Question 1 (9 marks)

a)   Simplify $\dfrac{3y^3}{5x} \times \dfrac{(3x)^3}{18y^3}$

b)   Arrange in ascending order of size:

$\{\ 51\%,\ \ 0.502,\ \ 0.\dot{5},\ \ \dfrac{51}{99}\ \}$

c)   Evaluate $\sqrt[3]{\dfrac{8.3 \times 4.1}{5.4 - 1.3}}$ correct to 3 significant figures.

d)   Find the value of x if:

$\sqrt{3} + \sqrt{75} = \sqrt{x}$

e)   Find the value of a and b if $\dfrac{\sqrt{3}-4}{2-\sqrt{3}} = a + b\sqrt{3}$.

Question 2 (9 marks)

a)   Which of the following are rational numbers:

$\{\ \sqrt{3},\ \ -0.34,\ \ 2,\ \ 3\pi,\ \ 1.\dot{5},\ \ 0,\ \ \dfrac{3}{7}\ \}$

b)   Solve:

(i) $x^2 + 8x = 0$

(ii) $2x + 3 \le 2$

(iii) $\dfrac{x+1}{2} - \dfrac{3x-1}{3} = 6$

c)   Factorise: $2 - 16p^3$

Question 3 (9 marks)

a)   Simplify $\dfrac{2xy + 2x - 6 - 6y}{4x^2 - 16x + 12}$

b)   Expand and simplify: $(m-2)^2 - (m-2)(m+2)$

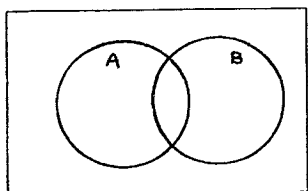c)   Simplify: $4\sqrt{48} + 3\sqrt{147} + 5\sqrt{12}$

Question 4 (9 marks)

a) Define a rational number and then show that

$$\frac{1}{3-\sqrt{2}} + \frac{1}{3+\sqrt{2}}$$ is a rational number

b) Solve: $\dfrac{5x+7}{x-1} = 3x+2$

c) Copy the diagram onto your answer page twice and shade the area indicated.

(i) $A \cap B$

(ii) $\bar{A} \cup \bar{B}$

Question 5 (9 marks)

a) Simplify: $\dfrac{2}{x^2+5x+6} - \dfrac{3}{x^2+6x+9}$

b) Factorise: $a^3b + 2a^3 - 4b - 8$

c) Solve simultaneously: $x^2 + y^2 = 9$ and $x + y = 3$

Question 6 (9 marks)

a) Solve $x^2 + 8x + 1 = 0$ giving your answer in exact simplified form.

b) One hundred and fifty Year 11 students were surveyed to find out how many liked algebra or geometry. 85 said they liked algebra and 53 said they liked geometry while 37 said they liked both. With the aid of a Venn diagram find out how many students liked:

(i) algebra or geometry or both

(ii) algebra but not geometry

(iii) only one of the two topics

(iv) neither of the topics

Question 7 (9 marks)

a) Express $0.36\dot{7}$ as a fraction in simplest form.

b) If $x^2 + y^2 + 6x - 8y + 30 = 0$ show that $(x+3)^2 + (y-4)^2 = 5$

c) Solve: $3x \le 5 < -7x$

QUESTION 1: (9 MARKS)

(a) $\dfrac{3y^2}{5x} \times \dfrac{(3x)^2}{18y^3}$

$= \dfrac{3y^2}{5x} \times \dfrac{9x^2}{18y^3}$

$= \dfrac{3x}{10y}$ ——— (1 MARK)

(b) $\{0.51, 0.502, 0.555, 0.515..\}$

$\therefore$ order is $\left\{0.502, 51\%, \dfrac{51}{99}, 0.\dot{5}\right\}$ ——— (1 MARK)

(c) $\sqrt[3]{\dfrac{8.3 \times 4.1}{5.4 - 1.3}}$

$= 2.02 \left(\begin{array}{c}\text{correct to}\\\text{3 sig. figs}\end{array}\right)$

——— (1 MARK)

(d) $\sqrt{x} = \sqrt{3} + \sqrt{75}$

$\quad = \sqrt{3} + 5\sqrt{3}$

$\quad = 6\sqrt{3}$

$\quad = \sqrt{108}$

$\therefore x = 108.$ ——— (3 MARKS)

(e) $a + b\sqrt{3} = \dfrac{\sqrt{3} - 4}{2 - \sqrt{3}} \cdot \dfrac{2 + \sqrt{3}}{2 + \sqrt{3}}$

$\quad = \dfrac{2\sqrt{3} + 3 - 8 - 4\sqrt{3}}{4 - 3}$

$\quad = \dfrac{-2\sqrt{3} - 5}{1}$

$\quad = -2\sqrt{3} - 5$

——— (3 MARKS)

$\therefore \underline{a = -5, \ b = -2}$

QUESTION 2: (9 MARKS)

(a) Rational numbers are $\{-0.34, 2, 1.\dot{5}, 0, \frac{3}{7}\}$ (1 MARK)

(b) (i) $x^2 + 8x = 0$

$x(x+8) = 0$

$\therefore x = 0, -8$ — (1 MARK)

(ii) $2x + 3 \leq 2$

$2x \leq -1$

$\therefore x \leq -\frac{1}{2}$ — (2 MARKS)

(iii) $\frac{x+1}{2} - \frac{3x-1}{3} = 6$

$\frac{3(x+1) - 2(3x-1)}{6} = 6$

$\frac{3x+3-6x+2}{6} = 6$

$\frac{5-3x}{6} = 6$

$5 - 3x = 36$

$-3x = 31$

$x = -10\frac{1}{3}$ — (3 MARKS)

(c) $2 - 16p^3 = 2(1-8p^3)$

$= 2[1^3 - (2p)^3]$

$= 2(1-2p)(1+2p+4p^2)$ — (2 MARKS)

QUESTION 3: (9 MARKS)

(a) $\dfrac{2xy + 2x - 6 - 6y}{4x^2 - 16x + 12}$

$= \dfrac{2x(y+1) - 6(1+y)}{4(x^2 - 4x + 3)}$

$= \dfrac{(y+1)(2x-6)}{4(x-3)(x-1)}$

$= \dfrac{2(y+1)(x-3)}{4(x-3)(x-1)}$

$= \dfrac{(y+1)}{2(x-1)}$ — (4 MARKS)

(b) $(m-2)^2 - (m-2)(m+2)$

$= m^2 - 4m + 4 - (m^2 - 4)$

$= \cancel{m^2} - 4m + 4 - \cancel{m^2} + 4$

$= 8 - 4m$ — (3 MARKS)

(c) $4\sqrt{48} + 3\sqrt{147} + 5\sqrt{12}$

$= 4.\sqrt{16}.\sqrt{3} + 3.\sqrt{49}.\sqrt{3} + 5.\sqrt{4}.\sqrt{3}$

$= 16\sqrt{3} + 21\sqrt{3} + 10\sqrt{3}$

$= 47\sqrt{3}$ — (2 MARKS)

QUESTION 4: (9 MARKS)

a) A rational number is one which can be expressed in the form $\frac{a}{b}$ where a, b are integers, $b \neq 0$.

$$\frac{1}{3-\sqrt{2}} + \frac{1}{3+\sqrt{2}} = \frac{3+\sqrt{2}+3-\sqrt{2}}{(3-\sqrt{2})(3+\sqrt{2})}$$

$$= \frac{6}{7}$$

$$= \frac{a}{b} \quad \text{where } a = 6, \ b = 7$$

∴ a rational number. —— (3 MARKS)

1 each part

(b) $\frac{5x+7}{x-1} = 3x+2 \qquad x \neq 1$

$5x+7 = (x-1)(3x+2)$ —— /

$5x+7 = 3x^2 + 2x - 3x - 2$ —— /

$0 = 3x^2 - 6x - 9$

$0 = x^2 - 2x - 3$ —— /

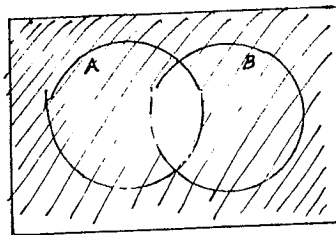$= (x-3)(x+1)$

∴ $x = 3, -1$ —— / —— (4 MARKS)

(c) (i)
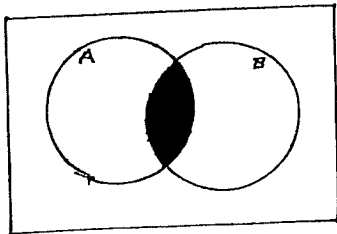
$A \cap B$

(ii)

$\bar{A} \cup \bar{B}$

—— (2 MARKS)

QUESTION 5: (9 MARKS)

(a) $\frac{2}{x^2+5x+6} - \frac{3}{x^2+6x+9}$

$$= \frac{2}{(x+2)(x+3)} - \frac{3}{(x+3)(x+3)}$$

$$= \frac{2(x+3) - 3(x+2)}{(x+2)(x+3)(x+3)}$$

$$= \frac{2x+6-3x-6}{(x+2)(x+3)(x+3)}$$

$$= \frac{-x}{(x+2)(x+3)^2}$$

—— (3 MARKS)

(b) $a^2 b + 2a^2 - 4b - 8$

$= a^2(b+2) - 4(b+2)$

$= (b+2)(a^2-4)$

$= (b+2)(a+2)(a-2)$

—— (2 MARKS)

(c) $x^2 + y^2 = 9$ —— ①

$x + y = 3$ —— ②

From ②, $y = 3-x$ sub in ①

∴ $x^2 + (3-x)^2 = 9$

$x^2 + 9 - 6x + x^2 = 9$

$2x^2 - 6x = 0$

$2x(x-3) = 0$

$x = 0, 3$ sub in ②

$y = 3, 0$

∴ $\begin{matrix} x = 0 \\ y = 3 \end{matrix} \Big] \quad \begin{matrix} x = 3 \\ y = 0 \end{matrix} \Big]$

—— (4 MARKS)

QUESTION 6: (9 MARKS)

(a)
$$x^2 + 8x + 1 = 0$$
$$\Rightarrow x^2 + 8x = -1$$
$$x^2 + 8x + 16 = 15$$
$$(x+4)^2 = 15$$
$$\therefore x + 4 = \pm\sqrt{15}$$
$$x = -4 \pm \sqrt{15}$$ — 3 MARKS

(b)
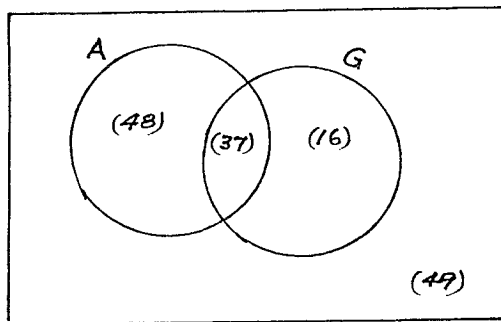
— 2 MARKS

(i) $n = 101$

(ii) $n = 48$

(iii) $n = 64$

(iv) $n = 49$. — 4 MARKS

QUESTION 7: (9 MARKS)

(a) let $x = 0.3\dot{6}\dot{7}$

ie $x = 0.367\ 367\ 367\ldots$ —①

then $1000x = 367.367\ 367\ 367\ldots$ —②

②－① $\Rightarrow$ $999x = 367$
$$x = \frac{367}{999}$$
$$\therefore 0.3\dot{6}\dot{7} = \frac{367}{999}.$$ — 3 MARKS

(b)
$$x^2 + y^2 + 6x - 8y + 30 = 0$$
$$\Rightarrow x^2 + 6x + y^2 - 8y = -30$$
$$\Rightarrow (x^2 + 6x + 9) + (y^2 - 8y + 16) = -30 + 9 + 16$$
$$\Rightarrow (x+3)^2 + (y-4)^2 = -5$$ — 3 MARK

(c)
$$3x \leq 5 \leq -7x$$
$$\Rightarrow 3x \leq 5 \text{ and } 5 \leq -7x$$
$$x \leq \frac{5}{3} \text{ and } \frac{-5}{7} \geq x$$
$$\therefore x \leq -\frac{5}{7}$$ — 3 MARKS

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