Angelica Tjokrowidjaj
Miss O'Dea

St George Girls High School

Year 11

Common Test – 1

2006

Mathematics

Time Allowed: 75 minutes

Instructions

1. All questions should be attempted.
2. Show all working.
3. START EACH QUESTION ON A NEW PAGE.
4. Marks will be deducted for careless work or poorly presented solutions.
5. On the cover sheet of the answer booklet clearly show:

   a) your name
   b) your mathematics class and teacher

Question 1 (10 marks) – **Start a New Page**   Marks

a) Evaluate correct to 2 decimal places.

$$\sqrt[4]{\frac{624.7}{5.2^3 - 2.1^2}}$$

                    2

b) What percentage is 35¢ of $1.12.   1

c) The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$. If a cone has a volume of 22cm^3 and a height of 2.5cm, find its radius correct to 2 significant figures.   2

d) Solve $2x + 3 = 12 - 3x$ 2

e) Simplify:

   (i) $2a^2 - 7a - 5a^2 - 3a$ 2

   (ii) $\frac{15ab}{4} \div \frac{5a^2 b}{8}$

f) Subtract $3x^2 y - 3xy^2 + x^3 - y^3$ from $x^3 + 3xy^2 + 3x^2 y + y^3$ 1

Question 2 (12 marks) – **Start a New Page**

Marks

a) Express $\dfrac{328}{495}$ as a recurring decimal.

1

b) Expand and simplify $(x-3)(x^2+7x-2)$

2

c) Factorise:

4

    (i) a^3+27

    (ii) $9x^2-25x^4$

    (iii) $6x^2-11x+4$

d) (i) Simplify $\sqrt{98}+2\sqrt{12}-3\sqrt{32}$

    (ii) Expand and simplify $(2\sqrt{3}+1)(2-\sqrt{3})$

e) Express $\dfrac{2}{\sqrt{8}}$ in simplified form with a rational denominator.

1

Question 3 (10 marks) – **Start a New Page**

Marks

a) Express $0.2\dot{7}$ as a rational number (show all your working).

2

b) Solve the equations:

4

    (i) $\dfrac{3x}{x+1}=\dfrac{4}{5}$

    (ii) $x^2-11x+30=0$

c) If $\dfrac{\sqrt{2}+\sqrt{5}}{\sqrt{2}-\sqrt{5}}-\dfrac{\sqrt{2}-\sqrt{5}}{\sqrt{2}+\sqrt{5}}=p\sqrt{10}$, find the value of p.

4

Question 4 (10 marks) – Start a New Page

Marks

a) Simplify:

4

(i) $\dfrac{x+1}{3} - \dfrac{x-1}{2}$

(ii) $\dfrac{2}{x^2+x} + \dfrac{x}{x+1}$

b) Solve:

4

(i) $x^2 = \sqrt{2}x$

(ii) $x^2 + 3x + 1 = 0$ leaving your answer in exact form.

c) If $x = \dfrac{1-\sqrt{5}}{2}$, show that $1 + \dfrac{1}{x} = x$

2

Question 5 (10 marks) – Start a New Page

Mark

a) Solve simultaneously:

3

$$y = 2x$$
$$x^2 + y^2 = 5$$

b) For the line $x + 2y - 10 = 0$

5

(i) Find the x-intercept.

(ii) Find the y-intercept.

(iii) Sketch the line (make your diagram 8 – 10 lines).

On the same diagram add the line

(iv) $y = 2x$

(v) Justify mathematically that $(2, 4)$ is the intersection point of the two lines.

c) If $\sqrt{24} + \sqrt{54} = \sqrt{x}$ find the value of x.

2

Question 6 (10 marks) – Start a New Page

Marks

a) Simplify $\dfrac{3x^2 + 4x - 4}{x^2 - 4}$

2

b) For the function $y = 3x - x^2$

6

    (i) Find the y-intercept.

    (ii) Find the x-intercepts.

    (iii) Find the coordinates of the vertex.

    (iv) Sketch the graph of the function.

    (v) State the range of the function.

c) If $F(x) = x^2 + 5$, find in simplest form $\dfrac{F(x+h) - F(x)}{h}$

2

Question 7 (10 marks) – Start a New Page

Marks

a) Solve giving your answer to 2 decimal places.

2

$$2x^2 - 3x - 1 = 0$$

b) Sketch the following graphs, clearly showing x and y intercepts.

6

    (i) $x^2 + y^2 = 16$

    (ii) $y = (x + 2)^2$

    (iii) $y = -\sqrt{x}$

c) (i) Find the value of c if $x^2 - 6x + c$ is a perfect square.

2

    (ii) Hence solve $x^2 - 6x = 1$ by completing the square.

Question 8 (10 marks) – Start a New Page   Marks

a) Einstein's equation relating the change of mass and energy is $E = mc^2$.
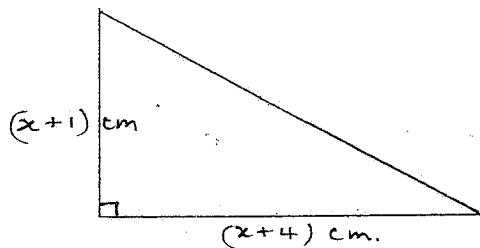   Find E if $m = 1.5 \times 10^3$ and $c = 3 \times 10^8$ 1

b) Make a the subject of $\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{1}{c}$ 2

c) Factorise $x^2 - y^2 + 2x - 2y$ 2

d)   4

The area of the triangle above is given as 20cm^2. Find the value of x.

e) State the natural domain of the function $y = \dfrac{x}{x^2 - 2x}$ 1

End of Paper

YR 11   COMMON TEST #1   2006 – 2 UNIT SOLUTIONS

Question 1:

(a) 1.46 (two decimal places)

(b) 31.25%

(c) $V = \frac{1}{3}\pi r^2 h$

$22 = \frac{1}{3}\pi r^2 \times 2.5$ then $\dfrac{66}{2.5 \times \pi} = r^2$

$r = \sqrt{\dfrac{66}{2.5 \times \pi}}$ $(>0, \text{measure})$

$r = 2.9$ (two significant fig)

(d) $2x + 3 = 12 - 3x$
      $5x + 3 = 12$
      $5x = 9$
      $x = \frac{9}{5}$

(e) (i) $2a^2 - 7a - 5a^2 - 3a = -3a^2 - 10a$
        $[-a(3a + 10)]$

   (ii) $\dfrac{15ab}{4} \times \dfrac{8}{5a^2 b} = \dfrac{6}{a}$

(f) $\dfrac{x^3 + 3xy^2 + 3x^2y + y^3 - (x^3 - 3xy^2 + 3x^2y - y^3)}{6xy^2 + 2y^3}$

Question 2:

(a)
$$\frac{328}{495} = 0.66\dot{2}$$

(b)
$$(x-3)(x^2+7x-2) = x^3+7x^2-2x-3x^2-21x+6$$
$$= x^3+4x^2-23x+6$$

(c) (i) $\quad a^3+3^3 = (a+3)(a^2-3a+9)$

(ii) $\quad x^2(3^2-(5x)^2) = x^2(3-5x)(3+5x)$

(iii) $\quad 6x^2-8x-3x+4 = 2x(3x-4)-(3x-4)$
$$= (3x-4)(2x-1)$$

(d) (i) $\quad \sqrt{49\times2} + 2\sqrt{4\times3} - 3\sqrt{16\times2}$
$$= \sqrt{49}\times\sqrt{2} + 2\times\sqrt{4}\times\sqrt{3} - 3\sqrt{16}\times\sqrt{2}$$
$$= 7\sqrt{2} + 4\sqrt{3} - 12\sqrt{2}$$
$$= 4\sqrt{3} - 5\sqrt{2}$$

(ii) $\quad (2\sqrt{3}+1)(2-\sqrt{3}) = 4\sqrt{3} - 2\times3 +2 - \sqrt{3}$
$$= 3\sqrt{3} - 4$$

(e)
$$\frac{2}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}} = \frac{2\sqrt{8}}{8}$$
$$= \frac{2\sqrt{4\times2}}{8}$$
$$= \frac{4\sqrt{2}}{8}$$
$$= \frac{\sqrt{2}}{2}$$

or $\quad \dfrac{2}{\sqrt{4}\times\sqrt{2}} = \dfrac{k}{2\sqrt{2}}$
$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{\sqrt{2}}{2}$$

Question 3

(a) Let $x = 0.272727\ldots$ ——①
Then $100x = 27.272727\ldots$ ——②

②−① $\quad 99x = 27$
$$x = \frac{27}{99}$$
$$x = \frac{3}{11}$$

(b) (i) $\quad 3x\times5 = 4(x+1)$
$$15x = 4x+4$$
$$11x = 4$$
$$x = \frac{4}{11}$$

(ii) $\quad (x-6)(x-5) = 0$
Let $x-6=0$ or $x-5=0$
$$x=6 \quad \text{or} \quad x=5$$

(c) Common Denominator - conjugate
$$\frac{(\sqrt{2}+\sqrt{5})^2 - (\sqrt{2}-\sqrt{5})^2}{(\sqrt{2})^2 - (\sqrt{5})^2} = \frac{(2+2\sqrt{10}+5) - (2-2\sqrt{10}+5)}{2-5}$$
$$= \frac{4\sqrt{10}}{-3}$$
$$= -\frac{4\sqrt{10}}{3}$$

So $\quad p = -\frac{4}{3}$

Question 4:

(a). (i) $\dfrac{2(x+1)}{6} - \dfrac{3(x-1)}{6} = \dfrac{2x+2-3x+3}{6}$

$$= \dfrac{5-x}{6}$$

(ii) $\dfrac{2}{x(x+1)} + \dfrac{x}{x+1} = \dfrac{2}{x(x+1)} + \dfrac{x \times x}{x(x+1)}$

$$= \dfrac{2+x^2}{x^2+x}$$

(b) (i) $x^2 = \sqrt{2}\cdot x$

$x^2 - \sqrt{2}x = 0$

$x(x-\sqrt{2}) = 0$

$\therefore x = 0 \quad \text{or} \quad x = \sqrt{2}$

(ii) $\left.\begin{array}{l} a=1 \\ b=3 \\ c=1 \end{array}\right\}$ $\quad x = \dfrac{-3 \pm \sqrt{3^2 - 4 \times 1 \times 1}}{2 \times 1}$

$$= \dfrac{-3 \pm \sqrt{5}}{2}$$

$x_1 = \dfrac{-3-\sqrt{5}}{2} \quad \text{and} \quad x_2 = \dfrac{-3+\sqrt{5}}{2}$

(c) $\dfrac{1}{x}$ reciprocal of x

$= \dfrac{2}{1-\sqrt{5}} \times \dfrac{(1+\sqrt{5})}{(1+\sqrt{5})}$

$= \dfrac{2(1+\sqrt{5})}{1-5}$

$= \dfrac{2(1+\sqrt{5})}{-4}$

$= -\dfrac{(1+\sqrt{5})}{2}$

So $\quad 1 + \dfrac{1}{x}$

$= 1 + \dfrac{-(1+\sqrt{5})}{2}$

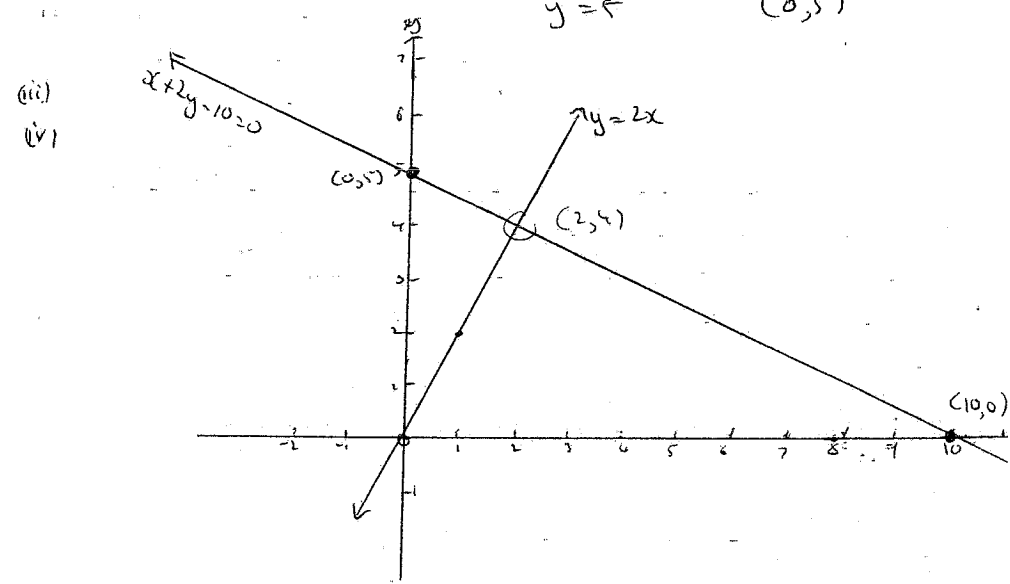$= 2 - \dfrac{(1+\sqrt{5})}{2}$

$= \dfrac{1-\sqrt{5}}{2}$

—4—

Question 5:

(a) $y = 2x \quad \cdots \text{①}$

$x^2 + y^2 = 5 \quad \cdots \text{②}$

Sub ① into ② $\quad x^2 + (2x)^2 = 5$

$x^2 + 4x^2 = 5$

$5x^2 = 5$

$5(x^2-1) = 0$

$5(x-1)(x+1)$

So $\quad x = 1, -1 \qquad (1, 2)$

th $\quad y = 2, -2 \qquad (-1, -2)$

(b) (i) let $y=0$, $\quad x + 0 - 10 = 0$

$x = 10 \qquad (10, 0)$

(ii) let $x=0$, $\quad 0 + 2y - 10 = 0$

$y = 5 \qquad (0, 5)$

(iii)

(iv)

(v) $2 + 2 \times 4 - 10 = 0 \qquad \& \qquad 4 = 2 \times 2$

$(2,4)$ lies on the $\qquad (2,4)$ lies on the

So $(3,4)$ is point of intersection

(c) $\quad \sqrt{4 \times 6} + \sqrt{9 \times 6} = 2\sqrt{6} + 3\sqrt{6}$
$$= 5\sqrt{6}$$
$$= \sqrt{25} \times \sqrt{6}$$
$$= \sqrt{150}$$
$$\therefore x = \underline{150}$$

Question 6

(a) $\quad \dfrac{(3x-2)(x+2)}{(x-2)(x+2)} = \dfrac{3x-2}{x-2}$

(b) (i) let $x=0$, $\quad y = 0-0$
$$y = 0$$
$$(0,0)$$

(ii) let $y=0$, $\quad x(3-x)=0$
$$x=0 \text{ and } x=3$$
$$\text{at } (0,0) \text{ and } (3,0)$$

(iii) Vertex lies on axis $\qquad$ or $\quad x = \dfrac{-3}{2 \times -1}$
$$x = \dfrac{0+3}{2}$$
$$x = \dfrac{3}{2}$$
$$y = 3 \times \tfrac{3}{2} - \left(\tfrac{3}{2}\right)^2 \quad x = \tfrac{3}{2}$$
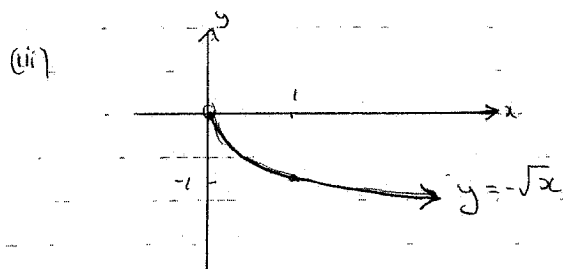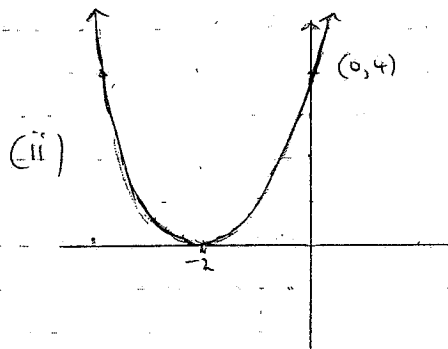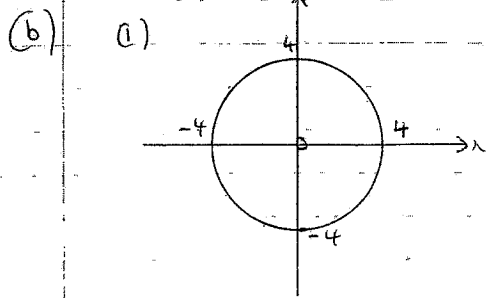$$\text{So Vertex } \left(\tfrac{3}{2}, \tfrac{9}{4}\right)$$

$(\tfrac{3}{2}, \tfrac{9}{4})$

$x = \tfrac{3}{2}$

(v) Domain: all real x
Range: $y \leq \dfrac{9}{4}$

(c) $F(x+h) = (x+h)^2 + 5$
$$= x^2 + 2xh + h^2 + 5$$

$\therefore \dfrac{F(x+h) - F(x)}{h} = \dfrac{x^2 + 2xh + h^2 + 5 - (x^2 + 5)}{h}$
$$= \dfrac{h(2x+h)}{h}$$
$$= 2x + h$$

QUESTION 7

(a) $a=2$, $b=-3$, $c=-1$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 2 \times -1}}{2 \times 2}$$

$$= \frac{3 \pm \sqrt{17}}{4}$$

$$x_1 = \frac{3 - \sqrt{17}}{4} \quad \text{and} \quad x_2 = \frac{3 + \sqrt{17}}{4}$$

$$\doteq -0.28 \qquad\qquad \doteq 1.78$$

(two dec. places)   (two dec. places)

(b) (i)

(ii)

(0,4)

(iii)

$y = -\sqrt{x}$

(c) (i) $x^2 - 6x + (-3)^2 = x^2 - 6x + 9 \qquad \therefore c = 9$

(ii) $x^2 - 6x + 9 = 1 + 9$

$(x - 3)^2 = 10$

$x - 3 = \pm \sqrt{10}$

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QUESTION 8

(a)
$$E = (1.5 \times 10^3) \times (3 \times 10^8)^2$$
$$= 1.5 \times 10^3 \times 9 \times 10^{16}$$
$$= 13.5 \times 10^{19}$$
$$= 1.35 \times 10^{20}$$

(b)
$$\frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$
$$\frac{1}{a} = \frac{b - c}{bc}$$
$$\therefore a = \frac{bc}{b - c}$$

(c)
$$(x-y)(x+y) + 2(x-y)$$
$$= (x-y)(x+y+2)$$

(d) $A = \frac{1}{2}bh \qquad \therefore \quad 20 = \frac{1}{2}(x+1)(x+4)$

$40 = x^2 + 5x + 4$

Solve $x^2 + 5x - 36 = 0$

$(x + 9)(x - 4) = 0$

$x > 0$, since measurement

So $x = 4$

(e) Domain: $x^2 - 2x \neq 0$

$x(x - 2) \neq 0$

$x \neq 0 \quad , \quad x \neq 2$

Domain all real x ; $x \neq 0$, $x \neq 2$.

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