St George Girls' High School

Year 11

Common Test #1

March 2002

Mathematics Extension 1

Time Allowed: 75 minutes

Instructions

1. All questions are of equal value.
2. All questions should be attempted.
3. Start each question on a new page.
4. Write on one side of each page only.
5. Marks may be deducted for poorly presented work.

Question 1 – (Start a new page) **Marks**

a) Without using a calculator determine which is larger: (show all working) **2**

$3\sqrt{3}+4$ or $7\sqrt{3}-3$

b) Sketch the following curves:

(i) $y=\sqrt{\frac{9}{4}-x^2}$ **2**

(ii) $y=\left(\frac{1}{3}\right)^x$ **2**

(iii) $y=\log_3 x$ **2**

c) Write down the domain and range for the curves in b). **3**

d) Determine whether the following curves are odd, even or neither:

(i) $y=x^4+2x^2+3$ **1**

(ii) $y=2^x$ **1**

(iii) $y=\dfrac{x}{x^3+1}$ **1**

Question 4 – (Start a new page)

Marks

a) Find the inverse function $y = f^{-1}(x)$ if:

(i) $f(x) = 2x + 1$ **1**

(ii) $f(x) = \dfrac{2x+1}{x-4}$ **2**

b) (i) For $f(x) = 2x + 1$, verify that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$. **2**

(ii) Having sketched $f(x) = \dfrac{2x+1}{x-4}$, explain how one could readily obtain the sketch of $y = f^{-1}(x)$. (Do not sketch either graph). **1**

c) By firstly sketching $y = x^2 - 1$, draw a sketch of $y = \dfrac{1}{x^2-1}$, showing all important features. **4**

d) (i) Write down the equations representing this function without using absolute value notation:

$y = 2x - 3 + |2x - 1|$ **2**

(ii) Hence, sketch the above function. **2**

Question 5 – (Start a new page)

Marks

a) Solve this inequality:

$\dfrac{2x-1}{x+3} \le 1$ **4**

b) Solve this pair of simultaneous equations. **4**

$2x + y = -3$ ①

$x^2 + y^2 = 5$ ②

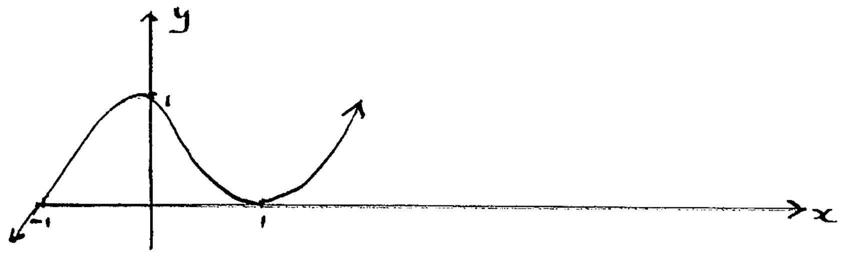
c) (i) By considering the behaviour of y as $x \to \pm\infty$, and considering whether the function is odd, even or neither, draw a sketch of $y = 2^{-x^2}$. **2**

(ii) By referring to the above sketch, discuss whether the inverse relation to the above function is itself a function. **1**

d) Prove, by contradiction, that $\log_2 3$ is irrational. **3**

Question 2 – (Start a new page) **Marks**

a) Shown here is a sketch of $y = f(x)$



On separate graphs, draw sketches of:

(i) $y = f(x) + 1$ **1**

(ii) $y = f(x+1)$ **1**

(iii) $y = -f(x)$ **1**

(iv) $y = f(-x)$ **1**

(v) $y = 2f(x)$ **1**

(vi) $y = |f(x)|$ **1**

(vii) $y = |f(x)| + 1$ **1**

b) (i) By firstly factorising, sketch the curve $y = x^3 + x^2 - 12x$, showing clearly the x and y intercepts. **3**

(ii) Hence, solve the inequality $x^3 + x^2 - 12x \geq 0$ **2**

(iii) Determine the domain of the function $y = \dfrac{1}{\sqrt{x^3 + x^2 - 12x}}$ **2**

Question 3 – (Start a new page) **Marks**

a) Solve this set of simultaneous equations for a, b and c: **4**

$$2a + b - c = 6$$
$$3a - b + 2c = 5$$
$$a - b - c = -2$$

b) Solve the following inequations:

(i) $-2 \leq 3 - x < 5$ **2**

(ii) $-1 < \log_3 x < 2$ **2**

(iii) $x^2 + 4x - 21 \geq 0$ **2**

(iv) $|2x+1| < 3$ **2**

(v) $2^{-x} > 32$ **2**

SOLUTIONS:-

QUESTION 1 - (a) Check sign of difference

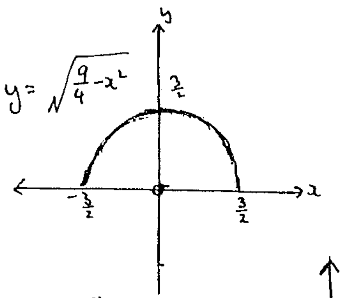
$(7\sqrt{3}-3)-(3\sqrt{3}+4)$

$= 4\sqrt{3}-7$

$= \sqrt{48}-\sqrt{49}$

< 0 $\quad \therefore 3\sqrt{3}+4 > 7\sqrt{3}-3$

(b) (i)



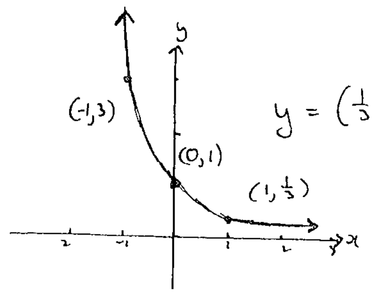
Semi-circle
centre origin
radius $\frac{3}{2}$ units

(ii) $y=\left(\frac{1}{3}\right)^x$

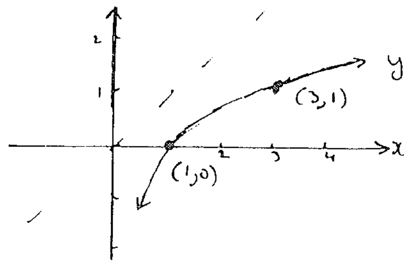
$y=(3^{-1})^x$

$y=3^{-x}$

Exponential



(iii)



(c) (i) D: $-\frac{3}{2} \le x \le \frac{3}{2}$ $\quad$ (ii) D: x, all reals

R: $0 \le y \le \frac{3}{2}$ $\quad$ R: $y > 0$

(iii) D: $x > 0$

R: y, all reals

(d) (i) let $f(x) = x^4+2x^2+3$

then $f(-x) = (-x)^4 + 2(-x)^2+3$

$= x^4+2x^2+3$

$\therefore f(x) = f(-x)$, this curve is an EVEN function

(ii) let $g(x) = 2^x$

then $g(-x) = 2^{-x}$

and $g(x) \neq g(-x)$

$g(x) \neq -g(-x)$

neither odd nor even

(iii) let $f(x) = \dfrac{x}{x^3+1}$

then $f(-x) = \dfrac{-x}{(-x)^3+1}$

$= \dfrac{-x}{-(x^3-1)}$

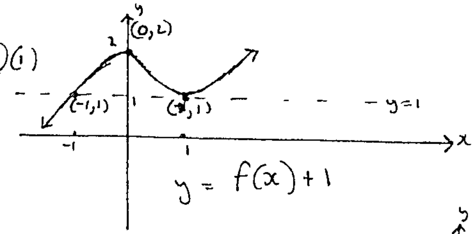
$= \dfrac{x}{x^3-1}$ $\quad \neq f(x)$

$\neq -f(x)$

neither odd nor even

QUESTION 2 :-

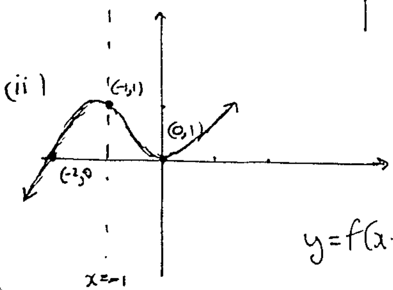
(a) (i)



"up one"

$y = f(x) + 1$

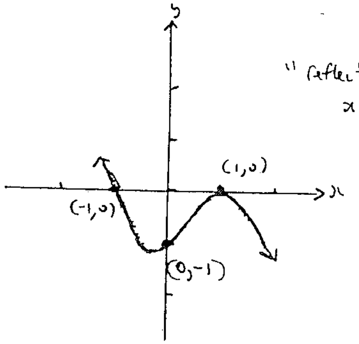
(ii)



$y = f(x+1)$

"left one"

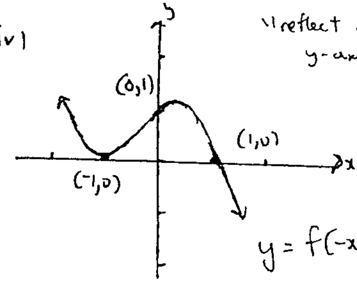
(iii)



"reflect across x-axis"

$y = -f(x)$

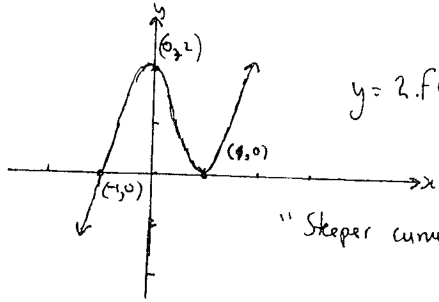
(iv)



"reflect across y-axis"

$y = f(-x)$

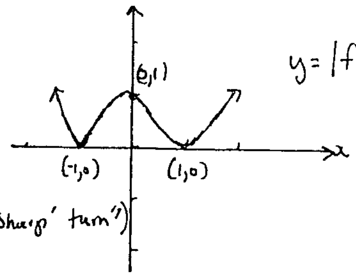
(v)



$y = 2.f(x)$

"Steeper curve"

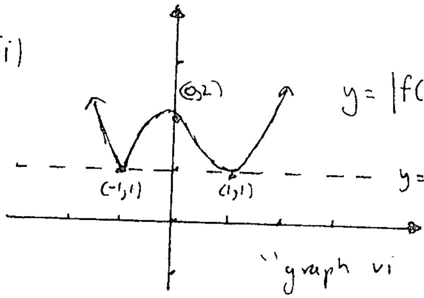
(vi)



$y = |f(x)|$

('Sharp' turn')

(vii)



$y = |f(x)| + 1$

"graph vi up 1"

(b) (i) $y = x(x^2 + x - 12)$

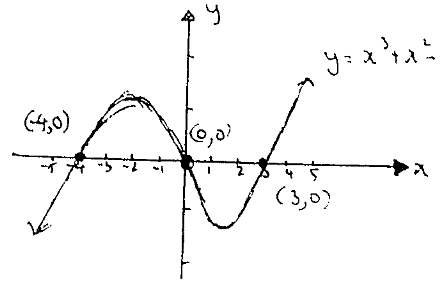
$y = x(x+4)(x-3)$

When $x = 0$, $y = 0$ → passes through the origin $(0,0)$

When $y = 0$, $x = 0, -4, 3$

x-intercepts $(-4,0)$, $(0,0)$, $(3,0)$

x	-5	-4	-3	0	1	3	4
y	-40 -ve	0	18 +ve	0	-10 -ve	0	32 +ve



(ii) From the graph: $x^3 + x^2 - 12x \geq 0$

for $-4 \leq x \leq 0$ and $x \geq 3$

(iii) Require $x^3 + x^2 - 12x > 0$

then Domain: $-4 < x < 0$ and $x > 3$

QUESTION 3:- (a) $2a + b - c = 6 \quad \ldots (i)$

$3a - b + 2c = 5 \quad \ldots (ii)$

$a - b - c = -2 \quad \ldots (iii)$

$(i) + (ii) \rightarrow 5a + c = 11 \quad \ldots A$

$(ii) - (iii) \rightarrow 2a + 3c = 7 \quad \ldots B$

From (A) $c = 11 - 5a$

Substitute into (B) $2a + 3(11-5a) = 7$

$33 - 13a = 7$

$-13a = -26$

$a = 2$

by substitution $c = 11 - 10$

$c = 1$

and $a - b - c = -2$

give $2 - b - 1 = -2$

$b = 3$

Solution $a = 2,\ b = 3,\ c = 1$

(b) (i) $-5 \leq -x \leq 2$

then $5 \geq x \geq -2$ i.e. $-2 \leq x \leq 5$

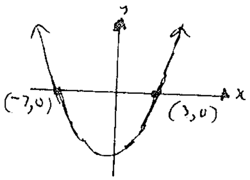
(ii) $3^{-1} < x < 3^{2}$

$\frac{1}{3} < x < 9$

(iii) $(x+7)(x-3) \geq 0$

From diagram

$x \leq -7$ and $x \geq 3$



(iv) $2x+1 > -3$ and $2x+1 < 3$

$2x > -4 \qquad 2x < 2$

$x > -2 \qquad x < 1$

Soln $-2 < x < 1$

(v) $\log_2 2^{-x} > \log_2 32$

$-x \log_2 2 > 5 \log_2 2$

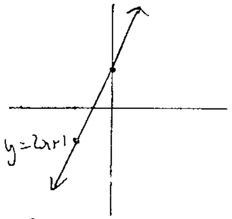
$-x > 5$

$\therefore x < -5$

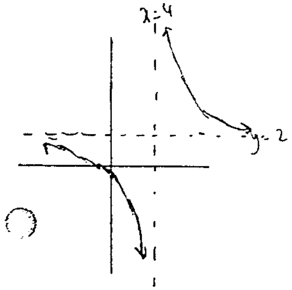
QUESTION 4:- (a) (i) $x = 2y+1$

then $y = \frac{1}{2}(x-1)$

$\therefore \; f^{-1}(x) = \frac{1}{2}(x-1)$



Satisfies "horizontal line test", ∴ has inverse as a function



Satisfies 'horizontal line test' except $x \neq 4$

(ii) $x = \frac{2y+1}{y-4}$

$xy - 4x = 2y + 1$

$y(x-2) = 4x+1$

$y = \frac{4x+1}{x-2}$

$\therefore f^{-1}(x) = \frac{4x+1}{x-2} \quad x \neq 2$

(b) (i) $f(x) = 2x+1$

$f^{-1}(x) = \frac{1}{2}(x-1)$

Now $f\left[f^{-1}(x)\right]$

$= 2\left[\frac{1}{2}(x-1)\right]+1$

$= x-1+1$

$= x$

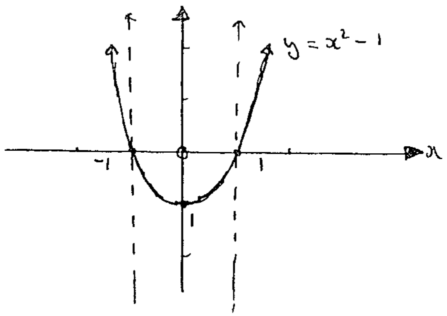
and $f^{-1}\left[f(x)\right] = \frac{1}{2}\left[(2x+1)-1\right]$

$= \frac{1}{2} \times 2x$

$= x$

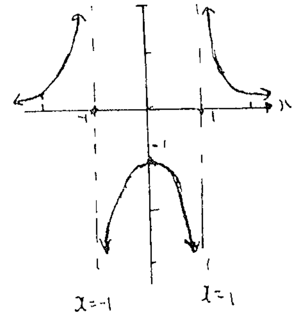
(ii) From the sketch of $y=f(x)$ reflect the curve across the line $y=x$.

(c)



- asymptotes at $x=-1$ & $x=1$

(carry over page)



Note: $x \to 0$, $\frac{1}{x} \to \infty$

and $x \to \infty$, $\frac{1}{x} \to 0$

$y = \frac{1}{x^2-1}$

(d) (i) for $2x-1<0$ then $y = 2x-3+-(2x-1)$

$x < \frac{1}{2}$ $\qquad y = -2$

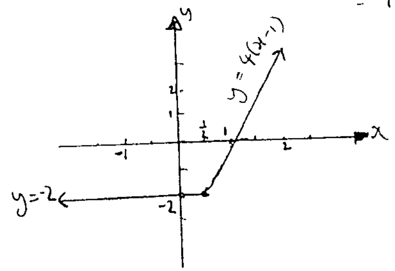
and

for $2x-1 \geq 0$ then $y = 2x-3+2x-1$

$x \geq \frac{1}{2}$ $\qquad = 4x-4$

$= 4(x-1)$

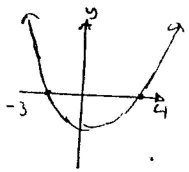
(ii)



When $x = \frac{1}{2}$, $y = -2$

$x = 1$, $y = 0$

QUESTION 5 :-



(a) $(x+3)^2 \times \frac{2x-1}{(x+3)} \leq 1 \times (x+3)^2$

$(x+3)(2x-1) - (x+3)^2 \leq 0$

$(x+3)\left[(2x-1) - (x+3)\right] \leq 0$

$(x+3)(x-4) \leq 0$

Soln. $x \neq -3 \quad \therefore -3 < x \leq 4$

(b) From ① $y = -3-2x$

Sub in ② $x^2 + \left[-(3+2x)\right]^2 = 5$

$x^2 + 9 + 12x + 4x^2 = 5$

$\therefore 5x^2 + 12x + 4 = 0$

$(5x+2)(x+2) = 0$

Then $x = -\frac{2}{5} \quad , \quad x = -2$

$y = \frac{-11}{5} \quad , \quad y = 1$

Soln $\left(-\frac{2}{5}, -\frac{11}{5}\right)$ and $(-2, 1)$

(c) (i) let $y = f(x) = 2^{-x^2}$

then $f(-x) = 2^{-(-x)^2}$

$= 2^{-x^2}$

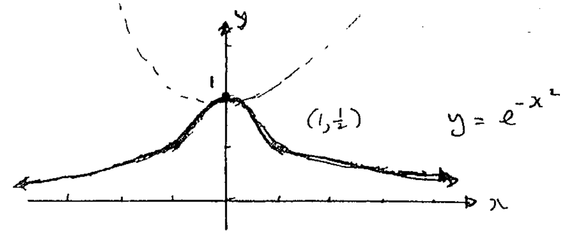
$= f(x)$ this curve has symmetry about the y-axis – EVEN

Domain all real x

& as $x \to \pm\infty$, $y = \frac{1}{2^{\infty}}$

$\to 0$

at $x = 0$, $y = 1$



(ii) This curve does not satisfy horizontal line test, therefore, it's inverse would not be a function.

(d) Assume $\log_2 3$ is rational.

Let $\log_2 3 = \frac{p}{q}$ • where p, q are non-zero integers with H.C.F of 1

then $3 = 2^{\frac{p}{q}}$

• raise to power of q

So $3^q = 2^p$

Now, powers of 3 and powers of 2 have no common value [odd values ≠ even values]

This result is a contradiction, hence, our initial assumption that $\log_2 3$ is rational must be incorrect $\therefore \log_2 3$ is irrational.

See over,