St George Girls High School

Year 11

Common Test – 1

2006

Mathematics

Time Allowed: 75 minutes

Instructions

1. All questions should be attempted.
2. Show all working.
3. START EACH QUESTION ON A NEW PAGE.
4. Marks will be deducted for careless work or poorly presented solutions.
5. On the cover sheet of the answer booklet clearly show:
   a) your name
   b) your mathematics class and teacher

Question 1 (10 marks) – **Start a New Page** Marks

a) Evaluate correct to 2 decimal places. 2

$$\sqrt[4]{\frac{624.7}{5.2^3 - 2.1^2}}$$

b) What percentage is $35¢$ of $1.12. 1

c) The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$. If a cone has a volume of 22cm^3 and a height of 2.5cm, find its radius correct to 2 significant figures. 2

d) Solve $2x + 3 = 12 - 3x$ 2

e) Simplify:

(i) $2a^2 - 7a - 5a^2 - 3a$ 2

(ii) $\frac{15ab}{4} \div \frac{5a^2b}{8}$

f) Subtract $3x^2y - 3xy^2 + x^3 - y^3$ from $x^3 + 3xy^2 + 3x^2y + y^3$ 1

Question 2 (12 marks) – **Start a New Page** Marks

a) Express $\frac{328}{495}$ as a recurring decimal. 1

b) Expand and simplify $(x-3)(x^2+7x-2)$ 2

c) Factorise: 4

(i) a^3+27

(ii) $9x^2-25x^4$

(iii) $6x^2-11x+4$

d) (i) Simplify $\sqrt{98}+2\sqrt{12}-3\sqrt{32}$

(ii) Expand and simplify $(2\sqrt{3}+1)(2-\sqrt{3})$

e) Express $\frac{2}{\sqrt{8}}$ in simplified form with a rational denominator. 1

Question 3 (10 marks) – **Start a New Page** Marks

a) Express $0.\dot{2}\dot{7}$ as a rational number (show all your working). 2

b) Solve the equations: 4

(i) $\frac{3x}{x+1}=\frac{4}{5}$

(ii) $x^2-11x+30=0$

c) If $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}-\sqrt{5}}-\frac{\sqrt{2}-\sqrt{5}}{\sqrt{2}+\sqrt{5}}=p\sqrt{10}$, find the value of p. 4

Question 4 (10 marks) – **Start a New Page** **Marks**

a) Simplify: 4

(i) $\frac{x+1}{3}-\frac{x-1}{2}$

(ii) $\frac{2}{x^2+x}+\frac{x}{x+1}$

b) Solve: 4

(i) $x^2=\sqrt{2}x$

(ii) $x^2+3x+1=0$ leaving your answer in exact form.

c) If $x=\frac{1-\sqrt{5}}{2}$, show that $1+\frac{1}{x}=x$ 2

Question 5 (10 marks) – **Start a New Page** **Mark**

a) Solve simultaneously: 3

$y=2x$
$x^2+y^2=5$

b) For the line $x+2y-10=0$ 5

(i) Find the x-intercept.

(ii) Find the y-intercept.

(iii) Sketch the line (make your diagram 8 – 10 lines).

On the same diagram add the line

(iv) $y=2x$

(v) Justify mathematically that (2, 4) is the intersection point of the two lines.

c) If $\sqrt{24}+\sqrt{54}=\sqrt{x}$ find the value of x. 2

Question 6 (10 marks) – **Start a New Page** **Marks**

a) Simplify $\dfrac{3x^2+4x-4}{x^2-4}$ 2

b) For the function $y=3x-x^2$ 6

(i) Find the y-intercept.

(ii) Find the x–intercepts.

(iii) Find the coordinates of the vertex.

(iv) Sketch the graph of the function.

(v) State the range of the function.

c) If $F(x)=x^2+5$, find in simplest form $\dfrac{F(x+h)-F(x)}{h}$ 2

Question 7 (10 marks) – **Start a New Page** **Marks**

a) Solve giving your answer to 2 decimal places. 2

$$2x^2-3x-1=0$$

b) Sketch the following graphs, clearly showing x and y intercepts. 6

(i) $x^2+y^2=16$

(ii) $y=(x+2)^2$

(iii) $y=-\sqrt{x}$

c) (i) Find the value of c if x^2-6x+c is a perfect square. 2

(ii) Hence solve $x^2-6x=1$ by completing the square.

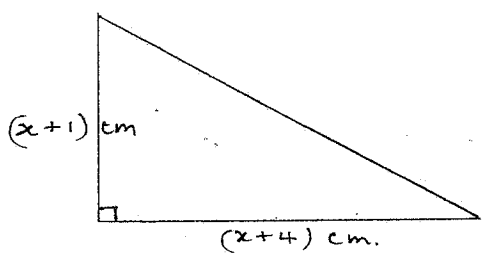
Question 8 (10 marks) – **Start a New Page** — Marks

a) Einstein's equation relating the change of mass and energy is $E = mc^2$.
Find E if $m = 1.5\times10^3$ and $c = 3\times10^8$ — 1

b) Make a the subject of $\frac{1}{a}+\frac{1}{b}=\frac{1}{c}$ — 2

c) Factorise $x^2 - y^2 + 2x - 2y$ — 2

d) — 4



The area of the triangle above is given as 20cm^2. Find the value of x.

e) State the natural domain of the function $y = \frac{x}{x^2 - 2x}$ — 1

End of Paper

YR 11 Common Test #1 2006 – 2 UNIT SOLUTIONS

Question 1:

(a) 1.46 (two decimal places)

(b) 31.25%

(c) $V = \frac{1}{3}\pi r^2 h$

$22 = \frac{1}{3}\pi r^2 \times 2.5$ then $\frac{66}{2.5\times\pi} = r^2$

$r = \sqrt{\frac{66}{2.5\times\pi}}$ (>0, measure

$r = 2.9$ (two significant fig)

(d) $2x + 3 = 12 - 3x$

$5x + 3 = 12$

$5x = 9$

$x = \frac{9}{5}$

(e) (i) $2a^2 - 7a - 5a^2 - 3a = -3a^2 - 10a$

$[-a(3a+10)]$

(ii) $\frac{15ab}{4} \times \frac{8}{5a^2b} = \frac{6}{a}$

(f) $x^3 + 3xy^2 + 3x^2y + y^3$

$-(x^3 - 3xy^2 + 3x^2y - y^3)$

$6xy^2 + 2y^3$

Question 2:

(a) $\frac{328}{495} = 0.66\dot{2}$

(b) $(x-3)(x^2+7x-2) = x^3+7x^2-2x-3x^2-21x+6$
$= x^3+4x^2-23x+6$

(c) (i) $a^3+3^3 = (a+3)(a^2-3a+9)$

(ii) $x^2(3^2-(5x)^2) = x^2(3-5x)(3+5x)$

(iii) $6x^2-8x-3x+4 = 2x(3x-4)-(3x-4)$
$= (3x-4)(2x-1)$

(d) (i) $\sqrt{49\times2}+2\sqrt{4\times3}-3\sqrt{16\times2}$
$= \sqrt{49}\times\sqrt{2}+2\times\sqrt{4}\times\sqrt{3}-3\sqrt{16}\times\sqrt{2}$
$= 7\sqrt{2}+4\sqrt{3}-12\sqrt{2}$
$= 4\sqrt{3}-5\sqrt{2}$

(ii) $(2\sqrt{3}+1)(2-\sqrt{3}) = 4\sqrt{3}-2\times3+2-\sqrt{3}$
$= 3\sqrt{3}-4$

(e) $\frac{2}{\sqrt{8}}\times\frac{\sqrt{8}}{\sqrt{8}} = \frac{2\sqrt{8}}{8}$ or $\frac{2}{\sqrt{4}\times\sqrt{2}} = \frac{2}{2\sqrt{2}}$

$= \frac{2\sqrt{4}\times\sqrt{2}}{8}$ $= \frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{4\sqrt{2}}{8}$ $= \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{2}$

Question 3

(a) let $x = 0.272727\ldots$ ... (1)

Then $100x = 27.272727\ldots$ ... (2)

(2) − (1) $99x = 27$

$x = \frac{27}{99}$

$x = \frac{3}{11}$

(b) (i) $3x\times5 = 4(x+1)$

$15x = 4x+4$

$11x = 4$

$x = \frac{4}{11}$

(ii) $(x-6)(x-5) = 0$

Let $x-6=0$ or $x-5=0$

$x = 6$ or $x = 5$

(c) Common denominator - conjugate

$\frac{(\sqrt{2}+\sqrt{5})^2-(\sqrt{2}-\sqrt{5})^2}{(\sqrt{2})^2-(\sqrt{5})^2} = \frac{(2+2\sqrt{10}+5)-(2-2\sqrt{10}+5)}{2-5}$

$= \frac{4\sqrt{10}}{-3}$

$= -\frac{4\sqrt{10}}{3}$

So $p = -\frac{4}{3}$

So $(2,4)$ is point of intersection

(c) $\sqrt{4\times6} + \sqrt{9\times6} = 2\sqrt{6} + 3\sqrt{6}$

$= 5\sqrt{6}$

$= \sqrt{25} \times \sqrt{6}$

$= \sqrt{150}$

$\therefore x = 150$

Question 6

(a) $\dfrac{(3x-2)(x+2)}{(x-2)(x+2)} = \dfrac{3x-2}{x-2}$

(b) (i) let $x=0$, $y = 0 - 0$

$y = 0$

$(0,0)$

(ii) let $y=0$, $x(3-x)=0$

$x=0$ and $x=3$

at $(0,0)$ and $(3,0)$

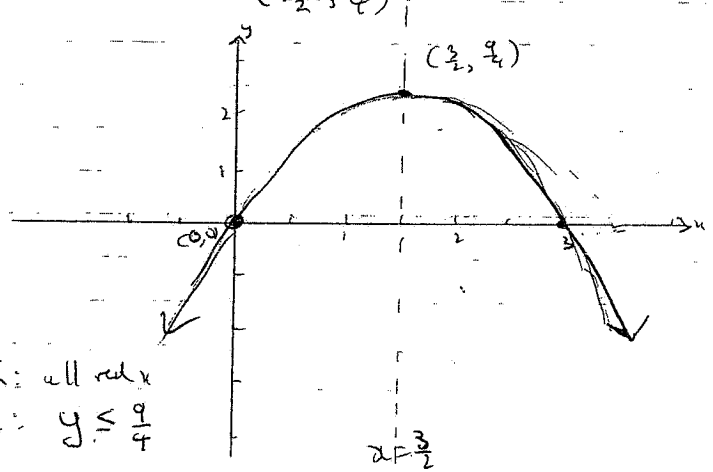
(iii) Vertex lies on axis or $x = \frac{-3}{2x-1}$

$x = \frac{0+3}{2}$

$x = \frac{3}{2}$ $x = \frac{3}{2}$

$y = 3\times\frac{3}{2} - \left(\frac{3}{2}\right)^2$

So Vertex $\left(\frac{3}{2}, \frac{9}{4}\right)$



(iv) Domain: all real x

Range: $y \le \frac{9}{4}$

(c) $F(x+h) = (x+h)^2 + 5$

$= x^2 + 2xh + h^2 + 5$

$\therefore \dfrac{F(x+h) - F(x)}{h} = \dfrac{x^2+2xh+h^2+5 - (x^2+5)}{h}$

$= \dfrac{h(2x+h)}{h}$

$= 2x + h$

QUESTION 7

(a) $a=2$, $b=-3$, $c=-1$

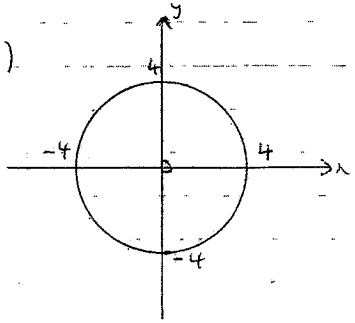
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4\times 2\times -1}}{2\times 2}$$

$$= \frac{3 \pm \sqrt{17}}{4}$$

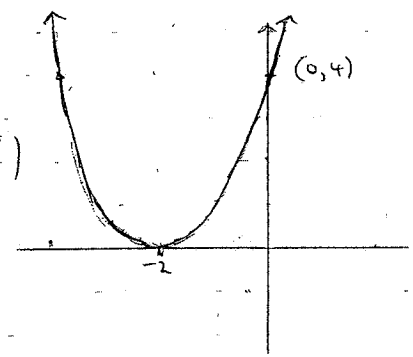
$$x_1 = \frac{3-\sqrt{17}}{4} \quad \text{and} \quad x_2 = \frac{3+\sqrt{17}}{4}$$

$\doteq -0.28$ (two dec. places) $\qquad \doteq 1.78$ (two dec. places)

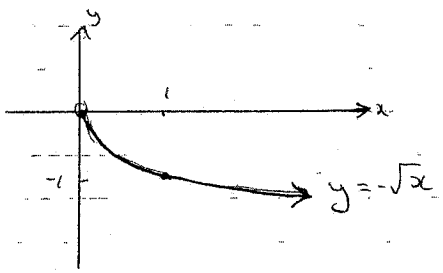
(b) (i)



(ii)



(iii)



(c) (i) $x^2 - 6x + (-3)^2 = x^2 - 6x + 9 \quad \therefore c = 9$

(ii) $x^2 - 6x + 9 = 1 + 9$

$(x-3)^2 = 10$

$x - 3 = \pm\sqrt{10}$

QUESTION 8

(a) $E = (1.5\times10^3) \times (3\times10^8)^2$

$= 1.5\times10^3 \times 9 \times 10^{16}$

$= 13.5 \times 10^{19}$

$= 1.35 \times 10^{20}$

(b) $\frac{1}{a} = \frac{1}{c} - \frac{1}{b}$.

$\frac{1}{a} = \frac{b-c}{bc}$

$\therefore a = \frac{bc}{b-c}$

(c) $(x-y)(x+y) + 2(x-y)$

$= (x-y)(x+y+2)$

(d) $A = \frac{1}{2}bh \quad \therefore \quad 20 = \frac{1}{2}(x+1)(x+4)$

$40 = x^2 + 5x + 4$

Solve $x^2 + 5x - 36 = 0$

$(x+9)(x-4) = 0$

$x > 0$, since measurement

So $x = 4$

(e) Domain : $x^2 - 2x \neq 0$

$x(x-2) \neq 0$

$x \neq 0 \quad , \quad x \neq 2$

Domain all real x ; $x \neq 0$, $x \neq 2$.

Question 4:

(a) (i) $\frac{2(x+1)}{6} - \frac{3(x-1)}{6} = \frac{2x+2-3x+3}{6}$

$= \frac{5-x}{6}$

(ii) $\frac{2}{x(x+1)} + \frac{x}{x+1} = \frac{2}{x(x+1)} + \frac{x \times x}{x(x+1)}$

$= \frac{2+x^2}{x^2+x}$

(b) (i) $x^2 = \sqrt{2}.x$

$x^2 - \sqrt{2}x = 0$

$x(x-\sqrt{2}) = 0$

$\therefore x = 0$ or $x = \sqrt{2}$

(ii) $a=1$, $b=3$, $c=1$

$x = \frac{-3 \pm \sqrt{3^2-4\times1\times1}}{2\times1}$

$= \frac{-3 \pm \sqrt{5}}{2}$

$a_1 = \frac{-3-\sqrt{5}}{2}$ and $a_2 = \frac{-3+\sqrt{5}}{2}$

(c) $\frac{1}{x}$ reciprocal of x

$= \frac{2}{1-\sqrt{5}} \times \frac{(1+\sqrt{5})}{(1+\sqrt{5})}$

$= \frac{2(1+\sqrt{5})}{1-5}$

$= \frac{2(1+\sqrt{5})}{-4}$

$= -\frac{(1+\sqrt{5})}{2}$

So $1 + \frac{1}{x}$

$= 1 + -\frac{(1+\sqrt{5})}{2}$

$= 2 - \frac{(1+\sqrt{5})}{2}$

$= \frac{1-\sqrt{5}}{2}$

Question 5

(a) $y = 2x$ ... ①

$x^2+y^2 = 5$ ... ②

Sub ① into ② $x^2 + (2x)^2 = 5$

$x^2 + 4x^2 = 5$

$5x^2 = 5$

$5(x^2-1) = 0$

$5(x-1)(x+1)$

So $x = 1, -1$ $(1, 2)$

$y = 2, -2$ $(-1, -2)$

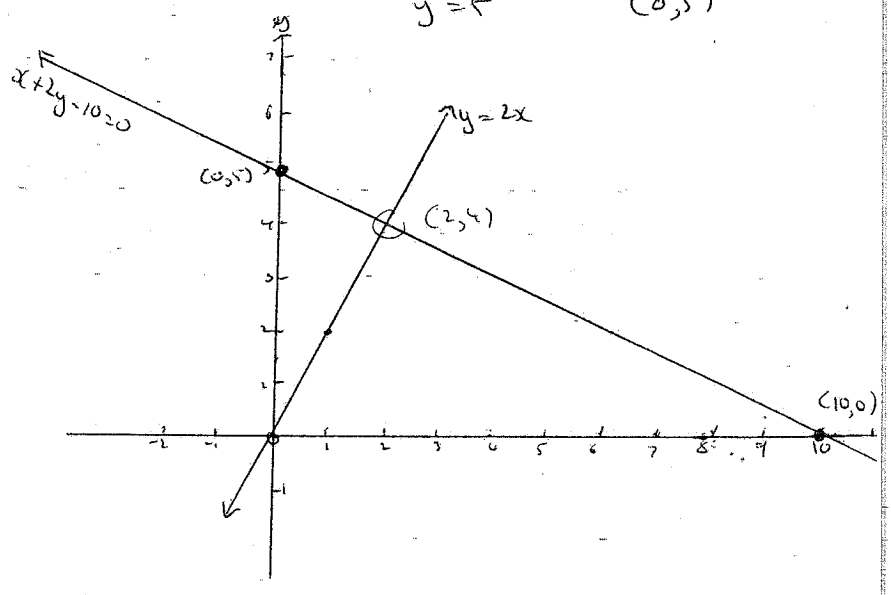
(b) (i) let $y=0$, $x + 0 - 10 = 0$

$x = 10$ $(10, 0)$

(ii) let $x=0$, $0 + 2y - 10 = 0$

$y = 5$ $(0, 5)$

(iii) (iv)



(v) $2 + 2\times4 - 10 = 0$ & $4 = 2\times2$

(2,4) lies on the (2,4) lies on the