St George Girls High School

Year 11

Common Test 1

March 2004

Mathematics Extension 1

Time Allowed: 75 minutes

Instructions

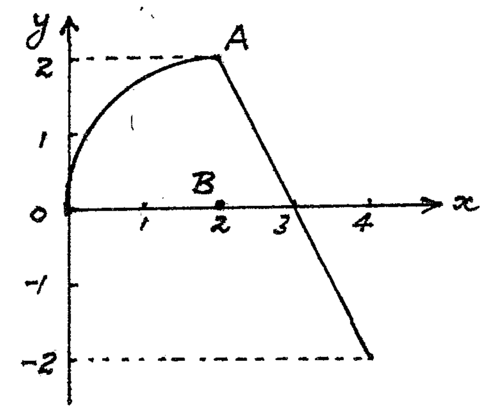
1. All 5 questions may be attempted.
2. Remove the graph sheet from the question booklet and submit it with your solutions.
3. Start each question on a new page.
4. All necessary working must be shown.

Question 1 – 12 marks – (Start a new page) **Marks**

a) Sketch the region satisfying both of the following: **2**

$$\left.\begin{array}{l} y > x^2 \\ x^2 + y^2 < 4 \end{array}\right\}$$

b) The function $y = f(x)$ is defined for $0 \le x \le 4$ as below.



(i) The quadrant OAB is part of a circle centred at B. State the equation of this circle. **1**

(ii) On the number planes provided, sketch each of the following:

(α) $y = f(x+1)$ **1**

(β) $y = f(2x)$ **1**

c) (i) Sketch $f(x) = \sqrt{4 - x^2}$ **1**

(ii) Hence sketch $y = \dfrac{1}{f(x)}$ **2**

d) Show that $\dfrac{1}{2+\sqrt{3}} + \dfrac{1}{2-\sqrt{3}}$ is rational. **2**

e) Solve for x: $-1 \le 3 - 2x < 1$ **2**

Question 2 – 12 marks – (Start a new page) **Marks**

a) Simplify each of the following:

(i) $\dfrac{3x^2-8x+5}{25-9x^2}$ **2**

(ii) $\dfrac{1}{x^2-3x}-\dfrac{1}{x^2-9}$ **2**

(iii) $(x+1)^3-(x-1)^3$ **2**

b) Find positive integers p and q such that $\dfrac{3^p}{2^q}=\dfrac{\left(\frac{3}{4}\right)^3\times\left(\frac{2}{3}\right)^5}{\left(\frac{8}{3}\right)^4}$ **3**

c) Sketch <u>any</u> function which has <u>all</u> of the following properties. **3**

(i) Domain is $-3<x<3$ ✓

(ii) Range is $y\le 2$

(iii) $f(1)=2$

(iv) $f(-x)=f(x)$ for all x in the domain.

(v) $x=3$ is a vertical asymptote. √

Question 3 – 12 marks – (Start a new page) **Marks**

a) Solve for x:

$$\frac{2x-1}{x+1} \geq 1$$ **3**

b) (i) Solve for x:

$$|2x-3| = x+2$$ **3**

(ii) On the same axes, sketch $y = |2x-3|$ and $y = x+2$ **2**

(iii) Hence solve for x: **2**

$$|2x-3| > x+2$$

c) Is it possible for a rectangular field to have an area of 100m^2 and perimeter 30m?

[NOTE: your explanation should include the use of equations and/or graphs] **2**

Question 4 – 12 marks – (Start a new page)

Consider the function $y = \dfrac{x^2 - 1}{x^2 - 4}$ and its graph.

(i) Find the x and y intercepts **3**

(ii) Find the vertical asymptotes **2**

(iii) Make x^2 the subject of this function **2**

(iv) State the horizontal asymptote **1**

(v) Sketch the graph of $y = \dfrac{x^2 - 1}{x^2 - 4}$ clearly showing essential features. **2**

(vi) Use the graph to determine the number of solutions of

$\dfrac{x^2 - 1}{x^2 - 4} = k$ where **2**

(α) $k < 0$

(β) $\dfrac{1}{4} < k \leq 1$

Question 5 – 12 marks – (Start a new page) **Marks**

Consider $f(x) = x^2 - 2x$

(i) Find the domain and range of $f(x)$ **3**

(ii) Prove that $f(x)$ is neither an even function nor an odd function **2**

(iii) The graph of $y = f(x)$ is said to pass the vertical line test. Explain carefully why this means that $f(x)$ is a function. **1**

(iv) State the largest positive domain of $f(x)$ for which $f^{-1}(x)$ exists as a function. **1**

(v) If the domain of $f(x)$ is restricted as in (iv) above, prove that $f^{-1}(x) = 1 + \sqrt{1 + x}$ clearly stating both the domain and range of $f^{-1}(x)$ **4**

(vi) Evaluate $f^{-1}[f(0)]$ **1**

Solutions. ST GEORGE GIRLS H.S.

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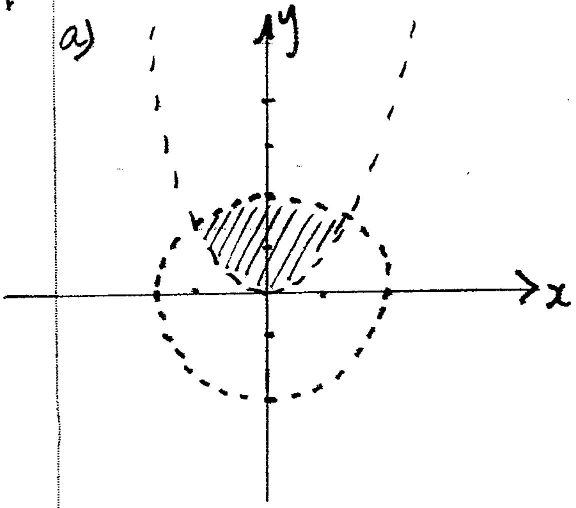
Q1

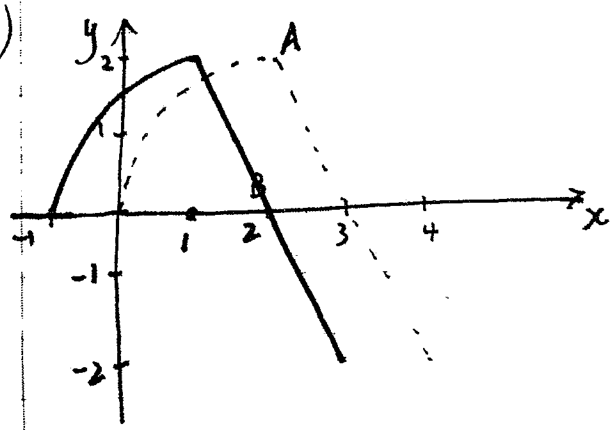
a)



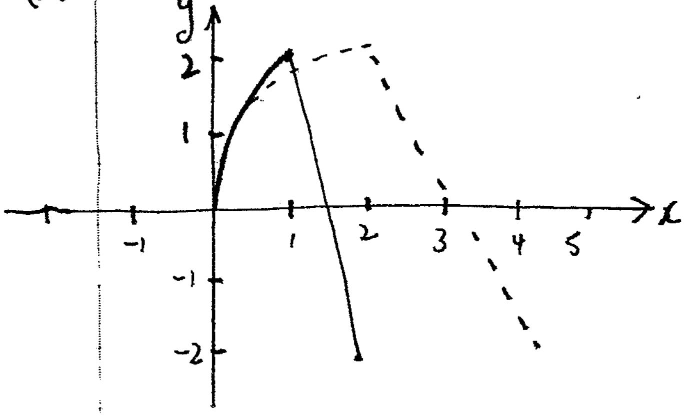
(b)

(i) $(x-2)^2 + y^2 = 4$

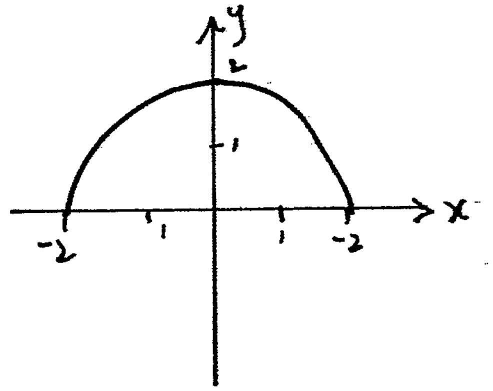
(ii) (α)



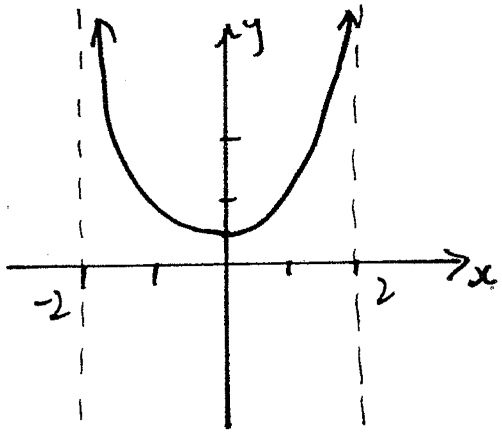
(β)



(c) (i)



(ii)



(d) $\frac{1}{2+\sqrt{3}} + \frac{1}{2-\sqrt{3}} = \frac{2-\sqrt{3}+2+\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}$

$= \frac{4}{2^2-(\sqrt{3})^2}$

$= \frac{4}{4-3}$

$= 4$

(e)

$-1 \le 3-2x < 1$

$-4 \le -2x < -2$

$\frac{-4}{-2} \geq x > \frac{-2}{-2}$

$2 \geq x > 1$

$\therefore \quad 1 < x \le 2$

Q2.

(a)(i) $\dfrac{3x^2-8x+5}{25-9x^2} = \dfrac{(3x-5)(x-1)}{(5+3x)(5-3x)}$

WRONG ANSWER ⇒

RIGHT = $\dfrac{x-1}{-3x-5}$.

$= \dfrac{x-1}{5+3x}$

(ii) $\dfrac{1}{x^2-3x} - \dfrac{1}{x^2-9}$

$= \dfrac{1}{x(x-3)} - \dfrac{1}{(x+3)(x-3)}$

$= \dfrac{x+3}{x(x+3)(x-3)} - \dfrac{x}{x(x+3)(x-3)}$

$= \dfrac{3}{x(x+3)(x-3)}$

(iii) $(x+1)^3-(x-1)^3$

$= [(x+1)-(x-1)][(x+1)^2+(x+1)(x-1)+(x-1)^2]$

$= 2(3x^2+1)$

$= 6x^2+2$

(b) $\dfrac{3^p}{2^q} = \left(\dfrac{3}{2^2}\right)^3 \times \left(\dfrac{2}{3}\right)^5 \div \left(\dfrac{2^3}{3}\right)^4$

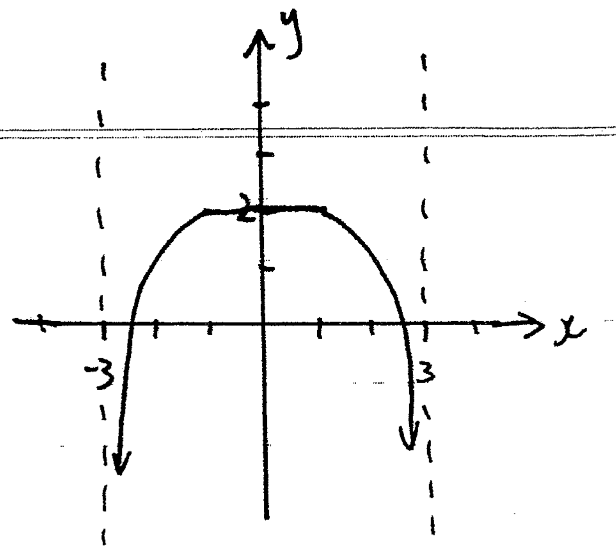
$= \dfrac{3^3}{2^6} \times \dfrac{2^5}{3^5} \times \dfrac{3^4}{2^{12}}$

$= \dfrac{3^7 . 2^5}{2^{18} . 3^5}$

$\therefore p = 2$

$= 3^2$ ... 13

(c)



~~1~~ ~~2~~ 3 ~~4~~ ~~7~~

Q3

(a)

$$\frac{2x-1}{x+1} \geq 1$$

$x \neq -1$

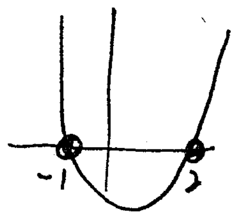
$$\frac{2x-1}{x+1} \geq 1$$

1 $(2x-1)(x+1) \geq (x+1)^2$

$2x^2+x-1 \geq x^2+2x+1$

$x^2-x-2 \geq 0$

1 $(x-2)(x+1) \geq 0$



$x \geq 2$

$x < -1$

(b) (i) $|2x-3| = x+2$

$2x-3 = x+2$

$x = 5$ 1

$2x-3 = -(x+2)$

$2x-3 = -x-2$

$3x = 1$

$x = \frac{1}{3}$ 1

check. 1

$x=5$

$|2x-3| = |10-3|$

$= 7$

$x+2 = 5+2$

$= 7$

$\therefore x=5$ is a solution

$x = \frac{1}{3}$

$|2x-3| = |\frac{2}{3}-3|$

$= |-2\frac{1}{3}|$

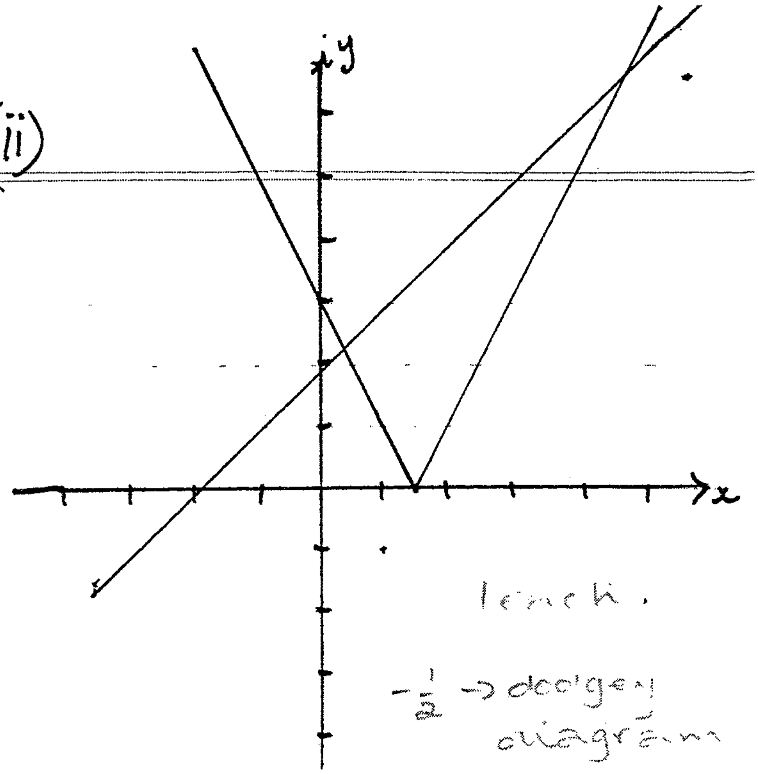
$= 2\frac{1}{3}$

$x+2 = \frac{1}{3}+2$

$= 2\frac{1}{3}$

$\therefore x = \frac{1}{3}$ is a sol

(ii)



leach.

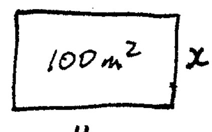
$-\frac{1}{2}$ → dodgey diagram

(iii) $|2x-3| > x+2$

$x > 5$ 1

$x < \frac{1}{3}$

(c)



$2x+2y = 30$

$x+y = 15$

$y = 15-x$

$\therefore x(15-x) = 100$ $\frac{1}{2}$

$15x - x^2 = 100$

$\therefore x^2 - 15x + 100 = 0$ $\frac{1}{2}$

$$x = \frac{15 \pm \sqrt{15^2 - 4.1.100}}{2}$$

$$= \frac{15 \pm \sqrt{-175}}{2}$$ $\frac{1}{2}$

$\therefore$ There is no solution to this problem $\frac{1}{2}$

$\therefore$ there is no rectangle with a perimeter of 30m and area of $100 m^2$.

Q4.

$$y = \frac{x^2-1}{x^2-4}$$

i) x-int when $y = 0$

$\therefore \frac{x^2-1}{x^2-4} = 0$

$\therefore x^2 - 1 = 0$

$x = \pm 1$

y-int when $x = 0$

$\therefore y = \frac{0-1}{0-4}$

$y = \frac{1}{4}$

ii) vertical asymptotes.

$x^2 - 4 \neq 0$

$x \neq \pm 2$

$\therefore$ vertical asymptotes

$x = 2$ and $x = -2$

ii) $y(x^2-4) = x^2-1$

$x^2y - 4y = x^2 - 1$

$x^2y - x^2 = -1 + 4y$

$x^2(y-1) = 4y - 1$

$x^2 = \frac{4y-1}{y-1}$

(iv) $\frac{x^2-1}{x^2-4} = \frac{x^2-4+3}{x^2-4}$

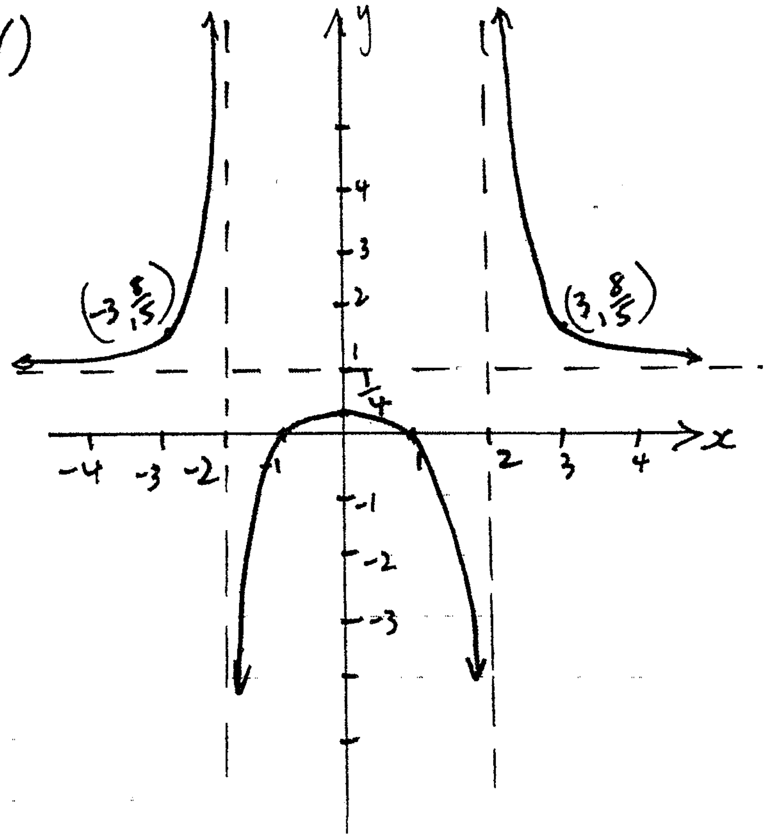
$= \frac{x^2-4}{x^2-4} + \frac{3}{x^2-4}$

$= 1 + \frac{3}{x^2-4}$

$\therefore$ horizontal asymptote

$y = 1$

(V)



(α) $\frac{x^2-1}{x^2-4} < 0$

2 solutions

(β) $\frac{1}{4} < \frac{x^2-1}{x^2-4} \leq 1$

no solutions.

Q5 (a) (i) axis $x = \frac{2}{2}$

$= 1$

vertex is $(1, -1)$

Domain all real numbers

Range $y \geq -1$

(ii) $f(x) = x^2 - 2x$

$f(-x) = (-x)^2 - 2(-x)$

$= x^2 + 2x$

$\neq f(x), -f(x)$

$\therefore f(x)$ is neither even nor odd

(iii) For a given value of x in the domain there is at most 1 y value

(iv) $x \geq 1$

(v) $f: \; y = x^2 - 2x$ $\qquad R: \; y \geq -1$

$f^{-1}: \; x = y^2 - 2y$ $\qquad D: \; x \geq -1$

$y \geq 1$

$y^2 - 2y - x = 0$

$y^2 - 2y = x$

$y^2 - 2y + 1 = 1 + x$

$(y-1)^2 = 1 + x$

$y - 1 = \pm\sqrt{1+x}$

$y = 1 \pm \sqrt{1+x}$ $\qquad$ but $y \geq 1$

$\therefore \; y = 1 + \sqrt{1+x}$

ie $f^{-1}(x) = 1 + \sqrt{1+x}$ $\qquad D: \; x \geq -1$

$R: \; y \geq 1$

(vi) $f^{-1}[f(0)]$

$= f^{-1}[0]$

$= 1 + \sqrt{1+0}$

$= 2$