St George Girls High School

Year 11 – Higher School Certificate Course

Assessment Task 1

December 2004

Mathematics Extension 1

Time Allowed: 75 Minutes
(plus 5 minutes reading time)

Instructions to Candidates

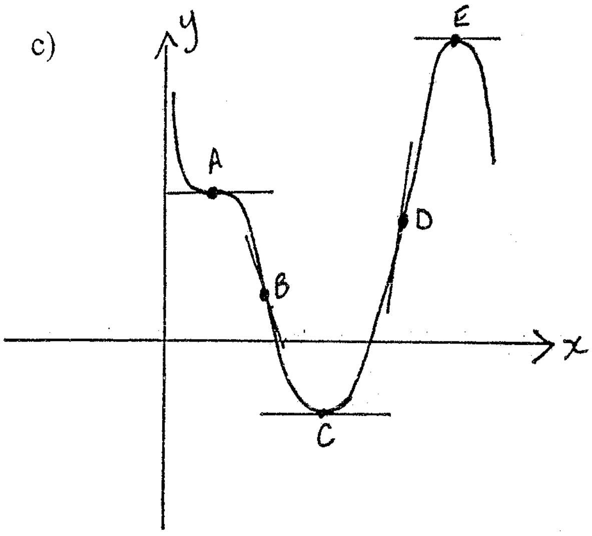
1. Write using black or blue pen.
2. Attempt all questions.
3. Start each question on a new page.
4. Show all necessary working.
5. Marks for each question are shown in the right column.
6. Complete cover sheet clearly showing:
   - your name
   - your mathematics class and teacher.

<u>Question 1</u> – (10 marks) – Start a New Page **Marks**

a) For a given function $y = f(x)$, $f(3) = 2$ and $f(7) = 6$. Sketch this function from $x = 3$ to $x = 7$ if over this domain $f'(x) > 0$ and $f''(x) < 0$ **2**

b) If $f(x) = 3x + \frac{1}{x^3}$ find $f''(2)$ **2**

c)



Given the graph of $y = f(x)$ drawn on the left, on separate axes sketch graphs of: **4**

(i) $y = f'(x)$

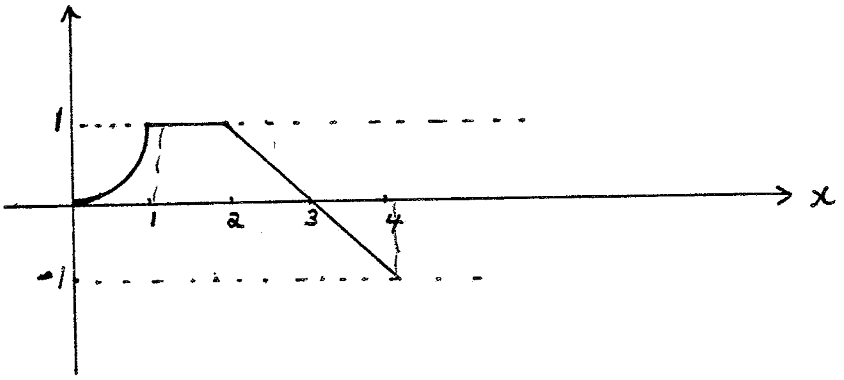
(ii) $y = f''(x)$

d) Find the primitive of $\sqrt{2x-7}$ **2**

Question 2 – (10 marks) – Start a new page **Marks**

a) The graph of $y = f(x)$ passes through $(2, 12)$ and $f'(x) = 9x^2 + 4$ find $f(x)$. 2

b) Use area formulae to evaluate $\int_0^4 f(x)\,dx$ given the sketch of $f(x)$ 2



c) Evaluate

(i) $\int_2^4 (3 - 2x)\,dx$ (ii) $\int_{-2}^{2} e^x + e^{-x}\,dx$ 4

d) Show that $\int x\sqrt{x}\,dx \neq \int x\,dx \cdot \int \sqrt{x}\,dx$ 2

Question 5 – (10 marks) – Start a new page **Marks**

a) For the curve $y = xe^{-x}$

(i) Show that the curve has one stationary point which is a maximum. **2**

(ii) Show that the curve passes through the origin. **1**

(iii) Find the point on the curve where the concavity changes. **2**

(iv) Find the value of the function when $x = -1$. **1**

(v) Show that the curve approaches the x-axis as x becomes very large. **1**

(vi) Sketch the curve. **2**

b) Find the minimum value of the function $y = 4 - x^2$ in the domain $-1 \le x \le 3$ **1**

Question 6 – (10 marks) – Start a new page **Marks**

a) Find: **6**

(i) $\int 4xe^{x^2}\,dx$

(ii) $\int \frac{2x}{e^{x^2}}\,dx$

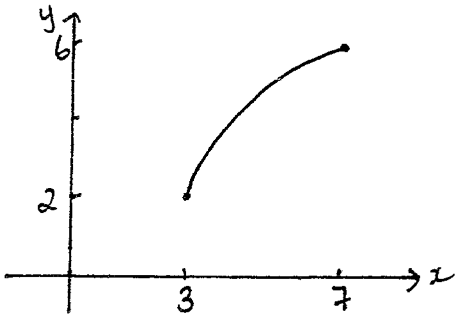
(iii) $\int e^{(2x+1)} + \frac{1}{x^2}\,dx$

b) The region bounded by the curve $y = 3^{x-1}$ and the x-axis between $x = 1$ and $x = 3$ is rotated about the x-axis. Use the Simpson's rule with 5 function values to approximate the volume of the solid formed. (Give your answer correct to two decimal places). **4**

End of Paper

Question 1

a.)



.) $f(x) = 3x + x^{-3}$

$f'(x) = 3 - 3x^{-4}$

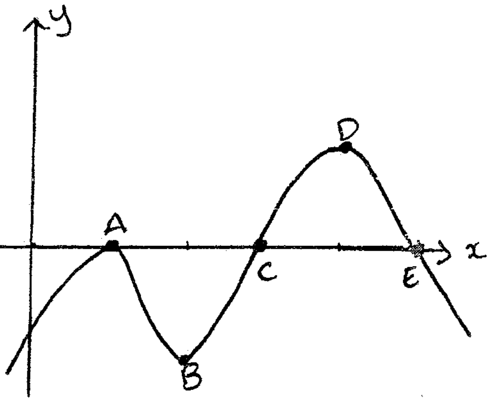
$f''(x) = 12x^{-5}$

$= \frac{12}{x^5}$

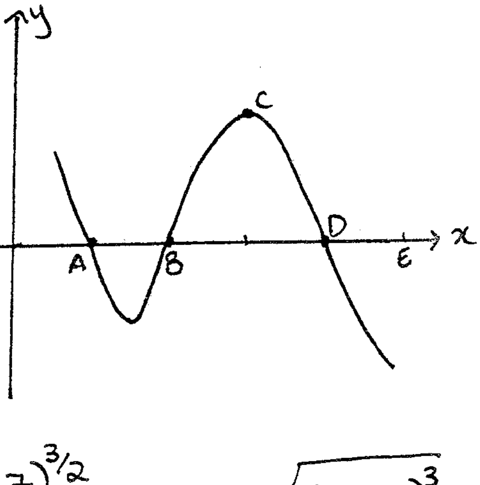
$''(2) = \frac{12}{32}$

$= \frac{3}{8}$

.) (i)



(ii)



d.) $\frac{(2x-7)^{3/2}}{3/2 \times 2} + c = \frac{\sqrt{(2x-7)^3}}{3} + c$

Question 2

a.) $f(x) = \frac{9x^3}{3} + 4x + c$ ①

when $x = 2$, $y = 12$

$12 = 3(2)^3 + 4(2) + c$

$\therefore c = -20$

$\therefore f(x) = 3x^3 + 4x - 20$ ①

b.) $A = \overset{1/2}{(1 - \frac{\pi}{4})} + \overset{1/2}{(1 \times 1)} + \overset{1/2}{(\frac{1}{2} \times 1 \times 1)} - \overset{1/2}{(\frac{1}{2} \times 1}$

$= 2 - \frac{\pi}{4}$ units^2

c.) (i) $\int_2^4 (3-2x)\,dx = \left[3x - \overset{\frac{(3-2x)^2}{-4}}{x^2}\right]_2^4$ ①

$= (12-16) - (6-4)$

$= -6$ ①

(ii) $\int_{-2}^{2} e^x + e^{-x}\,dx = \left[e^x - e^{-x}\right]_{-2}^{2}$ (

$= (e^2 - e^{-2}) - (e^{-2} - e$

$= 2e^2 - \frac{2}{e^2}$ ①

d.) LHS $= \int x \cdot x^{1/2}\,dx$

$= \int x^{3/2}\,dx$

$= \frac{2}{5}x^{5/2} + c$ ①

RHS $= \int x\,dx \cdot \int x^{1/2}\,dx$

$= \frac{x^2}{2} \cdot \frac{2}{3}x^{3/2} + c$

$= \frac{x^{7/2}}{3} + c$ ①

LHS ≠ RHS

$\therefore \int x\sqrt{x}\,dx \neq \int x\,dx \cdot \int \sqrt{x}\,dx$

Question 3

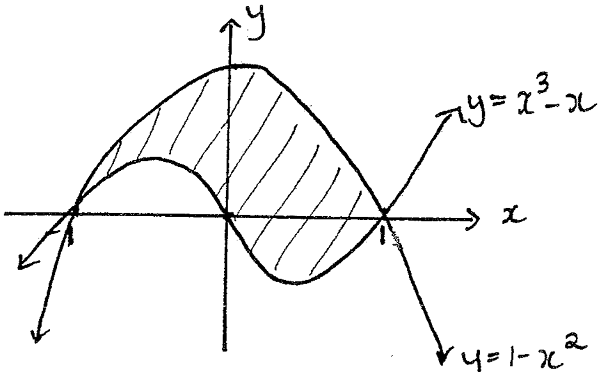
a.) (i) $\int x^2 + 2x + 1 \, dx = \frac{x^3}{3} + x^2 + x + c$

(ii) $\int x^{-3/2} + 2x^{-3} \, dx = -2x^{-1/2} - x^{-2} + c$

$= \frac{-2}{\sqrt{x}} - \frac{1}{x^2} + c$

b.) (i) $y = x(x-1)(x+1)$

$y = (1-x)(1+x)$



(ii) $A = \int_{-1}^{1} (1-x^2) - (x^3 - x) dx$

$= \int_{-1}^{1} 1 + x - x^2 - x^3 \, dx$

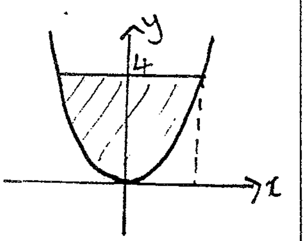
$= \left[x + \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4}\right]_{-1}^{1}$

$= (1 + \frac{1}{2} - \frac{1}{3} - \frac{1}{4}) - (-1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{4})$

$= 2 - \frac{2}{3}$

$= 1\frac{1}{3}$ units^2

c.) $x^2 = 2y$



$V = \pi \int_0^4 2y \, dy$

$= \pi \left[y^2 \right]_0^4$

$= \pi(16 - 0)$

$= 16\pi$ units^3

Question 4

a.) (i) $h + 2\pi r = 10$

$h = 10 - 2\pi r$

$V = \pi r^2 h$

$= \pi r^2 (10 - 2\pi r)$

(ii) $V = 10\pi r^2 - 2\pi^2 r^3$

$\frac{dV}{dr} = 20\pi r - 6\pi^2 r^2$

when $\frac{dV}{dr} = 0$

$2\pi r(10 - 3\pi r) = 0$

$r = 0$, $r = \frac{10}{3\pi}$ but $r > 0$

$\frac{d^2V}{dr^2} = 20\pi - 12\pi^2 r$

when $r = \frac{10}{3\pi}$, $\frac{d^2V}{dr^2} < 0$

$\therefore$ there is a max. when $r = \frac{10}{3\pi}$

when $r = 10/3\pi$

$V = 10\pi\left(\frac{10}{3\pi}\right)^2 - 2\pi^2\left(\frac{10}{3\pi}\right)^3$

$= \frac{1000}{9\pi} - \frac{2000}{27\pi}$

$= \frac{1000}{27\pi}$ units^3

b.) (i) $y' = e^x + 10x$

(ii) $y' = e^{3x}(2) + (1+2x)3e^{3x}$

$= e^{3x}(2 + 3 + 6x)$

$= e^{3x}(5 + 6x)$

(iii) $y' = \frac{(e^x - 1).e^x - (e^x + 1).e^x}{(e^x - 1)^2}$

$= \frac{e^x(e^x - 1 - e^x + 1)}{(e^x - 1)^2}$

$= \frac{-2e^x}{(e^x - 1)^2}$

) $\int_0^1 e^{3x}dx = \left[\frac{e^{3x}}{3}\right]_0^1$

$= \frac{e^3}{3} - \frac{e^0}{3}$

$= \frac{1}{3}(e^3 - 1)$

Question 5

a.) (i) $y = x.e^{-x}$

$y' = e^{-x}(1) + x(-e^{-x})$

$= e^{-x}(1-x)$

when $y' = 0$

$e^{-x}(1-x) = 0$

$e^{-x} \neq 0$, $1 - x = 0$

$\therefore x = 1$

$y'' = (1-x)(-e^{-x}) + e^{-x}(-1)$

$= -e^{-x}(2-x)$

when $x = 1$, $y'' < 0$

$\therefore$ max. when $x = 1$

when $x = 1$, $y = \frac{1}{e}$

$(1, \frac{1}{e})$ is a max. stat. pt

ii) when $x = 0$

$y = 0.e^{-0}$

$= 0$

$\therefore$ the curve passes through the origin

iii) $y'' = -e^{-x}(2-x)$

when $y'' = 0$

$-e^{-x}(2-x) = 0$

$-e^{-x} \neq 0$ $\therefore x = 2$

$\therefore$ possible point of inflexion when $x = 2$.

x	$1\frac{1}{2}$	2	3
y''	$-\frac{1}{2\sqrt{e}}$	0	$\frac{1}{e^3}$

Since concavity changes there is a point of inflexion at $(2, \frac{2}{e^2})$

(iv) when $x = -1$, $y = -1.e^1$

$= -e$

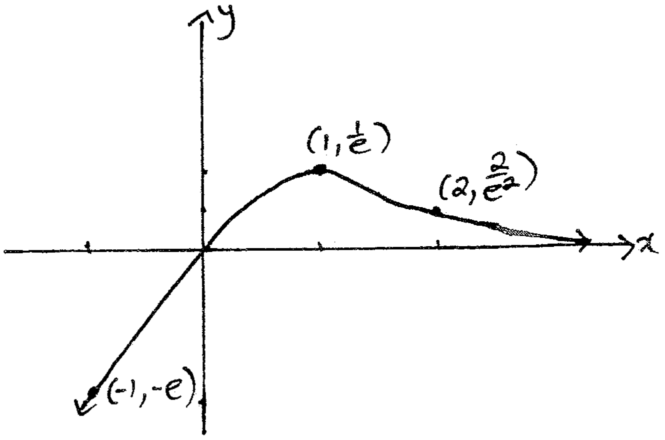
(v) $y = \frac{x}{e^x}$

as $x \to \infty$, $e^x \to \infty$

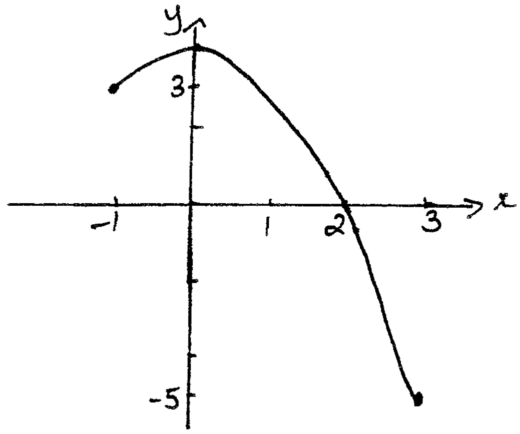
$\therefore \frac{x}{e^x} \to 0$ as $x \to \infty$

$\therefore$ the curve approaches the x-axis as x becomes large.

(vi)



b.)



$\therefore$ the minimum value is $y = 5$

Question 6

a.) (i) $\int 4xe^{x^2}dx = 2\int 2xe^{x^2}dx$

$= 2e^{x^2} + c$

(ii) $\int \frac{2x}{e^{x^2}}\, dx = \int 2xe^{-x^2}dx$

$= -e^{-x^2} + c$

(iii) $\int e^{(2x+1)} + x^{-2}dx = \frac{e^{(2x+1)}}{2} - \frac{1}{x} + c$

b.) $y = 3^{x-1}$

$y^2 = (3^{x-1})^2$

$= 3^{2x-2}$

$V = \pi \int_1^3 3^{2x-2}\, dx$

x	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
y^2	1	3	9	27	81

$V \doteqdot \pi \frac{\frac{1}{2}}{3}\left[1 + 81 + 4(3+27) + 2(9)\right]$

$= \frac{\pi}{6}\left[82 + 120 + 18\right]$

$= \frac{220\pi}{6}$

$= 115.19$ units^3 (2 d.p.)