Question 1 (10 marks)

a) Subtract $2x^4 + 4x^2 - x - 9$ from $5x^4 - 2x^3 + x^2 + 8$. **1**

b) Simplify $\dfrac{(-3x^2y^4)^3 \times (2xy^3)^2}{12x^{-2}y}$ **2**

c) Expand and simplify $(3c + 2)(c - 5) - (2c - 3)^2$ **3**

d) Factorise fully $18x^2 - 12x - 16$ **2**

e) Simplify $\dfrac{x + 2}{2} + \dfrac{x - 3}{3}$ **2**

Question 2 (10 marks)

a) Simplify $\dfrac{x^3 + 1}{x^2 + 2x + 1}$ **2**

b) Simplify $\dfrac{1}{x^2 + 2x - 8} - \dfrac{1}{x^3 - 8}$ **3**

c) Solve

i. $3(2x - 4) \leq 3(3x - 1) - x$ **3**

ii. $(3x+1)(2x+1) - 4x - 2 = 0$ **2**

Question 3 (10 marks)

a) Change the formula so that "p" is the subject.

$$\frac{1}{p} + \frac{1}{q} = 5$$ **3**

b) Find two consecutive numbers of which the sum of the squares is 145.
[First form an equation and then solve]. **3**

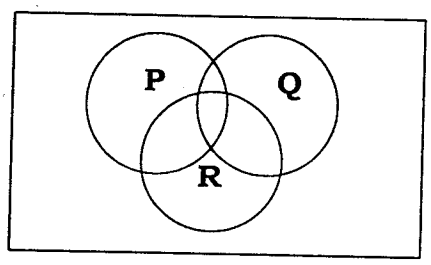
c) Solve

i. $\dfrac{1}{x-5} - \dfrac{1}{x-2} = 3$ **2**

d) Copy the following diagram twice and shade in the following sets.

i. $\overline{P \cup Q} \cap R$ **1**

ii. $P \cap (Q \cup R)$ **1**



Question 4 (10 marks)

a) Solve

i. $\left.\begin{aligned} 2c &= 3d + 11 \\ 5c + 2d &= 18 \end{aligned}\right\}$ **3**

ii. $\left.\begin{aligned} 2x^2 - 3y^2 &= 23 \\ x^2 - 2y^2 &= 7 \end{aligned}\right\}$ **3**

b) List all the subsets of the set $\{1, 3\}$. **2**

c) $|A| = 16,\ |B| = 5$

i. What is the greatest possible value of $|A \cap B|$? **1**

ii. What is the least possible value for $|A \cup B|$? **1**

Question 5 (10 marks)

a) 240 university students take part in a demonstration, all of them either carrying banners or sitting down in the road or both. 84 carry banners and 204 sit down in the road. By drawing a Venn Diagram and showing the relevant information, answer the following questions. **2**

i. How many students sit down in the road and carry banners? **1**

ii. How many students do only one of the activities? **1**

b) Write a clear definition of a rational number. **2**

c) Express $0.3\dot{4}\dot{5}$ as a fraction in its simplest form. **3**

d) From the following set of numerals list the subset including the irrational numbers: **1**

$$\left\{\sqrt[3]{27},\ \sqrt{0.09},\ \sqrt{0.144},\ \pi,\ -1.\dot{6}1\dot{5},\ -8^{\frac{1}{3}}\right\}$$

Question 6 (10 marks)

a) Expand and simplify **2**

$$\left(3\sqrt{5}+\sqrt{2}\right)\left(3\sqrt{5}-2\sqrt{2}\right)$$

b) Rationalise the denominator and simplify

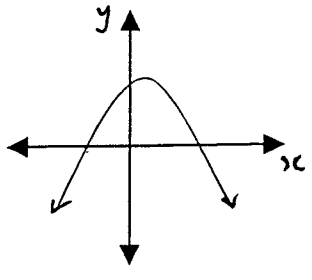
$$\frac{2\sqrt{x}-\sqrt{y}}{2\sqrt{x}+\sqrt{y}}$$

c) Find the values of a, b and c, given that c is not a perfect square, if **3**

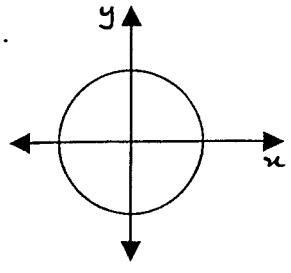
$$\left(2\sqrt{5}-\sqrt{3}\right)^2 = a+b\sqrt{c}$$

d) Do the following graphs represent functions? Give reasons for your answer. **3**

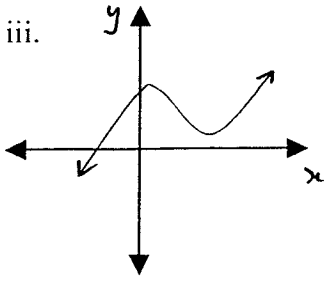
i.



ii.



iii.



Question 7 (10 marks)

a) Give the natural domain and range of

i. $y = \dfrac{1}{x - 3}$

ii. $y = \sqrt{x^2 - 3}$ **4**

b) Given $f(x) = 3x^2 + 2x - 1$

i. evaluate $f(-1)$ **1**

ii. and simplify $\dfrac{f(x + h) - f(x)}{h}$ **2**

c) Simplify $\dfrac{a - 2b}{6b^2 - ab - a^2}$ **1**

d) Simplify $\dfrac{4p^2 - 16}{p^2 + 2p + 4} \div \dfrac{p + 2}{p^4 - 8p}$ **2**

The End

ST GEORGE GIRLS HIGH SCHOOL

YEAR 11 MATHEMATICS COMMON TEST 1 2002

SOLUTIONS

Question 1

a) $5x^4 - 2x^3 + x^2 + 8 - (2x^4 + 4x^2 - x - 9)$

$= 5x^4 - 2x^3 + x^2 + 8 - 2x^4 - 4x^2 + x + 9$

$= 3x^4 - 2x^3 - 3x^2 + x + 17$

b) $\dfrac{-27x^6y^{12} \times 4x^2y^6 \times x^2}{12y}$

$= -9x^{10}y^{17}$

c) $3c^2 - 15c + 2c - 10 - (4c^2 - 12c + 9)$

$= 3c^2 - 15c + 2c - 10 - 4c^2 + 12c - 9$

$= -c^2 - c - 19$

d) $2(9x^2 - 6x - 8)$

$= 2(9x^2 + 6x - 12x - 8)$

$= 2((9x^2 + 6x) - (12x + 8))$

$= 2(3x(3x + 2) - 4(3x + 2))$

$= 2(3x + 2)(3x - 4)$

P : $-72x^2$

S : $-6x$

F : $6x, -12x$

e) $\dfrac{3(x+2)}{6} + \dfrac{2(x-3)}{6}$

$= \dfrac{3x + 6 + 2x - 6}{6}$

$= \dfrac{5x}{6}$

Question 2

a) $\dfrac{(x+1)(x^2 - x + 1)}{(x+1)^2}$

$= \dfrac{x^2 - x + 1}{x + 1}$

b) $\dfrac{1}{(x+4)(x-2)} - \dfrac{1}{(x-2)(x^2+2x+4)}$

$= \dfrac{x^2+2x+4}{(x+4)(x-2)(x^2+2x+4)} - \dfrac{(x+4)}{(x+4)(x-2)(x^2+2x+4)}$

$= \dfrac{x^2 + 2x + 4 - x - 4}{(x+4)(x-2)(x^2+2x+4)}$

$= \dfrac{x^2 + x}{(x+4)(x-2)(x^2+2x+4)}$

c)(i) $6x - 12 \leq 9x - 3 - x$

$-2x - 12 \leq -3$

$-2x \leq 9$

$x \geqslant -\dfrac{9}{2}$

(ii) $(3x + 1)(2x + 1) - 4x - 2 = 0$

$6x^2 + 3x + 2x - 4x - 2 = 0$

$6x^2 + x - 2 = 0$

$6x^2 + 4x - 3x - 2 = 0$

$(6x^2 + 4x) - (3x + 2) = 0$

$2x(3x + 2) - (3x + 2) = 0$

$(3x + 2)(2x - 1) = 0$

Either $3x + 2 = 0$ or $2x - 1 = 0$

$3x = -2$ $2x = 1$

$x = -\dfrac{2}{3}$ $x = \dfrac{1}{2}$

$(3x - 1)(2x + 1) = 0$

$x = \dfrac{1}{3}$ or $-\dfrac{1}{2}$

P : $-12x^2$

S : x

F : $4x, -3x$

Question 3

a) $\frac{1}{p} = 5 - \frac{1}{q}$

$\frac{1}{p} = \frac{5q - 1}{q}$

$p = \frac{q}{5q - 1}$

b) Let x and $x+1$ be two consecutive numbers

$x^2 + (x+1)^2 = 145$

$x^2 + x^2 + 2x + 1 = 145$

$2x^2 + 2x - 144 = 0$

$x^2 + x - 72 = 0$

$(x+9)(x-8) = 0$

Either $x + 9 = 0$ or $x - 8 = 0$

$x = -9$ $x = 8$

∴ the two consecutive numbers are either

-9 and -8 or 8 and 9

c) $\frac{1}{x-5} - \frac{1}{x-2} = 3$

$x - 2 - (x - 5) = 3(x-5)(x-2)$

$x - 2 - x + 5 = 3x^2 - 21x + 30$

$3 = 3x^2 - 21x + 30$

$3x^2 - 21x + 27 = 0$

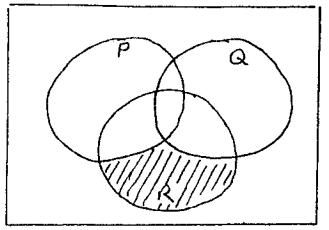
$x^2 - 7x + 9 = 0$

$x = \frac{7 \pm \sqrt{49 - 36}}{2}$

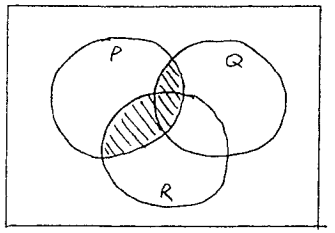
$= \frac{7 \pm \sqrt{13}}{2}$

Either $x = \frac{7 + \sqrt{13}}{2}$ or $x = \frac{7 - \sqrt{13}}{2}$

d) (i)



(ii)



Question 4

a) (i) $2c - 3d = 11$ ——— ①

$5c + 2d = 18$ ——— ②

① × 5

$10c - 15d = 55$ ——— ③

② × 2

$10c + 4d = 36$ ——— ④

③ − ④

$-19d = 19$

$d = -1$

sub. $d = -1$ into ①

$2c + 3 = 11$

$2c = 8$

$c = 4$

∴ $c = 4$ and $d = -1$

(ii) $2x^2 - 3y^2 = 23$ ——— ①

$x^2 - 2y^2 = 7$ ——— ②

② × 2

$2x^2 - 4y^2 = 14$ ——— ③

① − ③

$y^2 = 9$

$y = \pm 3$

sub. $y^2 = 9$ into ②

$x^2 - 18 = 7$

$x^2 = 25$

$x = \pm 5$

∴ the solutions are

$x = 5$, $y = 3$ or $x = -5$, $y = 3$

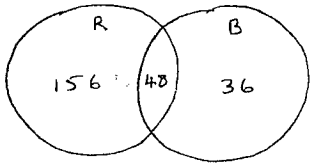
b) $\{1\}, \{3\}, \{1, 3\}, \emptyset$

c) (i) $|A \cap B| = 5$

(ii) $|A \cup B| = 16$

Question 5

a)



Let R be the set of students sitting on the road

Let B be the set of students carrying banners

(i) $|R \cap B| = 48$

(ii) $|A \cup B| - |A \cap B| = 192$

b) A rational number is any number that can be expressed as a fraction $\frac{a}{b}$ where a is an integer and b is a non zero integer

or

A rational number is a number that can be expressed as a terminating or recurring decimal

c) Let $x = 0.3454545\ldots$

$100x = 34.545454545\ldots$

$99x = 34.2$

$990x = 342$

$x = \frac{342}{990}$

$x = \frac{19}{55}$

d) $\{\sqrt{0.144}, \pi\}$ or any set that includes these two members

Question 6

a) $(3\sqrt{5})^2 - 6\sqrt{10} + 3\sqrt{10} - 4$

$= 45 - 3\sqrt{10} - 4$

$= 41 - 3\sqrt{10}$

b) $\frac{(2\sqrt{x} - \sqrt{y})(2\sqrt{x} - \sqrt{y})}{(2\sqrt{x} + \sqrt{y})(2\sqrt{x} - \sqrt{y})}$

$= \frac{4x - 2\sqrt{xy} - 2\sqrt{xy} + y}{4x - y}$

$= \frac{4x - 4\sqrt{xy} + y}{4x - y}$

c) $a + b\sqrt{c} = (2\sqrt{5} - \sqrt{3})^2$

$= 20 - 4\sqrt{15} + 3$

$= 23 - 4\sqrt{15}$

$\therefore a = 23$, $b = -4$ and $c = 15$

d) (i) Yes, every point on the curve has a unique x coordinate

(ii) No, fails the vertical line test, i.e. a vertical drawn through the graph at any value of x may cut the graph more than once

(iii) Yes, each point on the curve has a unique x coordinate

Question 7

a) (i) domain: all real x except $x = 3$ or $x \in \mathbb{R}\ x \neq 3$

range: all real y except $y = 0$ or $y \in \mathbb{R}\ y \neq 0$

(ii) domain: $x \leq -\sqrt{3}$ and $x \geq \sqrt{3}$ $x \in \mathbb{R}$

range: $y \in \mathbb{R},\ y \geq 0$

b) (i) $f(-1) = 3 - 2 - 1$

$= 0$

"

(ii) $f(x+h) = 3(x+h)^2 + 2(x+h) - 1$

$= 3(x^2 + 2xh + h^2) + 2x + 2h - 1$

$= 3x^2 + 6xh + 3h^2 + 2x + 2h - 1$

$$\frac{f(x+h) - f(x)}{h} = \frac{3x^2 + 6xh + 3h^2 + 2x + 2h - 1 - (3x^2 + 2x - 1)}{h}$$

$$= \frac{3x^2 + 6xh + 3h^2 + 2x + 2h - 1 - 3x^2 - 2x + 1}{h}$$

$$= \frac{6xh + 3h^2 + 2h}{h}$$

$$= 6x + 3h + 2$$

c) $$\frac{a - 2b}{6b^2 - ab - a^2}$$

P : $-6a^2b^2$

S : $-ab$

F : $-3ab,\ 2ab$

$$= \frac{a - 2b}{6b^2 - 3ab + 2ab - a^2}$$

$$= \frac{a - 2b}{(6b^2 - 3ab) + (2ab - a^2)}$$

$$= \frac{a - 2b}{3b(2b - a) + a(2b - a)}$$

$$= \frac{a - 2b}{(2b - a)(3b + a)}$$

$$= \frac{-1}{(3b + a)}$$

d) $$\frac{4(p^2 - 4)}{p^2 + 2p + 4} \times \frac{p(p^3 - 8)}{p + 2}$$

$$= \frac{4p(p - 2)(p + 2)(p - 2)(p^2 + 2p + 4)}{(p^2 + 2p + 4)(p + 2)}$$

$$= 4p(p - 2)^2$$