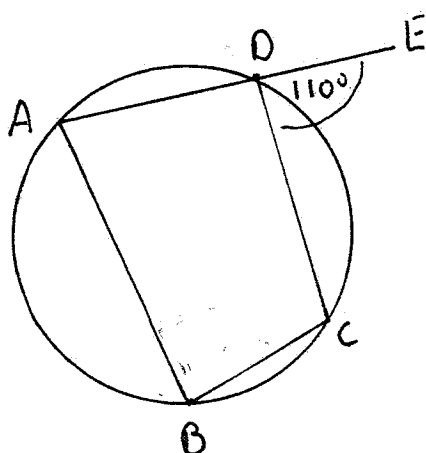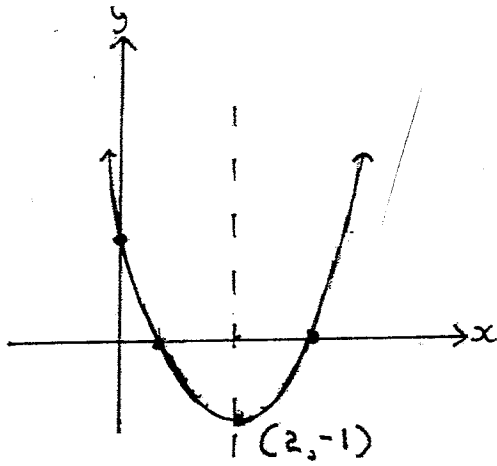
A.

B.

C.

D.

12.

[not to scale]

ABCD is a cyclic quadrilateral.

AD is extended to *E* such that $\angle CDE = 110°$. The size of angle $\angle ABC$ is:

A. 55°   B. 110°   C. 220°   D. 70°

13.

$(2, -1)$

Which of the following equations could describe this parabola.

A. $y = x^2 + x - 7$

B. $y = x^2 - 4x + 3$

C. $y = x^2 - 2x - 1$

D. $y = x^2 - x + 4$

14. Solve for $x : 3 - 2x \geq 5 + x$

A. $x \geq -\dfrac{2}{3}$ B. $x \leq \dfrac{8}{3}$ C. $x \leq -\dfrac{2}{3}$ D. $x \geq -\dfrac{8}{3}$

15. The volume of a cone is 157.08cm^3. If the base radius is 5cm then the height of the cone to the nearest centimetre is:

A. 15   B. 5   C. 6   D. 3

16. Which algebraic fraction when subtracted from $\dfrac{2x}{5}$ gives $-\dfrac{7x}{20}$

A. $\dfrac{9x}{25}$ B. $\dfrac{9x}{4}$ C. $\dfrac{3x}{24}$ D. $\dfrac{3x}{4}$

17.

$x = 1$
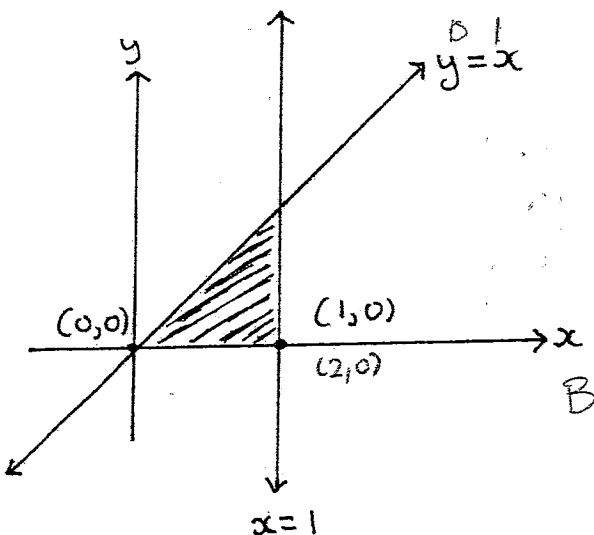
The shaded region represents all points given by

A. $y \geq x$ and $x \leq 1$ and $y \geq 0$
B. $y \leq x$ and $x \leq 1$ and $y \geq 0$
C. $y \leq x$ and $x \geq 1$ and $y \leq 0$
D. $y \geq x$ and $x \geq 1$ and $y \geq 0$

Note: Question 18 has more than one correct answer. Fill in every correct answer. [2 marks]
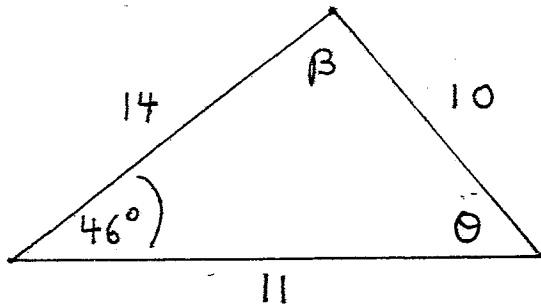
18.

Which of the following expressions are true for the given triangle.

[not to scale]

A. $\text{Area} = \dfrac{1}{2} \times 10 \times 11 \times \sin\theta$

B. $\theta = \dfrac{11^2 + 10^2 - 14^2}{2 \times 11 \times 10}$

C. $\sin\theta = \dfrac{11 \times \sin 46}{10}$

D. $\sin\beta = \dfrac{11 \times \sin 46}{10}$

19. Given $0 < b < a < 1$, which of the following inequalities is true?

A. $ab > 1$

B. $\dfrac{b}{a} > 1$
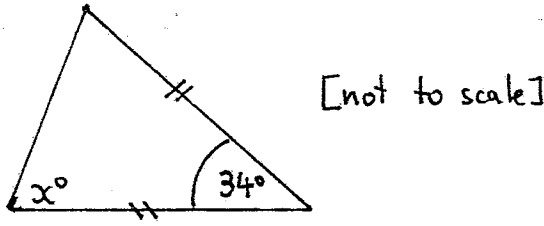
C. $a - b > 1$

D. $\dfrac{a}{b} > 1$
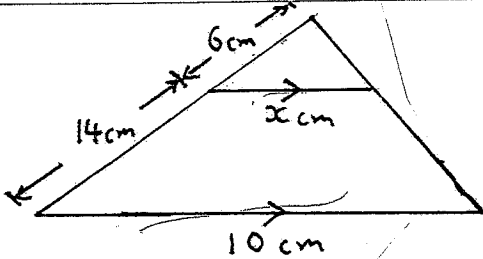
Section B

Answers only in answer column.
(1 mark each)
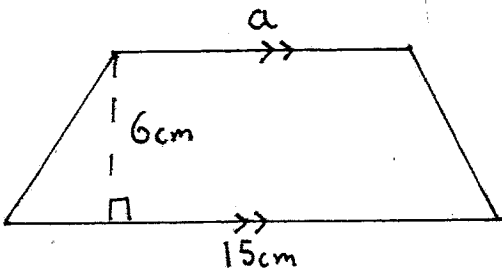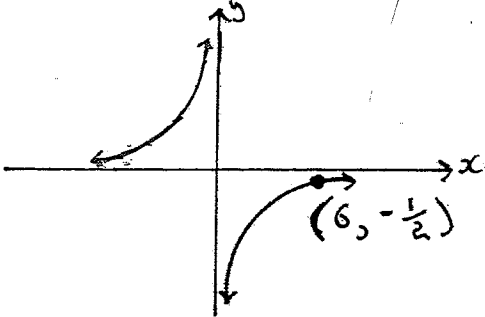
Question	Answer
1. What are the coordinates of the vertex for the parabola with equation $y = (x+1)^2 - 2$	
2. If $\cos\theta = \dfrac{2}{5}$ then $\cos(180° - \theta) =$	
3. [not to scale] The size of the angle marked $x°$ is	
4. Express as an equation "the product of x^2 and 5 is equivalent to the difference between twice x and 3".	
5. Rationalise the denominator $\dfrac{1}{3 + \sqrt{5}}$	
6. Find the value of $\dfrac{\sqrt[3]{2.1^2 - 3}}{3\pi}$ correct to 2 decimal places	
7. Write in the form $y = mx + b$ the equation of the line $2x - 3y + 7 = 0$	

Section B (cont'd)

Question	Answer
8. The exact curved surface area of a cone with base radius 3cm is 15π cm^2. What is the slant height of this case?	
9. Convert 20 m/sec to	km per hour
10. Find a general formula for this table of values. 	

x	-1	2	5	8
y	-8	1	10	19

Question	Answer
11. The value of x is	
12. Factorise $x^2 - 10x + 25$.	
13. Find the Simple Interest gained on $1500 at 9.6% p.a. for 30 months.	
14. Express in its simplest form $2^{2n} \times 3^{2n}$	
15. Angela has chosen 4 cards at random from a normal deck of cards. If three of the cards she has chosen are Aces, what is the probability that the 4th card is also an Ace?	

Section B (cont'd)

Question	Answer
16. The area of this trapezium is 66cm². Then 'a' has a value of	$a =$
(17.) The parabola with equation $y = (x - h)(x - 3)$ has its axis of symmetry as $x = -1$. Find h.	$h =$
18. The probability that Si-Si is late to school on Monday is $\dfrac{3}{8}$. The probability that she is late on Tuesday is $\dfrac{1}{5}$. Calculate the probability that she is late to school on both Monday and Tuesday.	
19. $xy = c$ is sketched below.	The value of c is _____
20. Convert 2560cm³ to m³	$=$ _____ m³

5

Section C

Show all necessary working.
Marks are as shown.

Marks

Question 1 (12 marks)

a)   For each of the following sequences find:

   (α) 5th term   (β) nth term

   (i)   1, 4, 7, ...   (ii)   3, 5, 9, 17, ...   2
                                                                           2

b)   Factorise:

   (i) $9a^2b - 3ab^3$ (ii) $x^2 - 4x + 4 - y^2$ 2
                                                                                   2

Question 1 (cont'd)

Marks

c)   Amanda must catch two trains on her way home. The first train she catches has a 0.7 chance of arriving on time. If it does arrive on time then Amanda has an even chance of making the connection with her next train.

If the first train is not on time, then Amanda only has 0.2 chance of making her connection.

(i)   Complete this tree diagram

2

First Train   **Second Train**

(0.5)   connect

on time

(0.5)   not connect

(0.7)

(0.2)   connect

late

not connect
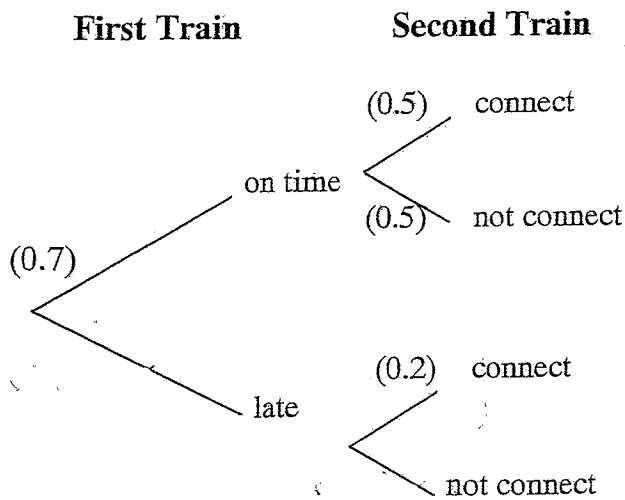
(ii)   Calculate the probability that Amanda does not make the connection with the second train

2

Question 2 (12 marks)

Marks

a) *AE* and *BD* are straight lines intersecting at *C*.

    (i)   Calculate the size of $\angle ACB = \theta$

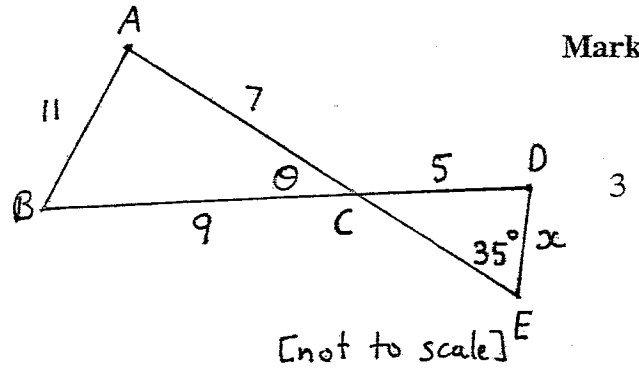   [Hint: Use the cosine rule]

3

[not to scale]

    (ii)   Hence, or otherwise, find the length of $DE = x$ 3
        [correct to one decimal place]

c) To calculate the amount of money in her savings account after 2½ years, Emma uses the compound interest formula $A = 2300\,(1.01)^{30}$

    (i)   Interest compounds monthly. What annual interest rate does Emma's saving account apply?   2

    (ii)   How much interest does Emma receive?   2

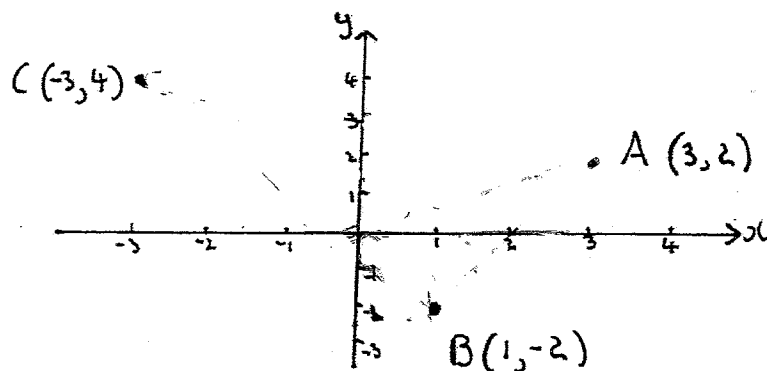d) Find the equation of the line passing through $(3, -2)$ and parallel to $y = 4x + 6$ 2

Question 3 (12 marks)

Marks

$A(3, 2)$, $B(1, -2)$ and $C(-3, 4)$ are three points on the number plane.

a)   Calculate the length of BC in $\triangle ABC$. [answer in exact form]

2

b)   Find the gradient of BC.

2

c)   Find the equation of the perpendicular from A to side BC in $\triangle ABC$.

2

Question 3 (cont'd)

Marks

d) If the area of $\triangle ABC$ is 13cm^2, find the length of the perpendicular from A to BC.

2

e) Give the coordinates of D such that $ABCD$ is a parallelogram.

2

f) The line from A to the midpoint of BC has equation $x - 4y + 5 = 0$. The line from C to the midpoint of AB has equation $4x + 5y - 8 = 0$.

Solve these equations simultaneously.

2

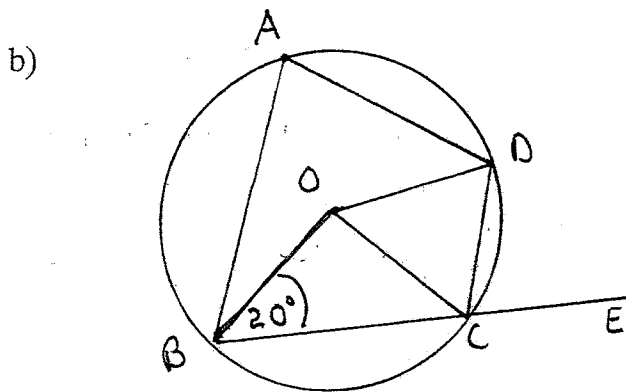## Question 4 (12 marks)

Marks

a) Comment on the solution(s) to the quadratic equation $ax^2 + bx + c = 0$ given that $b^2 - 4ac < 0$

b)

(not to scale)

ABCD is a cyclic quadrilateral with

(i)   *BC* extended to *E*.

(ii) $DC = OD$ and

(iii) $\angle OBC = 20°$

$6

(i)   Show that $\triangle ODC$ is an equilateral triangle.

(ii) Hence, or otherwise, give the size of $\angle ODC$.

Question 4 (cont'd)   **Marks**

(iii) Triangle OBC is an isosceles triangle since $OB = OC$ [radii of circle]

Therefore, $\angle BOC =$

(iv) Find, giving reasons, the size of $\angle BAD$

c)   Find the roots, correct to 2 decimal places, of $2x - 1 = \dfrac{2}{3x + 1}$ 3

d)   Show by the method of completing the square for $x^2 - x - 1 = 0$ 2

that $\left(x - \dfrac{1}{2} \right)^2 - \dfrac{5}{4} = 0$

Question 5 (12 marks)

Mar

a) Two dice are thrown. Would the chance of getting a total of seven by adding the numbers on the upper most faces be increased by the information that the first die showed a four?

2

Use a mathematical argument to support your response.

b) Find the area of an equilateral triangle which has a perimeter of $3x$ units.

3

Marks

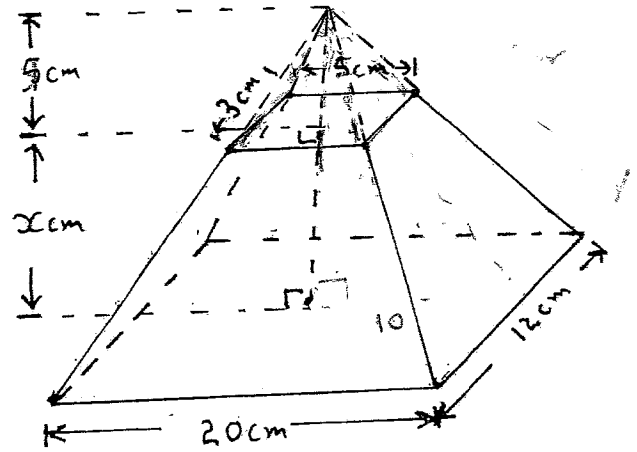## Question 5 (cont'd)

c)   (i)   Show that x, the height of the truncated pyramid is equal to 15cm in length.

2

(ii)   Hence, or otherwise, calculate the volume of the truncated Rectangular Pyramid.   3

d)   If $0 < x < 1$ show that $x^3 < x^2$

2

End of Paper

Name: _Angelica Tjokrowidjaja_   Class: _10 M 1_

St George Girls High School   SOLUTIONS

Year 10

Yearly Examination

2005

Advanced Mathematics

Time Allowed: 2 hours
(plus 5 minutes reading time)

Instructions

Section A: Multiple Choice.
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely on the answer sheet.
Section B: Answers only required.
Section C: Free response – all necessary working must be shown or full marks might not be awarded.

	Section A	19/20
	Section B	19/20
Section C	Question 1	11/12
	Question 2	12/12
	Question 3	12/12
	Question 4	12/12
	Question 5	10/12
	Total	95/100

Name: _Angelica Tjokrowidjaja_   Class: _10 M 1_

Multiple Choice Answer Sheet

Section A
Total marks (20)
Attempt Questions (1-20)

Use the multiple-choice answer sheet.
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 =$ (A) 2   (B) 6   (C) 8   (D) 9

A ◯   B ●   C ◯   D ◯

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ●   B ✖   C ◯   D ◯

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:
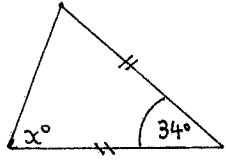
  ╱ correct
A ✖   B ✖   C ◯   D ◯

1.	A ◯	B ●	C ◯	D ◯ ✓
2.	A ●	B ◯	C ◯	D ◯ ✓
3.	A ◯	B ●	C ◯	D ● −
4.	A ◯	B ●	C ◯	D ◯ ✓
5.	A ◯	B ◯	C ●	D ◯ ✓
6.	A ◯	B ◯	C ●	D ◯ ✓
7.	A ◯	B ◯	C ●	D ◯ ✓
8.	A ◯	B ◯	C ●	D ◯ ✓
9.	A ◯	B ●	C ◯	D ◯ ✓
10.	A ●	B ◯	C ◯	D ◯ ✓
11.	A ◯	B ●	C ◯	D ◯ ✓
12.	A ◯	B ●	C ◯	D ◯ ✓
13.	A ◯	B ●	C ◯	D ◯ ✓
14.	A ◯	B ◯	C ●	D ◯ ✓
15.	A ◯	B ◯	C ●	D ◯ ✓
16.	A ◯	B ◯	C ◯	D ● ✓
17.	A ◯	B ●	C ◯	D ◯ ✓
18.	A ●	B ◯	C ◯	D ● ✓✓
19.	A ◯	B ◯	C ◯	D ● ✓
20.	A ◯	B ◯	C ◯	D ◯

19/20

Section B

$y = a(k-0)^2 + a$ *(handwritten)*

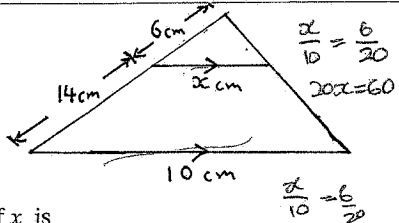Answers only in answer column.
(1 mark each)

Question	Answer
1. What are the coordinates of the vertex for the parabola with equation $y = (x+1)^2 - 2$	$(-1, -2)$ ✓
2. If $\cos\theta = \dfrac{2}{5}$ then $\cos(180° - \theta) =$ *(handwritten quadrant diagram: S/A/T/C)*	$-\dfrac{2}{5}$ $-66°25'$ (to nearest min) ✗
3. [not to scale] The size of the angle marked $x°$ is	$x = 73°$ ✓
4. Express as an equation "the product of x^2 and 5 is equivalent to the difference between twice x and 3". $5x^2 = 2x - 3$	$5x^2 = 2x - 3$ ✓
5. Rationalise the denominator $\dfrac{1}{3+\sqrt{5}} \times \dfrac{3-\sqrt{5}}{3-\sqrt{5}}$ $\dfrac{3-\sqrt{5}}{9-5}$	$\dfrac{3-\sqrt{5}}{4}$ ✓
6. Find the value of $\dfrac{\sqrt[3]{2.1^2 - 3}}{3\pi}$ correct to 2 decimal places	0.12 ✓
7. Write in the form $y = mx + b$ the equation of the line $2x - 3y + 7 = 0$ $3y = 2x + 7$ $3y = 2x + 7$ $y = \dfrac{2}{3}x + \dfrac{7}{3}$ $y = \dfrac{2}{3}x + \dfrac{7}{3}$	$y = \dfrac{2}{3}x + 2\dfrac{1}{3}$ ✓

Section B (cont'd)
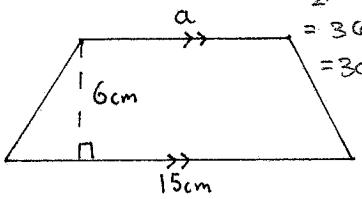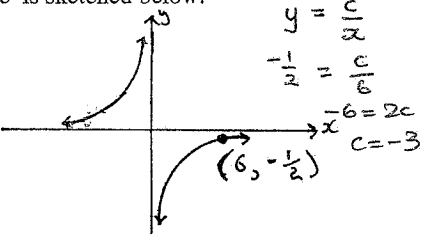
(handwritten top:) $\pi \times r \times l = \pi \times 3 \times l$; $15\pi = 3\pi l$; $\therefore l = 5$

Question	Answer
8. The exact curved surface area of a cone with base radius 3cm is 15π cm^2. *(handwritten: $\pi \times r \times l$; $(15\pi = (3 \times \pi) \times l$; $l = 5$)* What is the slant height of this case?	5 cm ✓
9. Convert 20 m/sec to *(handwritten: 1200 /min ; 7200/h)*	72 km per hour ✓
10. Find a general formula for this table of values. 	x	-1	2	5	8	 	y	-8	1	10	19		$y = 3x - 5$ ✓
11. *(handwritten: 6cm, 14cm, x cm, 10 cm; $\frac{x}{10} = \frac{6}{20}$; $20x = 60$; $\frac{x}{10} = \frac{6}{20}$)* The value of x is	$x = 3$ cm ✓
12. Factorise $x^2 - 10x + 25$. *(handwritten: $(x-5)^2$)*	$(x-5)^2$ ✓
13. Find the Simple Interest gained on $1500 at 9.6% p.a. for 30 months. *(handwritten: $I = \dfrac{1500 \times 9.6 \times 2.5}{100}$)*	$360 ✓
14. Express in its simplest form $2^{2n} \times 3^{2n}$ *(handwritten: $= (2\times3)^{2n}$; $= 6^{2n}$; $= (6^2)^n = 36^n$; $\frac{2^3 \times 3^3}{} = 216 = 6^3$)*	6^{2n} ✓ or 36^n
15. Angela has chosen 4 cards at random from a normal deck of cards. If three of the cards she has chosen are Aces, what is the probability that the 4th card is also an Ace? *(handwritten: $\dfrac{1}{49}$)*	$\dfrac{1}{49}$ ✓

(handwritten bottom of page 7: 1)

Section B (cont'd)

Question	Answer
16. a / 6cm / 15cm $\frac{1}{2} \times 6\,(a+5)$ $= 3\,(a+15)$ $= 3a + 45 = 66$ $3a = 21$ The area of this trapezium is 66cm². Then 'a' has a value of	$a = 7$cm ✓
17. The parabola with equation $y = (x-h)(x-3)$ has its axis of symmetry as $x = -1$. Find h. $\frac{-(3-h)}{2} = -1$ $-3+h = -2$ $x^2 - 3x - hx + 3h$ $x(3+h)$ $\frac{-(3x-hx)}{2}$ $-1 = \frac{3+h}{2}$ $-1 = -\frac{(3-h)}{2}$ $3+h = -2$ $\therefore h = -5$	$h = -5$ ✓
18. The probability that Si-Si is late to school on Monday is $\frac{3}{8}$. The probability that she is late on Tuesday is $\frac{1}{5}$. Calculate the probability that she is late to school on both Monday and Tuesday.	$\frac{3}{40}$ ✓
19. $xy = c$ is sketched below. $y = \frac{c}{x}$ $-\frac{1}{2} = \frac{c}{6}$ $-6 = 2c$ $c = -3$ $(6, -\frac{1}{2})$	The value of c is $\underline{-3}$ ✓
20. Convert 2560cm³ to m³ $\div 1\,000\,000$	$= \underline{0.00256}\text{ m}^3$ ✓

Section C

Show all necessary working.
Marks are as shown.

Marks

Question 1 (12 marks)

a) For each of the following sequences find:

(α) 5th term   (β) nth term

(i) $1, 4, 7, \ldots$ $\overset{+3\ +3}{\frown}$ ✓

$y = 3n - 2$

$\therefore$ 5th term $= 3 \times 5 - 2$
$= 13$ ✓

$T^n = 3n - 2$

(ii) $3, 5, 9, 17, \ldots$ $\overset{+2\ +4\ +8}{\frown}$ 2   2

$y = 2^n + 1$

$\therefore$ For nth term $= 2^n + 1$

5th term $= 2^5 + 1$
$= 33$

$T^n = 2^n + 1$

$6(n-1)-1$

$3n \quad 2(n-1) \times 3^{-1} \quad 2(3 \times n-1)$

$n^2 + n$

5th term? ✓

b) Factorise:

(i) $9a^2 b - 3ab^3$ 2
$= 3ab(3a - b^2)$ ✓

(ii) $x^2 - 4x + 4 - y^2$ 2
$= (x-2)^2 - y^2$
$= (x - 2 - y)(x - 2 + y)$ ✓

Question 1 (cont'd)   Marks
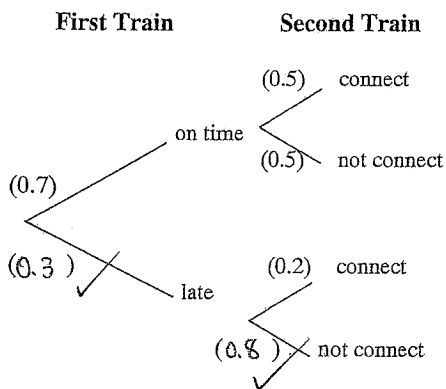
c)   Amanda must catch two trains on her way home. The first train she catches has a 0.7
     chance of arriving on time. If it does arrive on time then Amanda has an even chance
     of making the connection with her next train.

     If the first train is not on time, then Amanda only has 0.2 chance of making her
     connection.

     (i)   Complete this tree diagram   2

First Train   **Second Train**

(0.5) connect

on time

(0.5) not connect

(0.7)

(0.3) ✓

late (0.2) connect

(0.8) ✗   not connect $\mathcal{2}$ ✓

     (ii) Calculate the probability that Amanda does not make the connection with the
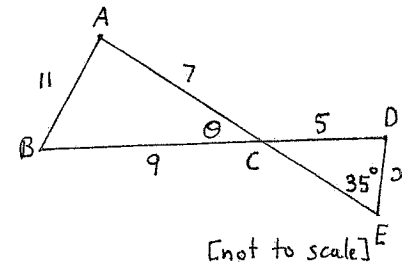   second train   2

$P(\text{no 2nd}) = 0.7 \times 0.5 + 0.3 \times 0.8$ ✓ ✓

$= 0.59$

$= \dfrac{59}{100}$ $\mathcal{2}$

Question 2 (12 marks)   Marks

a)   AE and BD are straight lines intersecting at C.

     (i)   Calculate the size of $\angle ACB = \theta$ 3

     [Hint: Use the cosine rule]

     $\cos \theta = \dfrac{7^2 + 9^2 - 11^2}{2 \times 7 \times 9}$

     $= 0.07\ldots$

     $\therefore \theta = 85°54'$ (to nearest min) ✓ $3.$

[not to scale]

     (ii) Hence, or otherwise, find the length of $DE = x$ 3
           [correct to one decimal place]

     $\angle DCE = 85°54'$ (vert. opp ∠s =)

     $\dfrac{x}{\sin 85°54'} = \dfrac{5}{\sin 35}$

     $x = \dfrac{5 \sin 85°54'}{\sin 35}$ ✓ $3.$

     $= 8.7$ units (to 1 dp)

c)   To calculate the amount of money in her savings account after 2½ years, Emma uses
     the compound interest formula $A = 2300\,(1.01)^{30}$

     (i)   Interest compounds monthly. What annual interest rate does Emma's saving
           account apply?   2

     Interest $= 1.01 - 1$

     $= 1\%$ per month.

     $\therefore$ Annual interest rate $= 1\% \times 12$

     $= 12\%$ ✓ $2.$

     (ii) How much interest does Emma receive?   2

     $A = 2300\,(1.01)^{30}$ $\therefore$ Interest $= 3100.05 - 2300$

     $= 3100.05$ $= \$800.05$ (to nearest ¢)

     ✓ $2.$

d)   Find the equation of the line passing through $(3, -2)$ and parallel to $y = 4x + 6$ 2

     $m = 4$ (gradients = when lines ∥)

     $\therefore y = 4x + b$

     Sub $(3, -2)$ $-2 = 4 \times 3 + b$ $\therefore y = 4x - 14$ ✓ $2.$
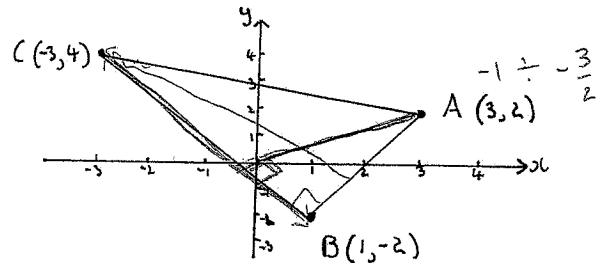
     $\therefore b = -14$

Question 3 (12 marks)   Marks

$A(3, 2)$, $B(1, -2)$ and $C(-3, 4)$ are three points on the number plane.

a) Calculate the length of BC in $\triangle ABC$. [answer in exact form]   2

$$BC = \sqrt{(1--3)^2 + (-2-4)^2}$$
$$= \sqrt{4^2 + 6^2}$$
$$= \sqrt{52}$$
$$= 2\sqrt{13}$$ ✓ 2

b) Find the gradient of BC.   2

$$m_{BC} = \frac{-2-4}{1--3}$$
$$= \frac{-6}{4}$$
$$= -\frac{3}{2}$$ ✓ 2

c) Find the equation of the perpendicular from A to side BC in $\triangle ABC$.   2

$$\perp m = -1 \div -\frac{3}{2}$$
$$= \frac{2}{3}$$ ✓

$$\therefore y = \frac{2}{3}x + b \qquad \therefore y = \frac{2}{3}x$$ ✓ 2

Sub $(3,2)$ $2 = \frac{2}{3} \times 3 + b$

$\therefore b = 0$

Question 3 (cont'd)   Marks

d) If the area of $\triangle ABC$ is 13cm^2, find the length of the perpendicular from A to BC.   2

$$A = \frac{1}{2} \times BC \times \perp A$$
$$13 = \frac{1}{2} \times 2\sqrt{13} \times \perp A$$
$$13 = \sqrt{13} \times \perp A$$
$$\therefore \perp A = \sqrt{13}$$ ✓ 2

$\therefore$ length of perpendicular is $\sqrt{13}$ units

e) Give the coordinates of D such that $ABCD$ is a parallelogram.   2

$mp\ AC = \frac{3-3}{2}, \frac{2+4}{2}$ $0 = \frac{x+1}{2}$ $3 = \frac{y-2}{2}$

$= (0, 3)$ $x+1 = 0$ $6 = y-2$

$\therefore x = -1$ $\therefore y = 8$ ✓ 2

$\therefore D$ is $(-1, 8)$

f) The line from A to the midpoint of BC has equation $x - 4y + 5 = 0$. The line from C to the midpoint of AB has equation $4x + 5y - 8 = 0$.

Solve these equations simultaneously.   2

$x - 4y + 5 = 0$ ①
$x = 4y - 5$ ②
$4x + 5y - 8 = 0$ ③

Sub ② in ③
$4(4y-5) + 5y - 8 = 0$
$16y - 20 + 5y - 8 = 0$
$21y = 28$
$y = \frac{4}{3}$ ④

① × 4

$x - 4y + 5 = 0$ ---①
$4x + 5y - 8 = 0$ ---②
$4x - 16y + 20 = 0$ ---③

② - ③

$4x + 5y - 8 = 0$
$4x - 16y + 20 = 0$
$21y - 28 = 0$
$21y = 28$
$\therefore y = \frac{4}{3}$ ---④

Sub ④ in ①
$x - 4 \times \frac{4}{3} + 5 = 0$
$x - 10\frac{1}{3} = 0$
$\therefore x = 10\frac{1}{3}$

Sub ④ in ②
$x = \frac{4}{3} \times 4 - 5$
$= \frac{1}{3}$

$\therefore x = \frac{1}{3}, y = \frac{4}{3}$ ✓ 2

Question 4 (12 marks)   Marks

a)   Comment on the solution(s) to the quadratic equation $ax^2 + bx + c = 0$ given that $b^2 - 4ac < 0$ 1

The quadratic equation $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is used to find the x solutions. However if the $b^2 - 4ac$ is less than zero there is no real solution as in the real number system there is no such thing as a negative ~~divided by~~ of a square root. e.g. $\sqrt{-x}$

∴ There are no solutions for x when $b^2 - 4ac < 0$ ✓

b)
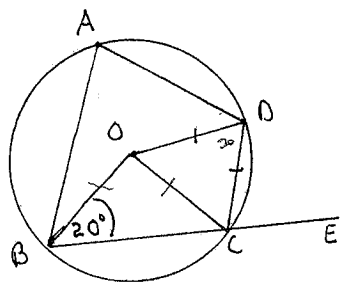
ABCD is a cyclic quadrilateral with

(i)   BC extended to E.

(ii) $DC = OD$ and

(iii) $\angle OBC = 20°$

(not to scale)

6

(i)   Show that $\triangle ODC$ is an equilateral triangle.

$OB = OC$ (radii of circle =) ✓
$OB = O\!D\!=\!DC$ (radii of circle =) not radius
∴ $OC = OD = DC$ ✓
∴ $\triangle ODC$ is equilateral as all 3 sides =   2

(ii)   Hence, or otherwise, give the size of $\angle ODC$.

∴ $\angle ODC + \angle DOC + \angle DCO = 180$ ($\angle$ sum $\triangle$)
∴ $3 \times \angle ODC = 180$ (= $\angle$'s opp = sides)
∴ $\angle ODC = 60°$ ✓   1

Question 4 (cont'd)   Marks

(iii)   Triangle OBC is an isosceles triangle since $OB = OC$ [radii of circle]
$\angle OCB = 20°$ (= $\angle$'s opp = sides)
Therefore, $\angle BOC = 180 - 20 - 20$ ($\angle$ sum $\triangle$)
$= 140°$ ✓   1

(iv)   Find, giving reasons, the size of $\angle BAD$

$\angle OCD = 60°$ (= $\angle$'s in equilateral $\triangle$, $\angle$ sum $\triangle$)
$\angle BOC = 140°$ (from above)
∴ $\angle BOD = 200°$ ✓   nice reasoning ☺
$\angle BAD = \frac{1}{2} \times 200°$ ($\angle$ at centre is twice $\angle$ at circumference subtended by same arc)
$= 100°$ ✓   2

c)   Find the roots, correct to 2 decimal places, of $2x - 1 = \dfrac{2}{3x+1}$ 3

$(2x-1)(3x+1) = 2$ ✓
$6x^2 + 2x - 3x - 1 = 2$
$6x^2 - x - 3 = 0$
$x = \dfrac{1 \pm \sqrt{1 - 4 \times 6 \times -3}}{12}$ ✓
$= \dfrac{1 \pm \sqrt{73}}{12}$ ✓
∴ $x = 0.80, \ -0.63$ (to 2 dp) ✓   3

d)   Show by the method of completing the square for $x^2 - x - 1 = 0$ that $\left(x - \frac{1}{2}\right)^2 - \frac{5}{4} = 0$ 2

$x^2 - x - 1 = 0$
$x^2 - x = 1$
$x^2 - x + \left(\frac{1}{2}\right)^2 = 1 + \left(\frac{1}{2}\right)^2$
$\left(x - \frac{1}{2}\right)^2 = \frac{5}{4}$ ✓
∴ $\left(x - \frac{1}{2}\right)^2 - \frac{5}{4} = 0$ 2

Question 5 (12 marks)

Marks

a) Two dice are thrown. Would the chance of getting a total of seven by adding the numbers on the upper most faces be increased by the information that the first die showed a four?

$4 \nearrow 3$

2

Use a mathematical argument to support your response.   6 chances out of 36

No, the chance of getting a total of 7 would not be increased as if the first die showed a four, the second die may only show a three for it to total 7, meaning the chance of getting a seven is $\frac{1}{36}$ ✓ |

However, if the information that the first die showed a 4 was not given, we could assume that the first die could show a 1, 2, 3, 4, 5 or 6 and supplemented by the second die, would add up to 7

Therefore the chance of getting a total of 7 would not be increased by the information that the die showed a 4, it would be decreased ✗

b) Find the area of an equilateral triangle which has a perimeter of $3x$ units.

3
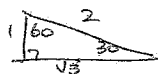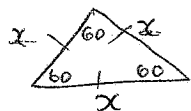
length of 1 side $= 3x \div 3$ (sides in equilateral $\triangle =$)
$= x$ ✓

∴ Area $= \frac{1}{2} \times x \times x \times \sin 60$ ✓

$= \frac{x^2}{2} \times \frac{\sqrt{3}}{2}$

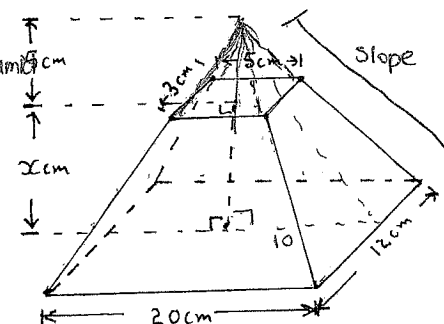$= \frac{\sqrt{3}x^2}{4}$ ✓

3.

Question 5 (cont'd)

Marks

c) (i) Show that x, the height of the truncated pyramid is equal to 15cm in length.

2

side ratio of smaller pyramid to untruncated pyramid

$= 5:20$
$= 1:4$

$\frac{1}{4} = \frac{5}{\text{total height}}$

∴ total height $= 20$

∴ truncated height $= 20 - 5$
$= 15\text{cm}$ ✓ 2

(ii) Hence, or otherwise, calculate the volume of the truncated Rectangular Pyramid.

3

Volume $= \frac{1}{3} \times 20 \times 12 \times 20 - \frac{1}{3} \times 3 \times 5 \times 5$

$= 1575 \text{cm}^3$ ✓ 3

d) If $0 < x < 1$ show that $x^3 < x^2$

2

let $x = \frac{1}{2}$ $0 < \frac{1}{2} < 1$

sub $\frac{1}{2}$ in $x^3 < x^2$ $= \frac{1}{8} < \frac{1}{4}$

∴ $x^3 < x^2$

∴ The inequality is true

∴ If $0 < x < 1$, $x^3 < x^2$

Any fraction between 1 and 0 cubed will be smaller than it is squared since the denominator is cubed, which makes it smaller than when it is squared.

you have shown it is true for only 1 eg. |

End of Paper