Su-Yee Chan

St George Girls' High School

Year 11

Assessment Task 1

March 2003

Mathematics Extension 1

Time Allowed: 70 minutes

Instructions

1. All questions are of equal value.
2. Attempt all questions.
3. Question 5 is to be answer on the sheet provided.
4. All other questions are to be completed in the answer booklet.
5. Start each question on a new page.

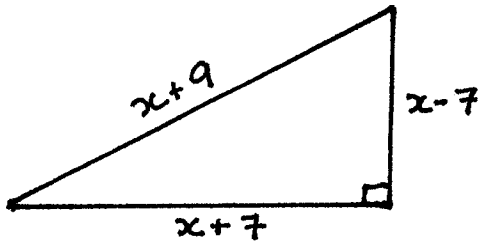
Question 1 – 10 marks – (Start a new page) **Marks**

a) Factorise each of the following: **4**

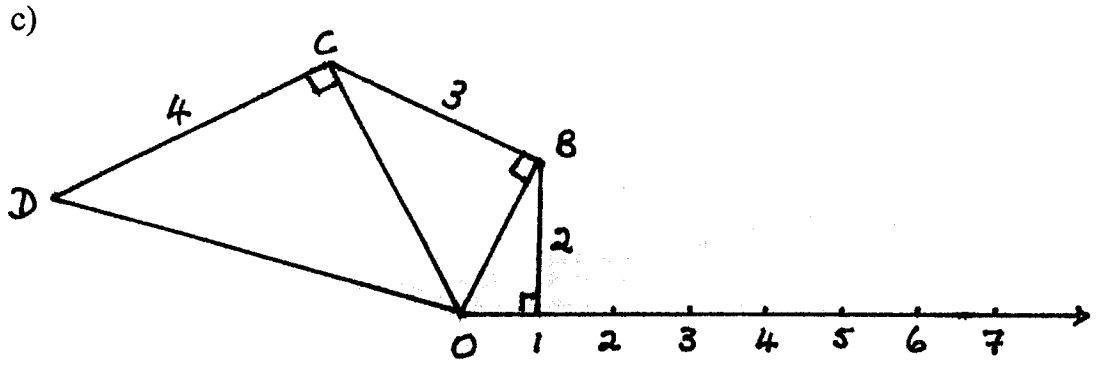
(i) $x^4 - 5x^2 - 36$

(ii) $y^2 - 6y + 9 - t^2$

b) Find the value of x. **4**



c)



With centre O and radius OD a circle is drawn. Between which 2 positive integers will this circle cut the number line? **2**

Question 2 – 10 marks – (Start a new page) **Marks**

a) Draw neat sketches of the graphs of: 8

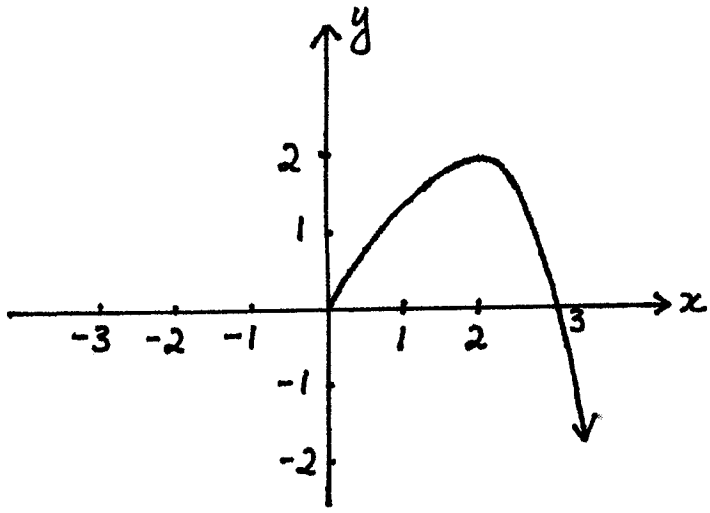
(i) $x^2 + y^2 = 5$

(ii) $y = x^2 - 2x + 2$

(iii) $y = 3^{-x}$

(iv) $x = -\sqrt{9 - y^2}$

b) 2



Half of the graph of $y = f(x)$ is shown in the diagram.

Copy and complete the graph given that $f(x)$ is an odd function.

Marks

Question 3 – 10 marks – (Start a new page)

Solve the following for x.

a) $|x-3|=4$ 2

b) $-2 \le 3-2x < 7$ 2

c) $x^2 - x - 20 \le 0$ 3

d) $\dfrac{4}{3-2x} \ge -1$ 3

Question 4 – 10 marks – (Start a new page)

a) What is the natural domain of the function $g(x) = \log_{10}(3-x)$? 1

b) Find the range of the function

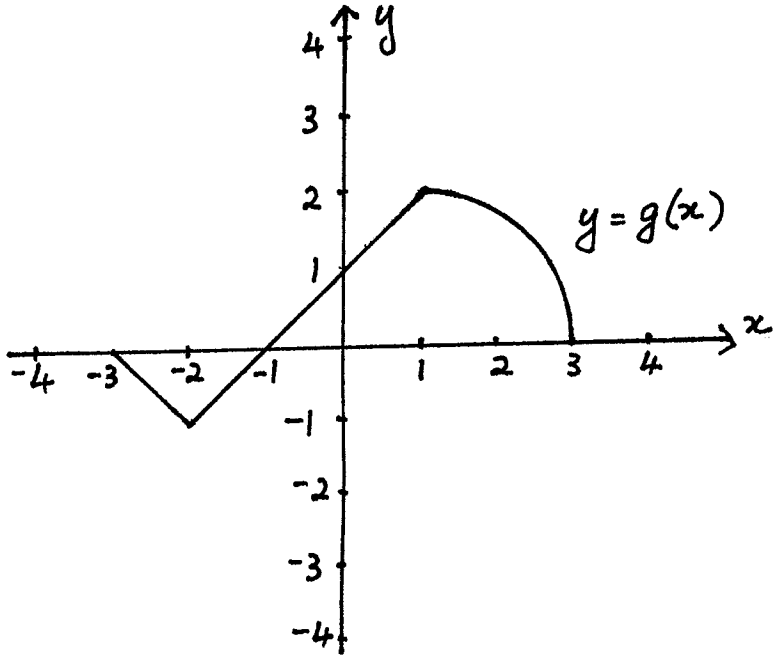
$f(x) = x^2 - 4x \qquad 0 \le x \le 6$ 3

c) Without using approximate values from your calculator, determine which of the numbers $17-3\sqrt{3}$ and $5+4\sqrt{3}$ is greater. 3

d) Solve and graph your solution on a number line $|2x-1| \le 5$. 3

Question 5 – 10 marks – (Answer on Sheet Provided) **Marks**

The graph of $y = g(x)$ is shown.



On the sheet provided sketch graphs of the following:

a) $y = g(x-1)$ 2

b) $y = g(x)+1$ 2

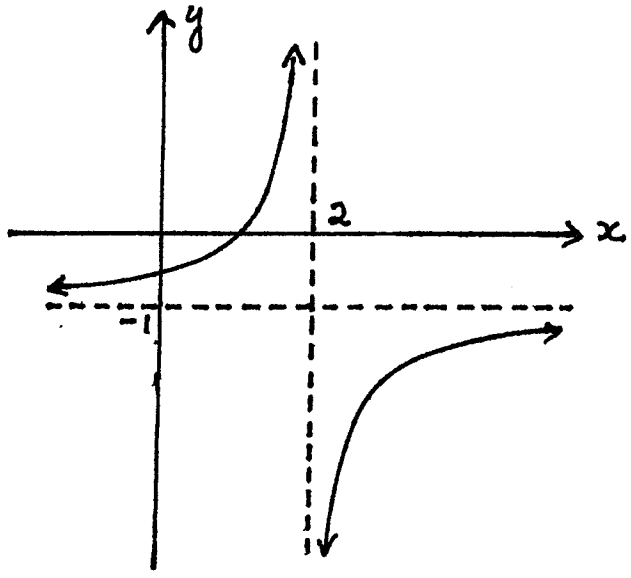
c) $y = 2g(x)$ 2

d) $y = -g(x)$ 2

e) $y = \dfrac{1}{g(x)}$ 2

Question 6 – 10 marks – (Start a new page) **Marks**

a)



The graph of $y = f(x)$ where $f(x) = \dfrac{x-1}{2-x}$ is shown.

(i) Explain why the inverse of $f(x)$ is a function. 1

(ii) Find $f^{-1}(x)$ 3

b) (i) Sketch the graphs of $y = |2x|$ and $y = x + 3$ on the same diagram 6

Hence

(ii) Write down the number of solutions of the equation $|2x| = x + 3$

and

(iii) Solve $|2x| \leq x + 3$

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Q1. a) i) $x^4 - 5x^2 - 36$

$= (x^2 - 9)(x^2 + 4)$

$= (x-3)(x+3)(x^2+4)$ 2

ii) $y^2 - 6y + 9 - t^2$

$= (y-3)^2 - t^2$

$= (y-3-t)(y-3+t)$ 2

b) $(x+9)^2 = (x+7)^2 + (x-7)^2$

$x^2 + 18x + 81 = 2x^2 + 49 \times 2.$

$x^2 - 18x + 17 = 0$

$(x-17)(x-1) = 0.$

$x = 17$ or 1

if $x = 1$ then length $x - 7$ is negative.

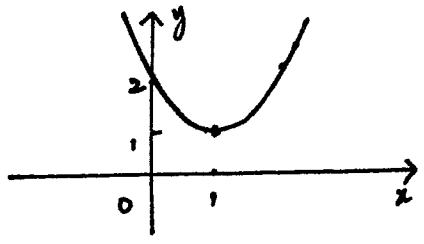
only solution is $x = 17$. 4

c) $OB = \sqrt{5}$

$OC = \sqrt{5+9} = \sqrt{14}$

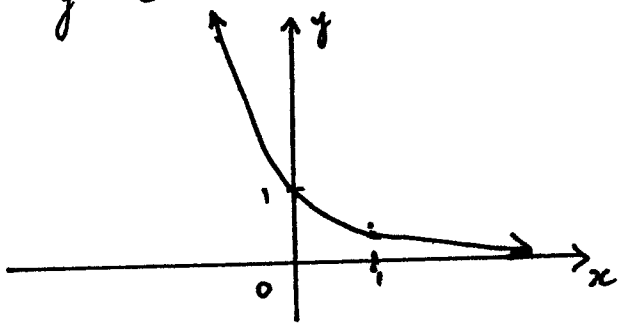
$OD = \sqrt{14+16} = \sqrt{30}.$

∴ Circle cuts between 5 and 6. 2



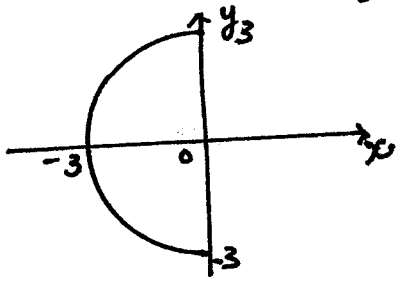
2

iii) $y = 3^{-x}$



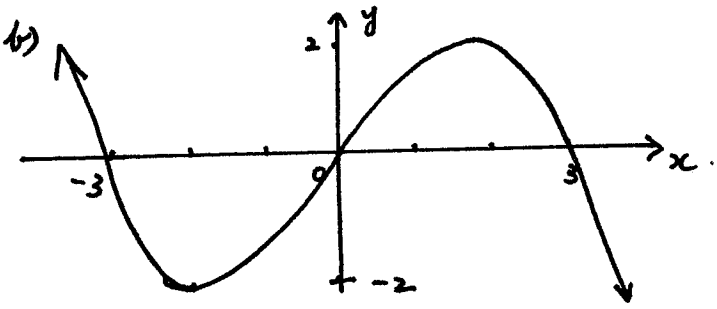
iv) $x = -\sqrt{9-y^2}$ cf $x^2 + y^2 = 9$.

D: $x \leq 0$.



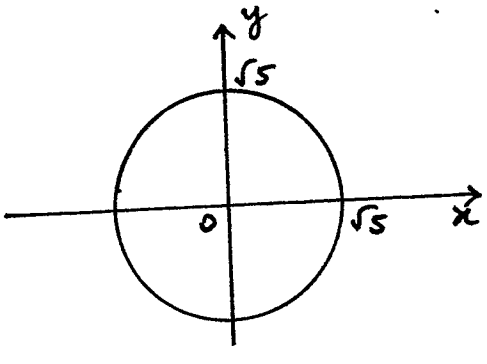
2

b)



Pt symm. about O.

Q2. a) i)



ii) $y = x^2 - 2x + 2$

$= (x-1)^2 + 1$ 2

Q3. a) $|x - 3| = 4$

$x - 3 = 4$ or $-(x-3) = 4$

$x = 7$ $-x + 3 = 4$

$x = -1$ 2

Soln: $x = 7, -1$

b) $-2 \leq 3 - 2x < 7$

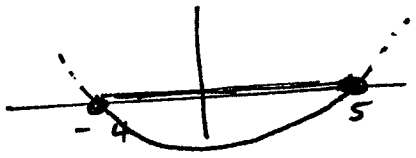
$-5 \leq -2x < 4$

$\frac{5}{2} \geqslant x > -2$ 2

Soln: $-2 < x \leq 5/2$

3. c) $x^2 - x - 20 \leq 0$

$(x-5)(x+4) \leq 0$



3

Soln : $-4 \leq x \leq 5$.

d) $\frac{4}{3-2x} \geq -1$, $3-2x \neq 0$

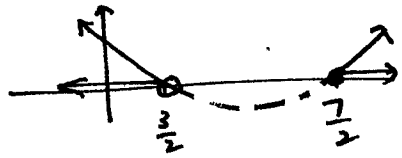
$x \neq \frac{3}{2}$

$x\ (3-2x)^2$

$4(3-2x) \geq -(3-2x)^2$

$(3-2x)^2 + 4(3-2x) \geq 0$.

$(3-2x)(7-2x) \geq 0$



3.

Soln: $x < \frac{3}{2}$, $x \geq \frac{7}{2}$.

4. a) $g(x) = \log_{10}(3-x)$

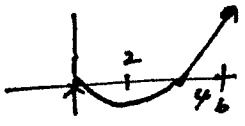
D : $3 - x > 0$

$x < 3$

1

b) $f(x) = x^2 - 4x \quad 0 \leq x \leq 6$

$= x(x-4)$



$x = 0$, $y = 0$

$x = 6$, $y = 12$

$x = 2$, $y = 4 - 8 = -4$

3.

$\therefore$ R : $-4 \leq y \leq 12$.

c) $17 - 3\sqrt{3} - (4\sqrt{3} + 5)$

$= 17 - 3\sqrt{3} - 4\sqrt{3} - 5$

$= 12 - 7\sqrt{3}$

$= 12 - \sqrt{147}$

< 0 as $\sqrt{147} > 12$

$\therefore$ $5 + 4\sqrt{3}$ is greater

d) $|2x-1| \leq 5$

$2x - 1 \leq 5$

$2x \leq 6$

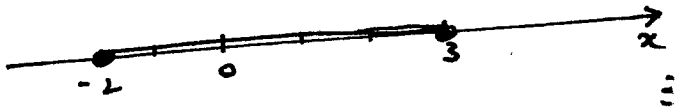
$x \leq 3$

$-(2x-1) \leq 5$

$-2x + 1 \leq 5$

$-2x \leq 4$

$x \geq -2$.



Soln : $-2 \leq x \leq 3$.

6. a) i) The inverse of $f(x)$ is a function as $f(x)$ is cut only once by a horizontal line.

ii) $y = \frac{x-1}{2-x}$

INV is $x = \frac{y-1}{2-y}$

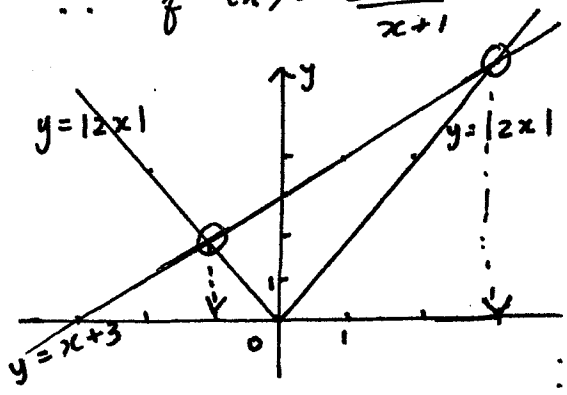
$2x - xy = y - 1$

$2x + 1 = y(1+x)$

$y = \frac{2x+1}{x+1}$

$\therefore$ $f^{-1}(x) = \frac{2x+1}{x+1}$

b) i)



ii) $|2x| = x + 3$ has 2 solu

$2x = x + 3$ and $-2x = x +$

$x = 3$

$-3x = 3$

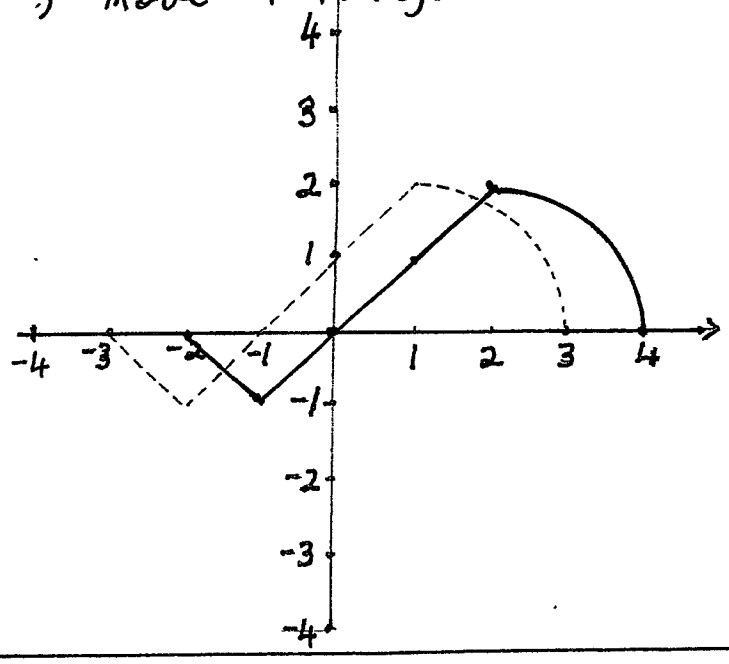
$x = -1$

iii) $|2x| \leq x + 3$

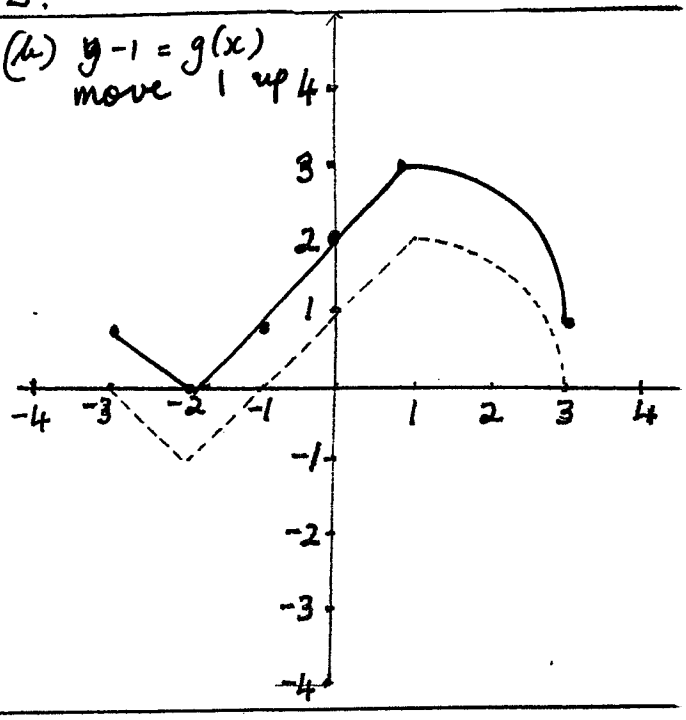
$-1 \leq x \leq 3$

QUESTION 5 NAME:

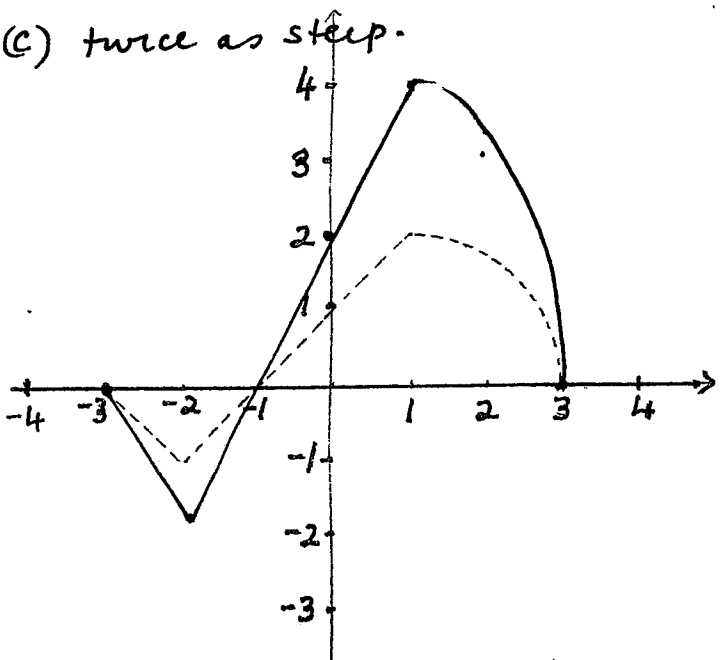
(a) move 1 to right



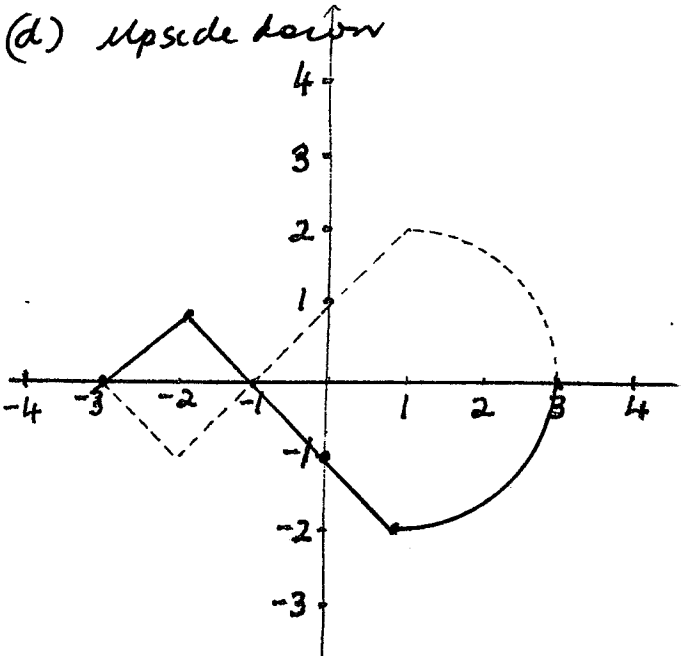
(b) $y-1=g(x)$
move 1 up



(c) twice as steep.



(d) upside down



(e)