St George Girls High School

Year 11/12 Higher School Certificate Course

Assessment Task 1

2006

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 75 minutes.
- Write using black or blue pen.
- Attempt all questions.
- Start each question on a new page.
- Show ALL working.
- Marks for each question are shown in right column
- Complete cover sheet clearly showing
  - your name
  - mathematics class and teacher

Question 1 (12 marks) – Start a New Page

Marks

a) Factorise $3x^2+x-2$ 2

b) Differentiate with respect to x 2

$$\frac{x}{x+1}$$

c) (i) Write down in factorised form the discriminant of the quadratic 2

$$2x^2+(k+2)x+(k+2)$$

(ii) Hence, or otherwise, find the values of k for which $2x^2+(k+2)x+(k+2)=0$ has no real solutions. 2

d) Find the equation of the locus of a variable point $P(x, y)$ such that it is 3 units from the y-axis. 2

e) Give the second derivative of $y=\frac{1}{x}$ 2

Question 2 (12 marks) – Start a New Page

Marks

a) Express x^2+2 in the form $A(x+1)^2+B(x+1)+C$ 3

b) Could $x+2y=4$ be the equation of a focal chord of the parabola $x^2=8y$? 3

Justify your answer.

c) Show, giving clear reasons, that the quadratic $2x^2-x+1$ is positive definite. 2

d) Find the equation of the normal to the parabola $y=4x-x^2$ at the point where the gradient of the tangent is -2. 4

Question 3 (12 marks) – Start a New Page

Marks

a) If α and β are the roots of $x^2+3x-5=0$, find the value of:

(i) $\alpha+\beta$ 1

(ii) $\alpha\beta$ 1

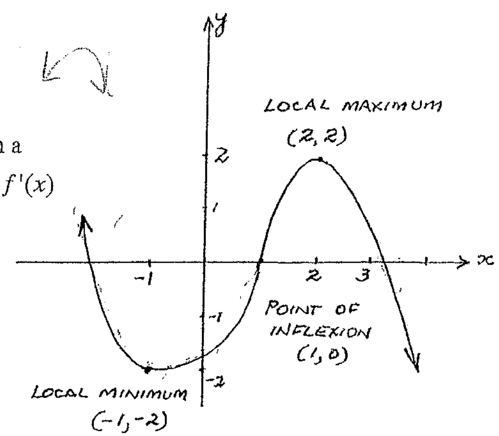
(iii) $\dfrac{1}{2\alpha}+\dfrac{1}{2\beta}$ 1

(iv) $\alpha^2+\beta^2$ 2

(v) $\alpha^3+\beta^3$ 2

b) Given is the sketch of the function $y=f(x)$. [fig. 1] 3

On at least $\frac{1}{3}$ of a page, sketch a possible curve to represent $y=f'(x)$



[Fig. 1]

c) Form the quadratic equation with roots $1-\sqrt{2}$ and $1+\sqrt{2}$ 2

Question 4 (12 marks) – Start a New Page

Marks

a) Sketch the curve $y = 2x^3 + 3x^2 - 12x + 7$ after finding: 6

(i) stationary points and determining their nature.

(ii) any points of inflexion.

(iii) the y-intercept

Clearly label these features on your sketch.

b) Express $y^2 + 10y - 12x + 61 = 0$ in the appropriate standard form of a parabola. 3

Hence, or otherwise, state the:

(i) coordinates of the vertex.

(ii) coordinates of the focus.

c) For what values of m is the line $y = mx - 12$ a tangent to the curve $y = 2x^2 - x - 10$? 3

Question 5 (12 marks) – Start a New Page

Marks

a) Find the second derivative of $f(x) = (3x-1)^4$. 4

Hence, evaluate $f''(1) - f'(1)$

b) A function $f(x)$ is continuous for all x. Draw a neat sketch of $f(x)$, displaying the essential features indicated by the following conditions. 4

$f(3) = 2$ and $f'(3) = 0$

$f'(x) > 0$ for $0 \le x < 3$

$f'(x) < 0$ for $x > 3$

$f(x)$ is an ODD function

c) A variable point $P(x, y)$ moves so that it is equidistant from $A(-1, 2)$ and the line $y = 4$ 4

Derive the equation describing the locus of P, and give a geometrical interpretation of this locus.

Question 6 (12 marks) – Start a New Page

Marks

a) Solve for x: $(x^2+x)^2-13(x^2+x)+42=0$ 4

b) For the function $y=\dfrac{x}{x^2-2x-3}$ 4

(i) Factor the denominator to find any discontinuities on the curve.

(ii) Show that the curve is decreasing for all possible values of x.

c) The function $y=ax^3+bx+c$ has a relative maximum at $(-2, 23)$ and a relative minimum at $(2,-9)$ 4

Find the values of a, b and c.

End of Paper

YR 11. ASSESSMENT TASK 1 2006

QUESTION 1

a) $3x^2+x-2$ S 3, P −6, N +3, −2

$=(3x-2)(x+1)$

b) $\frac{d}{dx}\left(\frac{x}{x+1}\right)$

$=\frac{(x+1).1-x.1}{(x+1)^2}$

$=\frac{1}{(x+1)^2}$

c) $2x^2+(k+2)x+(k+2)$

i) $\Delta=b^2-4ac$

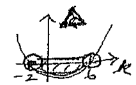
$=(k+2)^2-4\times2\times(k+2)$

$=(k+2)(k+2-8)$

$=(k+2)(k-6)$

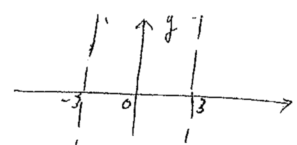
ii) No real soln if $\Delta<0$

$(k+2)(k-6)<0$



$-2<k<6$.

d)



locus $|x|=3$

or $x=\pm3$.

e) $y=x^{-1}$

$\frac{dy}{dx}=-x^{-2}$

$\frac{d^2y}{dx^2}=2x^{-3}$

QUESTION 2

a) $x^2+2\equiv A(x+1)^2+B(x+1)+C$

$x=-1$: $3=C$

By expanding $A=1$

at $x=0$: $2=A+B+C$

$B=2-A-C$

$=2-1-3$

b) $x+2y=4$

$x^2=8y$

vertex $(0,0)$

focal length $4a=8$

$a=2$

focus is $(0,2)$

Test $(0,2)$ in $x+2y=4$

$LS=0+2\times2$

$=4$

$=RS$

$\therefore$ $x+2y=4$ passes thro' the focus so it contains a focal chord.

c) $y=2x^2-x+1$

$a=2$ $\therefore a>0$ ∪

$\Delta=(-1)^2-4\times2\times1$

$=-7$ $\therefore$ no real roots

i.e. quadratic is POS. DEF.

d) $y=4x-x^2$

$\frac{dy}{dx}=4-2x$

For $\frac{dy}{dx}=-2$: $4-2x=-2$

$-2x=-6$

$x=3$

$y=3$.

gradient of normal $=\frac{1}{2}$

Eqn of normal:

$y-3=\frac{1}{2}(x-3)$

$2y-6=x-3$

$x-2y+3=0$.

QUESTION 3

a) $x^2+3x-5=0$

i) $\alpha+\beta=\frac{-b}{a}$

$=\frac{-3}{1}$

ii) $\alpha\beta=\frac{c}{a}$

$=-5$

iii) $\frac{1}{2\alpha}+\frac{1}{2\beta}=\frac{\beta+\alpha}{2\alpha\beta}$

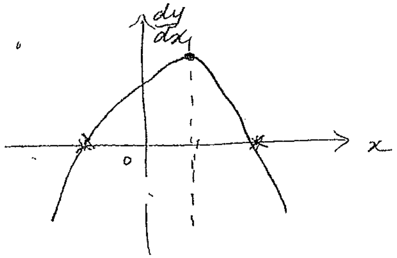
$=\frac{-3}{-10}$

$=\frac{3}{10}$

iv) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

v) $\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$
$= -3\,(19-(-5))$
$= -72$

b)



c) $\alpha = 1-\sqrt{2} \quad \beta = 1+\sqrt{2}$
$\alpha+\beta = 2$
$\alpha\beta = 1-2$
$= -1$

E quation:
$x^2 - (\alpha+\beta)x + \alpha\beta = 0$
$x^2 + 2x - 1 = 0.$

QUESTION 4

a) $y = 2x^3 + 3x^2 - 12x + 7$

i) $\frac{dy}{dx} = 6x^2 + 6x - 12$

$\frac{d^2y}{dx^2} = 12x + 6$

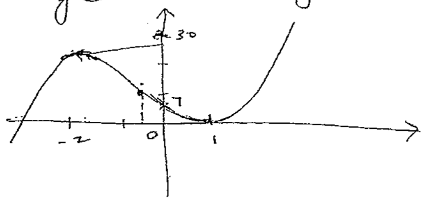
For stat pt $\frac{dy}{dx} = 0$
$x^2 + x - 2 = 0$
$(x+2)(x-1) = 0$
$x = 1, -2$
$y = 0, 27$

at $x = 1$, $\frac{d^2y}{dx^2} = 18$ ∪
∴ $(1,0)$ is MIN. T.P.
at $x = -2$, $\frac{d^2y}{dx^2} = -18$ ∩
∴ $(-2, 27)$ is MAX. T.P.

ii) For inflex. $\frac{d^2y}{dx^2} = 0$ and changes sign
$12x + 6 = 0$
$x = -\frac{1}{2}$

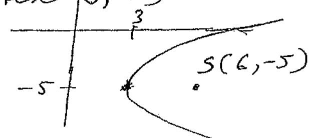
∴ $(-\frac{1}{2}, 12\frac{1}{2})$ is PT of INFLEX.

iii) y-int: $x = 0$, $y = 7$



b) $y^2 + 10y - 12x + 61 = 0$
$y^2 + 10y + 25 = 12x - 61 + 25$
$(y+5)^2 = 12(x-3)$

i) vertex $(3,-5)$



focal length $a = 3$

ii) focus $(6,-5)$

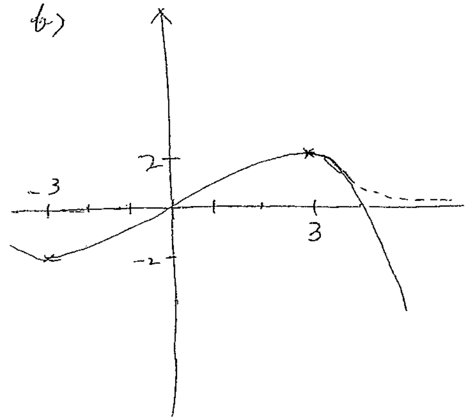
c) $y = mx - 12$
$y = 2x^2 - x - 10$
Solve sim.
$2x^2 - x - 10 = mx - 12$
$2x^2 - (m+1)x + 2 = 0$
For 1 solution $\Delta = 0$.
$(m+1)^2 - 4\times2\times2 = 0$
$(m+1)^2 = 16$
$m+1 = \pm 4$
$m = -1 \pm 4$
$= +3, -5.$
∴ Tangent if $m = 3, -5$.

QUESTION 5.

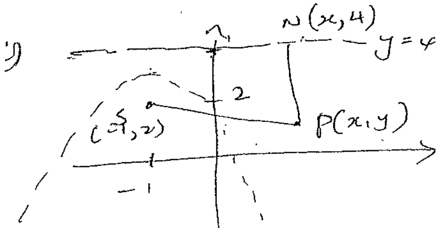
a) $f(x) = (3x-1)^4$
$f'(x) = 4(3x-1)^3 . 3$
$f''(x) = 12 . 3(3x-1)^2 . 3$
$= 108\,(3x-1)^2$

$f''(1) - f'(1) = 108(2)^2 - 12(2)^3$
$= 432 - 96$
$= 336.$

b)



i)



$PS = PN$
$PS^2 = PN^2$
$(x+1)^2 + (y-2)^2 = 0^2 + (y-4)^2$
$(x+1)^2 = y^2 - 8y + 16 - y^2 + 4y - 4$
$(x+1)^2 = -4y + 12$
$(x+1)^2 = -4(y-3)$

Locus is a parabola
vertex $(-1, 3)$, axis $x = -1$
focus $(-1, 2)$

QUESTION 6

i) $(x^2+x)^2 - 13(x^2+x) + 42 = 0$
$u = x^2 + x$
$u^2 - 13u + 42 = 0$
$(u-7)(u-6) = 0$
$u = 6, 7$

$x^2 + x = 6$
$(x+3)(x-2) = 0$
$x = 2, -3$

$x^2 + x - 7 = 0$
$x = \frac{-1 \pm \sqrt{1+28}}{2}$
$= \frac{-1 \pm \sqrt{29}}{2}.$

b) $y = \frac{x}{x^2 - 2x - 3}$

i) $x^2 - 2x - 3$
$= (x-3)(x+1)$
∴ discontinuous if $x = 3, -1$

ii) $\frac{dy}{dx} = \frac{(x^2-2x-3).1 - x(2x-2)}{(x^2-2x-3)^2}$
$= \frac{x^2 - 2x - 3 - 2x^2 + 2x}{(x-3)^2(x+1)^2}$
$= \frac{-3 - x^2}{(x-3)^2(x+1)^2}$

$x^2 \geq 0$ for all x
$-x^2 \leq 0$ for all x
∴ $-3 - x^2 < -3$ for all x
$(x-3)^2 > 0$ for all x in domain
$(x+1)^2 > 0$ for all x in domain
∴ $\frac{dy}{dx} = \frac{\text{negative}}{\text{positive} \times \text{positive}}$
< 0 for all x in domain
i.e. y is DECREASING for all x.

c) $y = ax^3 + bx + c$
$\frac{dy}{dx} = 3ax^2 + b$
$\frac{dy}{dx} = 0$ at $x = -2$
∴ $12a + b = 0$. — ①
$(-2, 23)$ on curve
$-8a - 2b + c = 23$ — ②
$(2, -9)$
$8a + 2b + c = -9$ — ③
② + ③ $\quad 2c = 14$
$c = 7$

∴ $8a + 2b + 7 = -9$
$4a + b = -8$ — ④

① − ④ $\quad 8a = 8$
$a = 1$
$b = -8 - 4a$
$b = -12$