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St George Girls High School

Year 11 – Higher School Certificate Course

Assessment Task 1

December 2004

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 75 minutes.
- Write using black or blue pen.
- Attempt all questions.
- Start each question on a new page.
- Show ALL working.
- Marks for each question are shown in right column
- Complete cover sheet clearly showing
  - your name
  - mathematics class and teacher

Question 1 (15 marks) – Start a New Page **Marks**

a) Find the derivatives of: **3**

(i) $y = 3x^5$

(ii) $y = x^3(x-3)$

(iii) $y = \dfrac{x^2 + 2x}{x^2}$

b) For the function $y = \dfrac{x}{x+2}$ **4**

(i) Show that the derivative is given by $\dfrac{2}{(x+2)^2}$

(ii) Find the equation of the tangent to the curve at the point $(-1, -1)$

c) Find the values of x on the curve $y = (x^2+1)(x+3)^2$ where the tangents to the curve are horizontal. **4**

d) For the function **4**

$$f(x) = \begin{cases} 4 - x^2, & x \le 1 \\ x + 2 & x > 1 \end{cases}$$

(i) Evaluate $f(1) + f(2)$

(ii) Give reasons to justify the statement "$f(x)$ is continuous at the point where $x = 1$"

Question 2 (15 marks) – Start a New Page **Marks**

a) Solve $x - \frac{6}{x} = 1$ **2**

b) If α and β are the roots of the quadratic equation $2x^2 - 6x + 1 = 0$, write down the values of: **6**

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\alpha^2 + \beta^2$

(iv) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

c) A ball is thrown vertically upwards and its height (h) in metres at time (t) seconds is given by $h = 5 + 14t - t^2$ **4**

(i) Express h in the form $A - (t + B)^2$

(ii) Hence or otherwise find the greatest height reached by the ball and the time when this occurs.

d) For the quadratic equation $4x^2 - 2kx + k - 1 = 0$ **3**

(i) Show that the discriminant (Δ) is equal to $4(k-2)^2$.

(ii) Explain why the roots of the quadratic equation must be rational if k is rational.

Question 3 (15 marks) – Start a New Page **Marks**

a) Find the centre and radius of the circle $x^2 + 10x + y^2 - 6y + 30 = 0$ **3**

b) Find in general form the equation of the locus of point $P\ (x, y)$ which moves so that it is equidistant from the points $A(-2, 3)$ and $B(4, 7)$. **3**

c) For the parabola $y^2 = -8x$ **3**

(i) Find the coordinates of the vertex and the focus.

(ii) Find the equation of the directrix.

d) For the curve $y = x^4 + 4x^3 - 3$ find the coordinates of any stationary points and determine what type of stationary points they are. **6**

Question 4 (15 marks) – Start a New Page **Marks**

a) Show that the quadratic given by $f(x) = x^2 - 2x + 3$ is positive definite. **3**

b) Solve $2(2^{2x}) - 9(2^x) + 4 = 0$ **3**

c) Find the values of the constants a, b and c such that **3**

$$x^2 + 6x - 5 \equiv ax(x+1) + b(x+1)^2 + cx$$

d) (i) Show that the equation of the normal to the parabola $x^2 = 8y$ at the point $(-4, 2)$ is $y = x + 6$. **6**

(ii) This normal meets the parabola again at the point Q. Find the coordinates of Q.

Question 5 (15 marks) – Start a New Page **Marks**

a) Differentiate **2**

(i) $x\sqrt{x}$

(ii) $(2x-5)^3$

b) By solving the equations simultaneously show that the line $x+2y-4=0$ is a tangent to the hyperbola $xy=2$. **3**

c) Find the values of x for which the curve $y=x^3-12x$ is decreasing. **4**

d) The curve $y=x^3+bx^2+cx+d$ has a maximum turning point at $(-1,\ 0)$ and a minimum turning point when $x=2$. **6**

(i) Explain why $b-c+d=1$

(ii) Show that $2b-c=3$ and that $4b+c=-12$

(iii) Hence find the equation of the curve.

End of Paper

Year 11 Mathematics (2U)

Assessment Task #1

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SOLUTIONS & MARKING SCALE

Q1

MARKS

a)(i) $\frac{dy}{dx} = \frac{d}{dx} 3x^5$

$= 15x^4$ — 1

(ii) $\frac{d}{dx}\left(x^3(x-3)\right)$

$= \frac{d}{dx}(x^4 - 3x^2)$

$= 4x^3 - 6x$ — 1

(iii) $\frac{d}{dx}\left(\frac{x^2+2x}{x^2}\right)$

$= \frac{d}{dx}(1 + 2x^{-1})$ — 1

$= -2x^{-2}$.

b)(i) $y = \frac{x}{x+2}$

$\therefore \frac{dy}{dx} = \frac{(x+2).1 - x(1)}{(x+2)^2}$ (quotient rule)

$= \frac{2}{(x+2)^2}$

4

(ii) at $x = -1$, $\frac{dy}{dx} = 2$, $y = -1$

$\therefore$ eqn of tangent:

$y + 1 = 2(x+1)$

ie $y = 2x + 1$

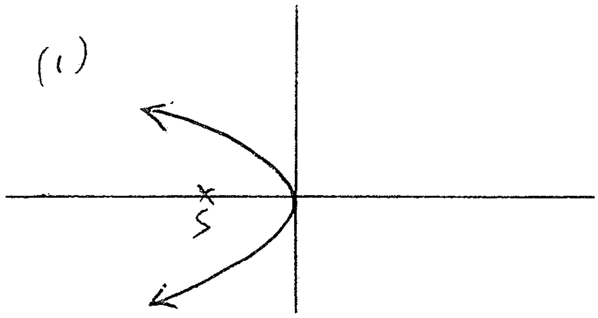
Squaring both sides gives:

$\therefore\ x^2 + 4x + 4 + y^2 - 6y + 9 = x^2 - 8x + 16 + y^2 - 14y + 49$

ie $12x + 8y - 52 = 0$

ie $3x + 2y - 13 = 0$ (3)

c) (i)



$y^2 = -8x$

"a" $= 2$

$\therefore$ vertex: $(0,0)$ (1)

focus $(-2,0)$ (1)

(ii) Directrix: $x = 2$ (1)

d) $y = x^4 + 4x^3 - 3$

$81 - 4 \times 27 - 3$
$=$

$\therefore\ y' = 4x^3 + 12x^2$
$= 4x^2(x+3)$

$\therefore$ st. pts at $(0,-3)$ and $(-3,-30)$

$y'' = 12x^2 + 24x = 12x(x+2)$ (6)

at $x = 0$, $y'' = 0$ inconclusive

$\therefore$ test:

x	0^-	0	0^+
y'	+	0	+

OR

x	0^-	0	0^+
y''	−	0	+

ie horizontal point of inflection at $(0,-3)$

(Note ACCEPT showing y'' changes sign at $x=0$)

at $x = -3$, $y'' = 36 > 0$ $\therefore$ min. turning point

Q4 a) $f(x) = x^2 - 2x + 3$.

$\Delta = 4 - 4 \times 3$

< 0.

& leading coefficient is positive

$\therefore f(x)$ is positive definite

(3)

b) $2 \cdot 2^{2x} - 9 \cdot 2^x + 4 = 0$

Let $u = 2^x$

$\Rightarrow 2u^2 - 9u + 4 = 0$

$2u \quad -1$

$u \quad -4$

$(2u - 1)(u - 4) = 0$

$\therefore u = \frac{1}{2}, 4$

(3)

$\therefore 2^x = \frac{1}{2}$ or 4

$\therefore x = -1, 2$.

c) $x^2 + 6x - 5 \equiv ax(x+1) + b(x+1)^2 + cx$

$\equiv ax^2 + ax + bx^2 + 2bx + b + cx$

$\equiv (a+b)x^2 + (a+2b+c)x + b$.

$\therefore$ $a + b = 1$

$a + 2b + c = 6$

$b = -5$

$\Rightarrow a = 6, b = -5, c = 10$

(3)

d)(i) $x^2 = 8y$

$\therefore y = \frac{1}{8}x^2$

$\frac{dy}{dx} = \frac{x}{4}$

at $x = -4$, $\frac{dy}{dx} = -1$

$\therefore$ slope of normal is 1

Equation of normal is

$y - 2 = 1(x + 4)$

ie $y = x + 6$.

(i) Solve $y = x + 6$ and $x^2 = 8y$ simultaneously

ie $x^2 = 8(x+6)$

$x^2 - 8x - 48 = 0$

$(x-12)(x+4) = 0$

$\therefore x = -4, 12.$

at Q, $x = 12$, $y = 18$

ie Q (12, 18)

Q5

a) (i) $\frac{d}{dx} x\sqrt{x} = \frac{d}{dx} x^{3/2}$ (1)

$= \frac{3}{2} x^{\frac{1}{2}}$ or $\frac{3\sqrt{x}}{2}$.

(ii) $\frac{d}{dx}(2x-5)^3 = 3(2x-5)^2 \cdot 2$ (1)

$= 6(2x-5)^2.$

b) $\left.\begin{aligned} x + 2y - 4 = 0 \\ xy = 2 \end{aligned}\right\} \rightarrow x = 4 - 2y$

$\Rightarrow (4-2y)y = 2$ (3)

& $4y - 2y^2 = 2$

$y^2 - 2y + 1 = 0$ ie $\Delta = 0.$

$(y-1)^2 = 0$

Since only one distinct root for y, & hence x, $x + 2y - 4 = 0$ is tangential to $xy = 2$.

c) $y = x^3 - 12x$

$\therefore y' = 3x^2 - 12$ (4)

$y' < 0 \Rightarrow 3(x^2 - 4) < 0$

$\therefore -2 < x < 2$, gives

d) $y = x^3 + bx^2 + cx + d$.

(i). Sub in $(-1, 0)$ since pt. lies on curve.

$\Rightarrow 0 = -1 + b - c + d$

ie $b - c + d = 1$.

(ii) $y' = 3x^2 + 2bx + c$.

when $x = -1$, $y' = 0$ (st. pt at $(-1, 0)$)

$\Rightarrow 0 = 3 - 2b + c$.

ie $2b - c = 3$

(6)

when $x = 2$, $y' = 0$ (st. pt at $x = 2$)

$\Rightarrow 0 = 12 + 4b + c$.

$\therefore 4b + c = -12$

(iii)

$b - c + d = 1$ ①

$2b - c = 3$ ②

$4b + c = -12$ ③.

② + ③ $\Rightarrow 6b = -9$

$\therefore b = -3/2$

$\therefore c = 2b - 3$

$= -3 - 3$

$= -6$.

① $\Rightarrow -3/2 + 6 + d = 1$.

$d = 1 - 4\frac{1}{2}$

$= -3\frac{1}{2}$.

$\therefore y = x^3 - 3/2\, x^2 - 6x - 3\frac{1}{2}$.