St George Girls' High School

Year 11

Common Test #1

March 2001

Mathematics Extension 1

Time Allowed: 75 minutes

Instructions to Candidates

- All questions are of equal value.
- All questions should be attempted.
- Start each question on a new page.
- Write on one side of each page only.
- Marks may be deducted for poorly presented work.

Marks

Question 1 (Start a new page)

(a) Without using decimal approximations show that **2**

$5\sqrt{3} - 8 > 4 - 2\sqrt{3}$

(b) If $f(x) = 2 - x^2$ find (i) $f(-1)$ **2**

(ii) $f(\sqrt{x})$

(c) If $g(x) = x + \frac{4}{x}$ show that $g(3 - \sqrt{5})$ is rational **2**

(d) If $x + 2y + \sqrt{x - y} = 1 + 2\sqrt{7}$ find the values of x and y given that they are rational. **3**

(e) Write down the natural domain of **3**

(i) $f(x) = \sqrt{3 - x}$

(ii) $g(x) = \frac{1}{x + 5}$

(iii) $h(x) = 10^x$

(f) If $C = \{x : -1 \le x < 5\}$ and $D = \{x : -4 \le x < 1\}$ graph on separate number lines **2**

(i) $C \cap D$

(ii) $C \cup D$

Question 2 (Start a new page) **Marks**

(a) Solve the simultaneous equations **4**

$x + 3y + z = 2$ (1)

$x - 2y + z = 12$ (2)

$2x + 5y - z = -9$ (3)

(b) Express as a simplified fraction **3**

$$\frac{3}{x^2-16} - \frac{2}{x^2+4x}$$

(c) What range of values does $y = x^2 - 6x + 2$ take if x takes values $0 < x \le 5$ **3**

(d) Find the inverse function $y = f^{-1}(x)$ if **4**

(i) $f(x) = x^3 - 2$

(ii) $f(x) = \dfrac{x+2}{3-2x}$

Question 3 (Start a new page) **M**

(a) Factorise $x^6 + x^5 + x^4 - 8x^3 - 8x^2 - 8x$

(b) Sketch graphs of the following relations/functions on separate diagrams. You must clearly show any intercepts with the coordinate axes.

(i) $3x + 2y = 12$

(ii) $y = 16 - x^2$

(iii) $x = \sqrt{9 - y^2}$

(iv) $y = 5^x$

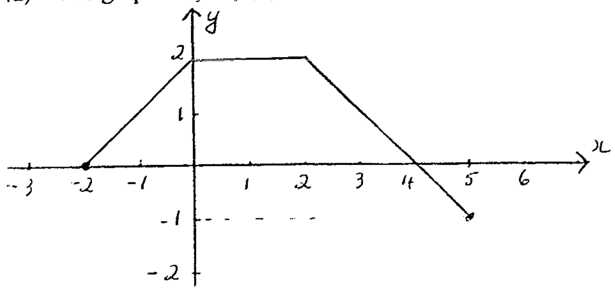
(c) (i) Expand $(x^2 + 2)^2$

(ii) Using your answer to part (i), or otherwise, express $x^4 + 4$ as the product of quadratic factors.

Question 4 (Start a new page)

Marks

(a) The graph of $y = f(x)$ is shown **10**



On separate diagrams draw neat sketches of

(i) $y = f(x) - 1$

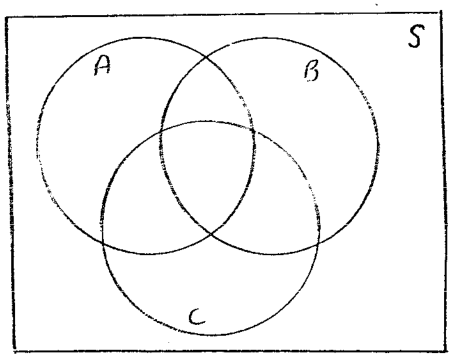
(ii) $y = f(x-1)$

(iii) $y = 2f(x)$

(iv) $y = -f(x)$

(v) $y = \dfrac{1}{f(x)}$

(b) A, B and C are subsets of the universal set S. **4**



$|S| = 100$

$|A| = 28$, $|B| = 26$ and $|C| = 37$

$|A \cap B| = 13$, $|B \cap C| = 9$, $|A \cap C| = 7$

$|\overline{A \cup B \cup C}| = 34$

By using the Venn diagram, or otherwise find $|A \cap B \cap C|$

Question 5 (Start a new page)

Marks

(a) Solve the following inequations **10**

(i) $-3 \le 5 - 4x < 7$

(ii) $2^{x+1} \le 16$

(iii) $x^2 - 2x - 3 > 0$

(iv) $\dfrac{3x+2}{x-3} \le 1$

(b) If $0 < a < b$ **4**

(i) prove that $\dfrac{1}{a} > \dfrac{1}{b}$

(ii) for what values of a and b is $\dfrac{1}{a-1} > \dfrac{1}{b-1}$?

Solutions to Maths Ext 1

Common test 1

2001

Q1

a) $5\sqrt{3}-8 > 4-2\sqrt{3}$

Consider $5\sqrt{3}-8-(4-2\sqrt{3})$

$= 7\sqrt{3}-12$

$= \sqrt{147}-\sqrt{144}$

> 0

$\therefore 5\sqrt{3}-8 > 4-2\sqrt{3}$ 2

b) $f(x) = 2-x^2$

(i) $f(-1) = 2-(-1)^2$

$= 1$ 1

(ii) $f(\sqrt{x}) = 2-(\sqrt{x})^2$

$= 2-x$ 1

c) $g(x) = x + \frac{4}{x}$

$g(3-\sqrt{5}) = 3-\sqrt{5} + \frac{4}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$

$= 3-\sqrt{5} + \frac{4(3+\sqrt{5})}{(9-5)}$

$= 6$ (rational). 2

d) $x+2y+\sqrt{xy} = 1+2\sqrt{7}$

$= 1+\sqrt{28}$

$x+2y = 1$ ①

$x \cdot y = 28$ ②

① - ② ⇒

$3y = -27$

$y = -9$

Sub into ① ⇒ $x = 19$.

$x = 19,\ y = -9$ 3

e) (i) $f(x) = \sqrt{3-x}$

$3-x \geq 0$

i.e. $x \leq 3$

(ii) $g(x) = \frac{1}{x+5}$

D: $x \in \mathbb{R},\ x \neq -5$

(iii) $h(x) = 10^x$

D $x \in \mathbb{R}$

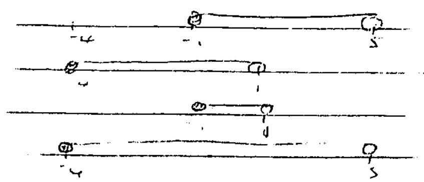
f)

C

D

$C \cap D$

$C \cup D$



Q2

a)

$x + 3y + z = 2$ ①

$x - 2y + z = 12$ ②

$2x + 5y - z = -9$ ③

① + ③ ⇒ $3x + 8y = -7$ ④

② + ③ ⇒ $3x + 3y = 3$ ⑤

④ - ⑤ ⇒ $5y = -10$

$y = -2$

Sub $y = -2$ into ⑤

⇒ $3x - 6 = 3$

$x = 3$

Sub $x = 3,\ y = -2$ into ①

⇒ $3 - 6 + z = 2$

$z = 5$

$x = 3,\ y = -2,\ z = 5$

Q2 b) $\frac{3}{x^2-16} - \frac{2}{x^2+4x}$

$= \frac{3}{(x+4)(x-4)} - \frac{2}{x(x+4)}$

$= \frac{3x - 2(x-4)}{x(x+4)(x-4)}$

$= \frac{x+8}{x(x^2-16)}$ 3.

c) test $f(0)$, $f(5)$, $f\left(\frac{-b}{2a}\right)$ – the y-coord of vertex

$f(0) = 2$

$f(5) = 25 - 30 + 2$
$= -3$

$f\left(-\frac{-6}{2}\right) = f(3) = 9 - 18 + 2$
$= -7$

$-7 \le f(x) \le 2$ 3.

d) (i) $f(x) = x^3 - 2$

$y = x^3 - 2$

inverse is $x = y^3 - 2$

$y = \sqrt[3]{x+2}$

ie $f^{-1}(x) = \sqrt[3]{x+2}$. 2

(ii) $y = f(x) = \frac{x+2}{3-2x}$

inverse is $x = \frac{y+2}{3-2y}$

$x(3-2y) = y+2$

ie $3x - 2xy = y + 2$

$y(1+2x) = 3x - 2$

$y = \frac{3x-2}{2x+1}$

ie $f^{-1}(x) = \frac{3x-2}{2x+1}$

2/14

Q3 a) $x^6 + x^5 + x^4 - 8x^3 - 8x^2 - 8x$

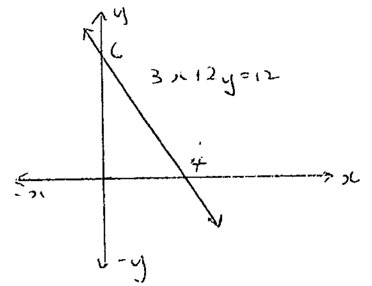
$= x(x^5 + x^4 + x^3 - 8x^2 - 8x - 8)$

$= x[x^3(x^2+x+1) - 8(x^2+x+1)]$

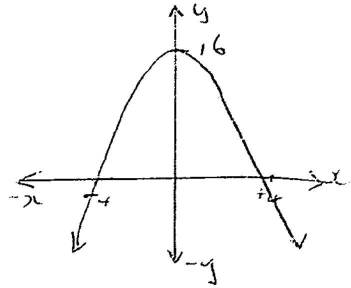
$= x(x^2+x+1)(x^3-8)$

$= x(x^2+x+1)(x-2)(x^2+2x+4)$

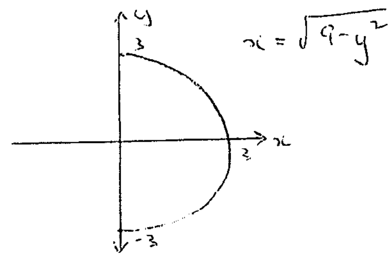
b) (i)



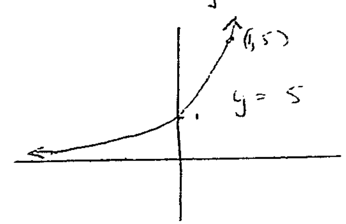
(ii)



(iii)



(iv)



2 each. = 8

c) (i) $(x^2+2)^2 = x^4 + 4x^2 + 4$ 1

(ii) $x^4 + 4 = (x^2+2)^2 - 4x^2$

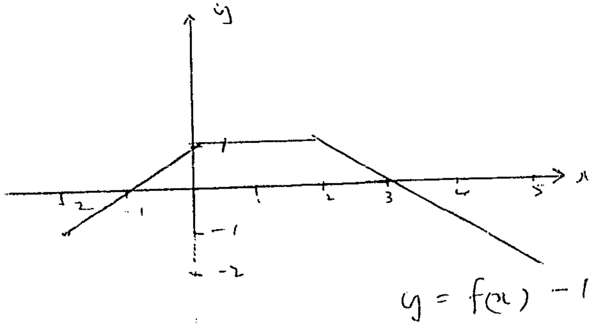
$= (x^2+2+2x)(x^2+2-2x)$

$= (x^2+2x+2)(x^2-2x+2)$

1

Q4

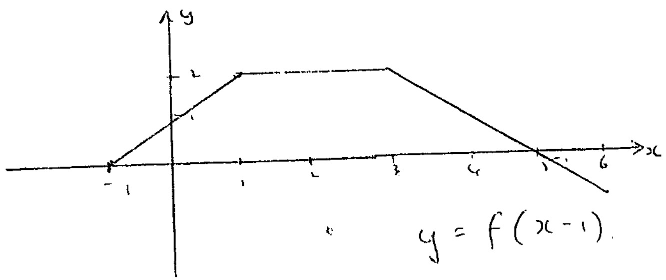
a) (i)



$y = f(x) - 1$

2

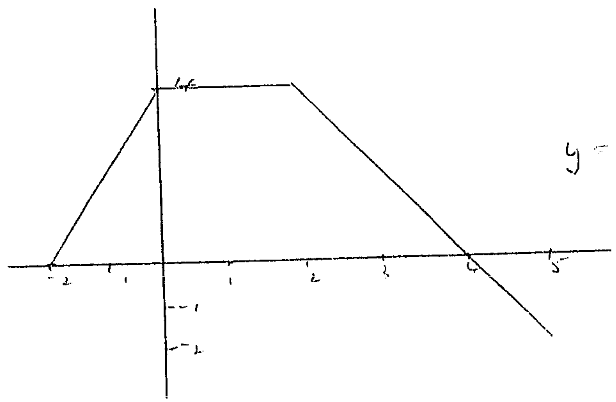
(ii)



$y = f(x-1)$

2

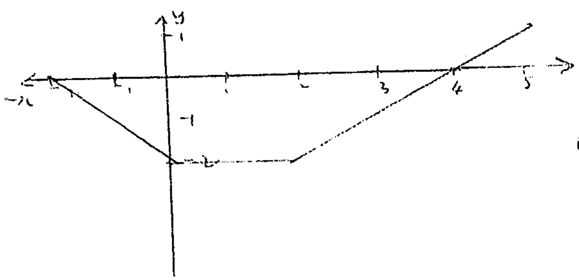
(iii)



$y = 2f(x)$

2

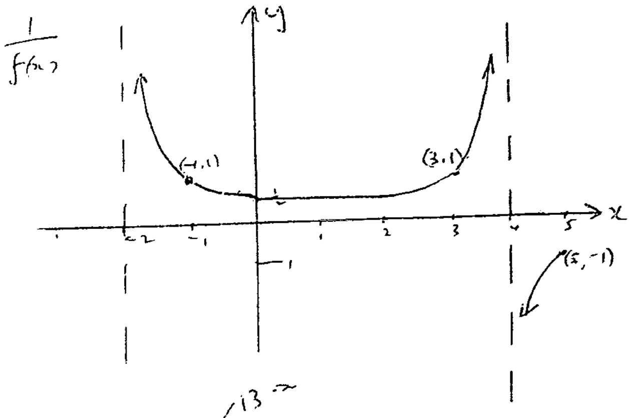
(iv)



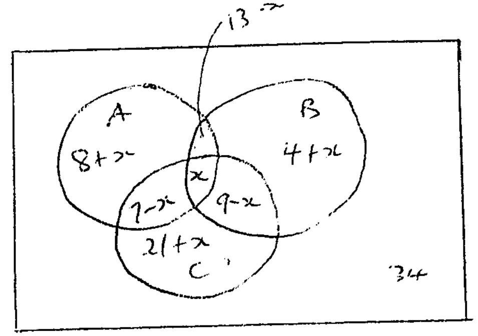
$y = -f(x)$

2

(v) $y = \frac{1}{f(x)}$



b)



$8+x+13-x+x+7-x+4+x+9-x+21+x+34 = 100$

$\therefore x + 96 = 100$

$x = 4$

ie. $|A \cap B \cap C| = 4$.

Q5

a) (i) $-3 \leq 5 - 4x < 7$

$-5 \quad -5 \quad -5$

$-8 \leq -4x < 2 \quad (\div -4)$

$\therefore 2 \geq x > -\frac{1}{2}$

(ii) $2^{x-3} \leq 16$

$x - 3 \leq \log_2 16$

ie. $x - 3 \leq 4$

$x \leq 7$

or $2^{x-3} \leq 2^4$

$\therefore x - 3 \leq 4$

$x \leq 7$

(iii) $x^2 - 2x - 3 > 0$

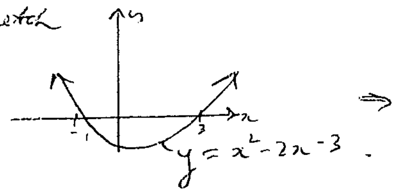
$(x-3)(x+1) > 0$

put LHS = 0 $\Rightarrow$ $x = -1, 3$

test $x = 0$ ✗

$\therefore$ $x > 3$, $x < -1$.

or sketch



$\Rightarrow$ $x > 3$, $x < -1$.

(iv) $\dfrac{3x+2}{x-3} \leq 1$

"Hole & test" or "$\times (x-3)^2$" or $\times (x-3)$ & consider any necessary change in inequality sign.

"Hole & test"

$x - 3 = 0 \Rightarrow x = 3$

solve $\dfrac{3x+2}{x-3} = 1$

$\therefore 3x + 2 = x - 3$

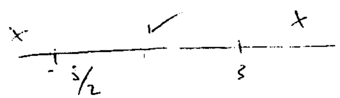
$2x = -5$

$x = -5/2$

test $x = -3$ $\quad \dfrac{-7}{6} \leq 1$ ✗

$x = 0$ $\quad -\dfrac{2}{3} \leq 1$ ✓

$x = 4$ $\quad \dfrac{14}{1} \leq 1$ ✗



$\therefore$ $-2\frac{1}{2} \leq x < 3$. ($x \neq 3$!)

OR $\dfrac{3x+2}{x-3} \leq 1$

$\dfrac{3x+2}{x-3} \times (x-3)^2 \leq (x-3)^2$

$\therefore (3x+2)(x-3) \leq (x-3)^2$

$(x-3)[3x+2-(x-3)] \leq 0$.

$(x-3)(2x+5) \leq 0$.

$\Rightarrow$ $-5/2 \leq x < 3$.

b) $0 < a < b$

(i) $\dfrac{1}{a} > \dfrac{1}{b}$. ?

Consider $\dfrac{1}{a} - \dfrac{1}{b} = \dfrac{b-a}{ab}$.

> 0 since $b > a$, $ab > 0$ since a, b ...

$\therefore$ $\dfrac{1}{a} > \dfrac{1}{b}$

(ii) $\dfrac{1}{a-1} - \dfrac{1}{b-1} = \dfrac{b-1-(a-1)}{(a-1)(b-1)}$

$= \dfrac{b-a}{(a-1)(b-1)}$

> 0 when $(a-1)(b-1) > 0$

[since $b - a > 0$]

ie $a > 1$ and $b > 1$.

or $a < 1$ & $b < 1$.

ie $0 < a < b < 1$.

Ans $a > 1$ and $b > 1$

or $0 < a < b < 1$.