St George Girls High School Angelica Tjokrowidjaja

Year 11

Common Test – 1

2006

Mathematics Extension 1

Time Allowed: 75 minutes

Marks: 60

Instructions

1. All questions should be attempted.
2. Show all working.
3. START EACH QUESTION ON A NEW PAGE.
4. Marks will be deducted for careless work or poorly presented solutions.
5. On the cover sheet of the answer booklet clearly show:
   a) your name
   b) your mathematics class and teacher

Question 1 (10 marks) – **Start a New Page** **Marks**

a) Factorise:

(i) $x^2 - 2x - y^2 + 1$ 2

(ii) $3y^4 - 9y^2 - 12$ 3

b) Simplify $\dfrac{x^3+1}{x^2-6x+5} \div \dfrac{x^3-x^2+x}{x^2-x}$ 3

c) Solve $2x^2 - 5x = 4$ by completing the square. 2

Question 2 (10 marks) – **Start a New Page** **Marks**

a) Simplify $\dfrac{2}{x^2-6x+8}+\dfrac{1}{x^2-5x+6}$ 2

b) Solve: 4

$$x+y+z=6$$
$$2x+3y+z=13$$
$$x+2y-z=5$$

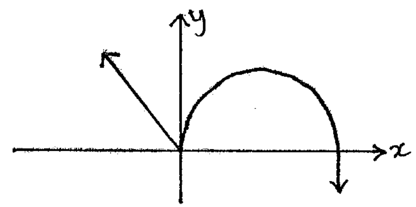
c) A swimming pool is filled by 2 pipes in 12 hours. The smaller pipe by itself takes 7 hours longer than the larger pipe by itself to fill the pool. How long does the larger pipe take to fill the pool? 4

Question 3 (10 marks) – **Start a New Page** **Marks**

a) For $\left(x+y\sqrt{3}\right)^2=37-20\sqrt{3}$ find the values of x and y, given they are rational. 3

b) Describe how the graph of $y=x^3$ can be shifted to give the graph of $y=4-(x-1)^3$ 2

c) Below is the graph of $y=f(x)$ 2



Sketch the graphs of:

(i) $y=-f(x)$

(ii) $y=f(-x)$

d) (i) Sketch $y=x^2+2x-1$ 3

(ii) Restrict the domain of $y=x^2+2x-1$ so that the inverse function $y=f^{-1}(x)$ exists.

Question 4 (10 marks) – **Start a New Page** **Marks**

a) (i) Given $f(x)=\sqrt[3]{x-1}$, find $f^{-1}(x)$ 6

(ii) Sketch the graphs of $y=f(x)$ and $y=f^{-1}(x)$ on the same number plane.

(iii) Explain why the inverse is a function.

b) Sketch the graph of $y=9-x^2$, showing all intercepts with the co-ordinate axes and the vertex. 2

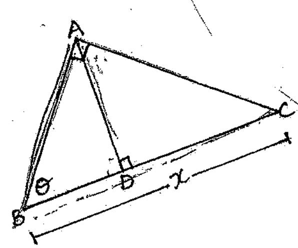
c) Find the domain and range of $y=\dfrac{1}{\sqrt{x^2-9}}$ 2

Question 5 (10 marks) – **Start a New Page** **Marks**

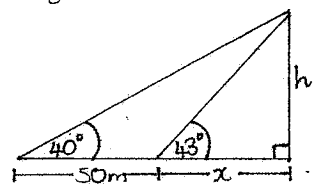
a) In ΔABC, $\angle BAC=90^\circ$, $\angle ABC=\theta$, $BC=x$ 3

(i) Show that $AB=x\cos\theta$

(ii) Hence, show $BD=x\cos^2\theta$



b) From a boat the window of a lighthouse is seen at an angle of elevation of 40°. After moving towards the lighthouse a distance of 50m, the angle of elevation is found to be 43°. 3



(i) Show that $h=x\tan 43^\circ$ and $h=(50+x)\tan 40^\circ$

(ii) Hence, find how far the boat is now from the lighthouse.

c) Find the exact value of: 2

(i) $\cos 510^\circ$

(ii) $\tan(-120^\circ)$

d) Form a cartesian equation by eliminating θ from this pair of equations:

$x=a\sec\theta,\ y=b\tan\theta$ 2

Question 6 (10 marks) – **Start a New Page** **Marks**

a) Solve, where $0^\circ \le x \le 360^\circ$: 5

(i) $4\sin^2 x - 3 = 0$.

(ii) $\cos(2x + 10^\circ) = \frac{-\sqrt{3}}{2}$

b) A ship sails from Port A for 50 nautical miles due east to Port B. It then proceeds a distance of 20 nautical miles in the direction 160°T to Port C. 5

Find the:

(i) Distance of Port C from Port A, correct to two decimal places.

(ii) Bearing of C from A, to the nearest degree.

End of Paper

-1-

Year 11 Maths Ext. 1
CT #1 2006
Solutions

Q1 a) (i) $x^2 - 2x - y^2 + 1$ (2)

$= x^2 - 2x + 1 - y^2$

$= (x-1)^2 - y^2$

$= (x - 1 + y)(x - 1 - y)$

(ii) $3y^4 - 9y^2 - 12$ (3)

$3y^2 \quad -12$

$y^2 \quad +1$

$= (3y^2 - 12)(y^2 + 1)$

$= 3(y^2 - 4)(y^2 + 1)$

$= 3(y+2)(y-2)(y^2+1)$

b) $\frac{x^3+1}{x^2-6x+5} \div \frac{x^3 - x^2 + x}{x^2 - x}$ (3)

$= \frac{(x+1)(x^2 - x + 1)}{(x-5)(x-1)} \times \frac{\cancel{x}(x-1)}{\cancel{x}(x^2 - x + 1)}$

$= \frac{x+1}{x-5}$

c) $2x^2 - 5x = 4$ (2)

i.e. $x^2 - \frac{5}{2}x = 2$

$x^2 - \frac{5}{2}x + \left(\frac{5}{4}\right)^2 = 2 + \left(\frac{5}{4}\right)^2$

$\left(x - \frac{5}{4}\right)^2 = 2 + \frac{25}{16} = \frac{57}{16}$

$\therefore x - \frac{5}{4} = \frac{\pm\sqrt{57}}{4}$ $\quad \therefore x = 5 \pm \sqrt{57}$

Q2 a) $\dfrac{2}{x^2-6x+8} + \dfrac{1}{x^2-5x+6}$

$= \dfrac{2}{(x-4)(x-2)} + \dfrac{1}{(x-3)(x-2)}$

$= \dfrac{2(x-3)+x-4}{(x-4)(x-2)(x-3)}$ (2)

$= \dfrac{3x-10}{(x-4)(x-2)(x-3)}$

b) $x+y+z=6$ (1)
$2x+3y+z=13$ (2)
$x+2y-z=5$ (3).

(1) + (3) $\Rightarrow$ $2x+3y=11$. (4) $\xrightarrow{\times 3}$ $6x+9y=33$ (6)
(2) + (3) $\Rightarrow$ $3x+5y=18$ (5) $\xrightarrow{\times 2}$ $6x+10y=36$ (7)

(7) − (6) $\Rightarrow$ $y=3$
Sub $y=3$ into (4) $\Rightarrow$ $2x+9=11$
$\therefore x=1$.

Sub $x=1$, $y=3$ into (1)
$\Rightarrow$ $z=2$

(1) ✓
$2+9+2=13$ (2) ✓
$1+6-2=5$ (3) ✓

ie $x=1$, $y=3$, $z=2$

c) Let V = volume of pool (in units^3)
Let T = time taken for larger pipe to fill pool.
$\therefore$ $T+7$ = time taken for small pipe to fill pool.

$\therefore$ In 1 hour, larger pipe fills $\frac{V}{T}$ units^3
smaller " " $\frac{V}{T+7}$ "

$\therefore$ $12\left(\frac{V}{T} + \frac{V}{T+7}\right) = V$ (12h for both to fill pool)

ie $\frac{12}{T} + \frac{12}{T+7} = 1$ $\times T(T+7)$

$\Rightarrow$ $12(T+7)+12T = T^2+7T$. (4)
$24T+84 = T^2+7T$
$T^2-17T-84=0$
$T = \dfrac{17 \pm \sqrt{625}}{2}$ or T −21, T +4
$= \dfrac{17 \pm 25}{2}$,

$\therefore T = \dfrac{17+25}{2} = 21.$

ie it takes larger pipe 21 hours to fill ~~the~~ pool.

Q3 a) $(x+y\sqrt{3})^2 = 37-20\sqrt{3}$.

LHS $= x^2+2xy\sqrt{3}+3y^2$.

$\therefore$ $x^2+3y^2 = 37$ ① on equating like parts.

$2xy = -20$ ② (1)

ie $y = \frac{-10}{x}$ from ②

sub $y = -\frac{10}{x}$ into ①

$\Rightarrow x^2 + 3 \cdot \frac{100}{x^2} = 37$.

$x^4 + 300 = 37x^2$

$x^4 - 37x^2 + 300 = 0$ (1) (3)

Let $u = x^2$

$\therefore u^2 - 37u + 300 = 0$

u -25

u -12

$\therefore u = 12, 25$

ie $x^2 = 12$ or 25.

Now, x is rational $\therefore x^2 = 25$

ie $x = \pm 5$. (1)

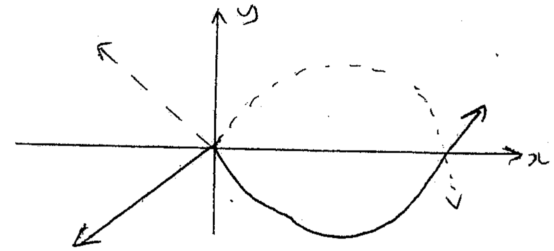
b) $y = x^3 \rightarrow y = \cancel{x}\ 4-(x-1)^3$.

$\frac{1}{2}$ Translation of 1 unit to right

1 Reflection in X-axis

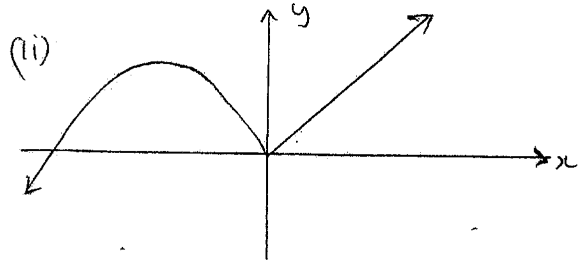
$\frac{1}{2}$ Translation of 4 units upwards $\}$ -1pm (2)

c) (i)



$y = -f(x)$ (1)

(ii)



(1)

$\frac{1}{2}$ → x-intercepts

$\frac{1}{2}$ → y-intercept

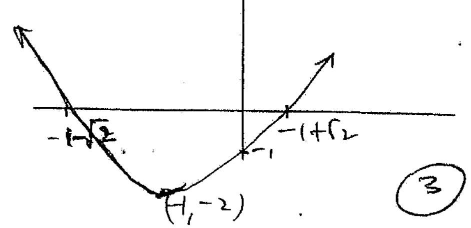
1 → vertex.

d)(i) $y = x^2+2x-1$

$= (x+1)^2 - 2$.

ie $y+2 = (x+1)^2$

V$(-1,-2)$



(3)

(2)

(ii) $x \geq -1$ or $x \leq -1$. (1)

Q4

a) (i) $f(x) = \sqrt[3]{x-1}$

ie $y = \sqrt[3]{x-1}$

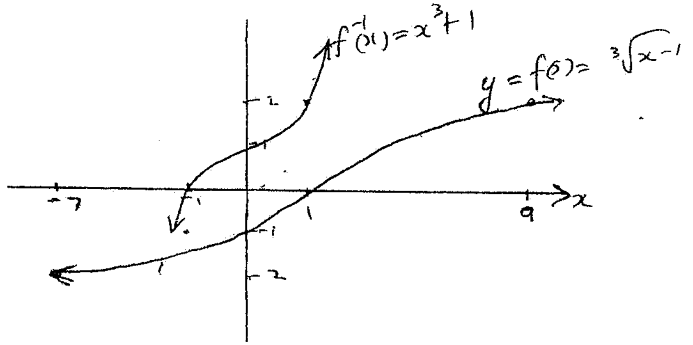
$\therefore$ Inverse is $x = \sqrt[3]{y-1}$

ie $x^3 = y-1$

$y = x^3+1$ (2)

$\therefore f^{-1}(x) = x^3+1$.

(ii)



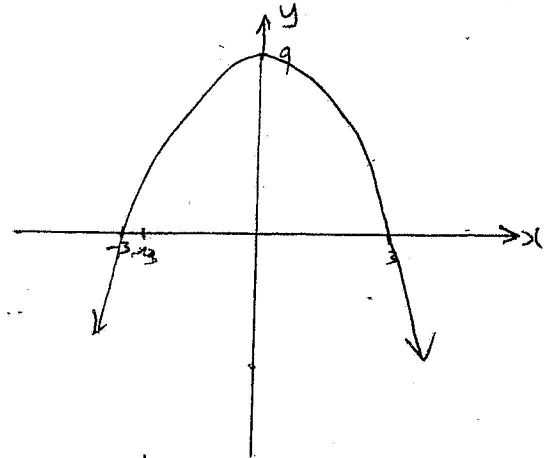
(2)

(6)

(iii) $y=f^{-1}(x)$ satisfies vertical line test ie for each x in the domain, there exists at most y value. (2)

OR $y=f(x)$ satisfies horizontal line test ie for each y value there is at most one x value.

b)



(2)

c) $y = \dfrac{1}{\sqrt{x^2-9}}$

$x^2-9>0$

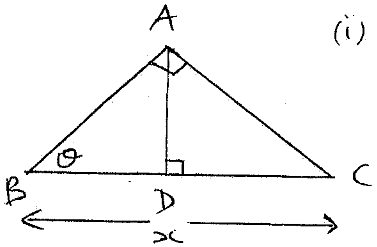
$\therefore x>3,\ x<-3$

D: $x>3,\ x<-3$

R: $y>0$.

(2)

Q5 (a)



(i) From $\triangle ABC$, $\cos\theta = \frac{AB}{x}$

$\therefore AB = x\cos\theta$

(1)

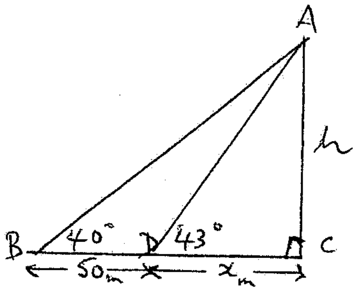
③

(ii) From $\triangle ABD$, $\cos\theta = \frac{BD}{AB}$

$\therefore BD = AB\cos\theta$ (2)

$= x\cos^2\theta$ using (1)

b)



(i) from $\triangle ADC$,

$\tan 43° = \frac{h}{x}$

$\therefore h = x\tan 43°$ (1

From $\triangle ABC$, $\tan 40° = \frac{h}{x+50}$

$\therefore h = (x+50)\tan 40°$

③

(ii) $h = x\tan 43° = (x+50)\tan 40°$

$x\tan 43° = x\tan 40° + 50\tan 40°$ (2

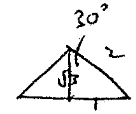
$\therefore x(\tan 43° - \tan 40°) = 50\tan 40°$

c) (i) $\cos 510° = \cos(510 - 360°)$

$= \cos 150°$

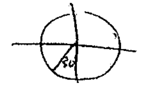
$= -\cos 30°$

$= -\frac{\sqrt{3}}{2}$ ①



(ii) $\tan(-120°) = +\tan 60°$

$= \sqrt{3}$ ①



d) $x = a\sec\theta$ ①

$y = b\tan\theta$ ②

From ①, $\sec\theta = \frac{x}{a}$ ②

② $\tan\theta = y/b$

$\therefore \tan^2\theta + 1 = \sec^2\theta$ becomes

$\left(\frac{y}{b}\right)^2 + 1 = \left(\frac{x}{a}\right)^2$

ie $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Q6

a) (i) $4\sin^2 x - 3 = 0$

$\therefore \sin x = \pm\frac{\sqrt{3}}{2}$

working angle $= 60°$

$\therefore x = 60°, 120°, 240°, 300°$

(2)

(ii) $\cos(2x + 10°) = -\frac{\sqrt{3}}{2}$

let $u = 2x+10$

$0 \le x \le 360°$

$10 \le 2x+10 \le 730$

ie $10° \le u \le 730°$

(3)

$\therefore \cos u = -\frac{\sqrt{3}}{2}$

working angle $= 30°$. u in quad. 2, 3.

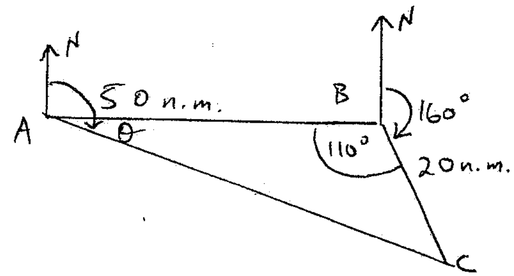
$\therefore u = 150°, 210°, 510°, 570°, \cancel{870}$

$\therefore 2x + 10 = 150, 210, 510, 570$

$2x = 140, 200, 500, 560$

$x = 70°, 100°, 250°, 280°$.

b) (i)



$b^2 = a^2 + c^2 - 2ac\cos B$

$= 20^2 + 50^2 - 2 \times 20 \times 50 \cos 110°$

$= 400 + 2500 - 2000\cos 110°$

$= 3584.040287$

$\therefore b = 59.87$ n.m. (to 2 dec. pl)

(2)

ie C is 59.87 n.m. from port A.

(ii) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$= \frac{59.87^2 + 50^2 - 20^2}{2 \times 59.87 \times 50}$

$= 0.9494$

(3)

$\therefore \angle A \doteqdot 18°\,18'$

ie $\theta = 18°$ to nearest degree.

$\therefore$ Bearing of C from A is $(\cancel{180}\ 90 + \theta)$

$\doteqdot \cancel{198°}$ 108° T (nearest deg)