St George Girls' High School

Year 11

Common Test – 2

June 2003

Mathematics

Time Allowed: 75 minutes

Instructions

1. All questions should be attempted.
2. Show all working.
3. START EACH QUESTION ON A NEW PAGE.
4. Marks will be deducted for careless work or poorly presented solutions.
5. On the cover sheet of the answer booklet clearly show:
   a) your name
   b) your mathematics class and teacher

Question 1 (8 marks) – Start a New Page

Marks

a) Factorise: $3+2x-x^2$ **2**

b) Find all values of θ such that $\cos 2\theta = 1$, $0° \leq \theta° \leq 360°$ **2**

c) If $f(x) = x^2 - 1$ and $g(x) = 1 - 3x$

Find:

(i) $f(-2)$ **1**

(ii) x if $g(x) = 4$ **1**

(iii) $f[g(x)]$ **2**

Question 2 (8 marks) – Start a New Page **Marks**

a) Evaluate: $\dfrac{\text{cosec}78^\circ 12'}{\sin 215^\circ 24'}$ **1**

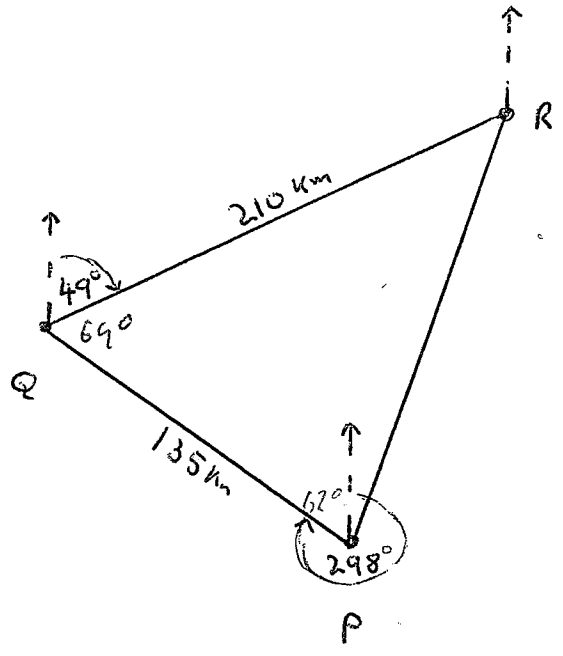
giving your answer correct to 3 decimal places.

b) Using EXACT ratios show that **3**

$$\sin^2 60.\cot 30^\circ + \sec 30^\circ = \frac{17\sqrt{3}}{12}$$

c) A ship sails from port P for 135km on a bearing of 298° to reach port Q. From port Q it sails on a bearing of 49° for 210km to port R. **4**

Find to the nearest km, the distance of port P from port R.



[not to scale]

Question 3 (8 marks) – Start a New Page **Marks**

a) Solve for θ where $-180^\circ \leq \theta^\circ \leq 180^\circ$ **3**

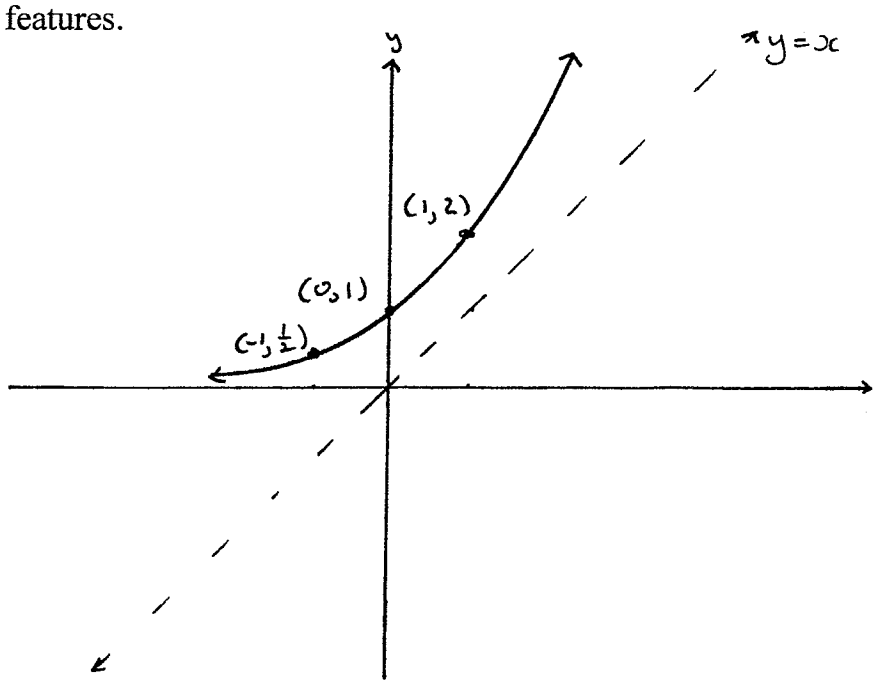
$$\tan^2\theta - \tan\theta = 0$$

b) Simplify: $\cos(180-\theta).\cot(90-\theta)$ **2**

c) A triangle has sides 12cm and 7cm with the angle opposite the 7cm side being 29°. Find TWO possibilities for the angle opposite the 12cm side (to nearest minute) and explain why there can be two answers. **3**

Question 4 (8 marks) – Start a New Page **Marks**

a) Copy this diagram into your answer booklet and sketch the inverse relation showing essential features. **2**



b) Determine algebraically the inverse of the function $y = \frac{1}{x} - 1$. **2**

[Give the inverse with y as the subject]

c) (i) Sketch the graph of the parabola $y = 1 - x^2$ showing the co-ordinates of the points where $x = -1$, $x = 0$ and $x = 1$ **2**

(ii) Using part (i), and by noting any observations about reciprocals, graph $y = \frac{1}{1 - x^2}$ **2**

Question 5 (8 marks) – Start a New Page

Mark

a) Solve: $|5x+3|=7$ **2**

b) Consider the function $g(x)=\sqrt{2-x}$

(i) State the domain and range **2**

(ii) Sketch $y=g(x)$ **1**

c) (i) On the same set of axes carefully graph

$y=|x+1|$ and $2x+y-1=0$ **2**

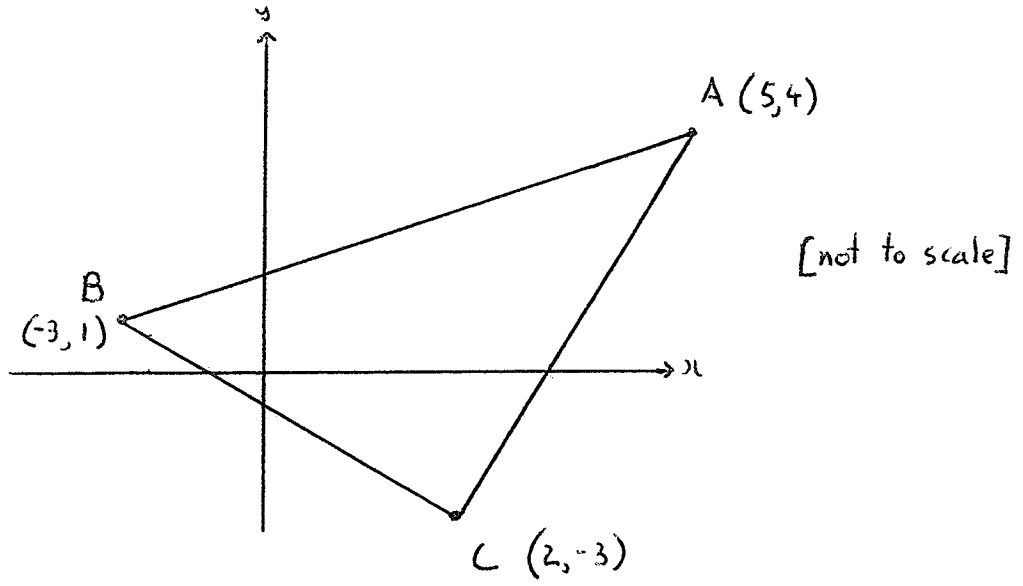
(ii) Use your graph to solve $|x+1|+2x\leq 1$ **1**

Question 6 (8 marks) – Start a New Page **Marks**

a) (i) Express $\frac{2}{7}x = 1 - \frac{5}{7}y$ in general form. **1**

(ii) Give the x and y intercepts of the line from (i). **1**

b)



(i) Find the EXACT distance of A to B. **1**

(ii) Show that the equation of the line through A and B is $3x - 8y + 17 = 0$ **2**

(iii) Find the length of the altitude of the triangle from AB to C. **2**

(iv) Hence, or otherwise, find the area of the triangle ABC. **1**

Question 7 (8 marks) – Start a New Page

Marks

a) Find the equation of the line through $(-1, -3)$ parallel to the line with equation $3x - 2y = 12$. **3**

b) (i) Shade the region defined by the intersection of

$x \geq -1$, $y \leq 2$ and $2x - y - 4 \leq 0$ **3**

(ii) Let the point of intersection of the lines $y = 2$ and $2x - y - 4 = 0$ be A. Find the size of the acute angle at A. (Correct to the nearest minute). **2**

Question 8 (8 marks) – Start a New Page **Marks**

a) If $\sin\theta = \frac{2}{3}$ and $\tan\theta < 0$

(i) give the EXACT ratio for $\cos\theta$ **2**

(ii) give the EXACT ratio for $\cot\theta$ **1**

(iii) by using the exact ratios above, show $\frac{\sin\theta}{\cos\theta} = \tan\theta$ **2**

b) If $2x - 3y - 3 = 0$ and $x + ay + b = 0$ are perpendicular to one another.
Find the values of a and b if the two lines also intersect at $x = 1$. **3**

YEAR 11 COMMON TEST 2 — MATHEMATICS — 2003

1. a) $3+2x-x^2 = (3-x)(1+x)$

b) $\cos 2\theta = 1 \qquad 0 \le \theta \le 360$

$\therefore \; 0 \le 2\theta \le 720$

$2\theta = 0, 360, 720$

$\therefore \theta = 0, 180, 360$

c) i) $f(-2) = (-2)^2 - 1$

$= 3$

ii) $1 - 3x = 4$

$-3x = 3$

$x = -1$

iii) $f(1-3x) = (1-3x)^2 - 1$

$= 1 - 6x + 9x^2 - 1$

$= 9x^2 - 6x$

---

2. b) -1.764 (to 3 d.p.)

b) $\sin^2 60 . \cot 30 + \sec 30 = \frac{17\sqrt{3}}{2}$

$LHS = \left(\frac{\sqrt{3}}{2}\right)^2 . \sqrt{3} + \frac{2}{\sqrt{3}}$

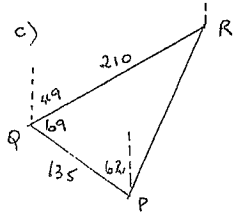
$= \frac{3\sqrt{3}}{4} + \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{3\sqrt{3}}{4} + \frac{2\sqrt{3}}{3}$

$= \frac{9\sqrt{3} + 8\sqrt{3}}{12}$

$= \frac{17\sqrt{3}}{12}.$

c)



$R\hat{Q}P = 69°$

$PR^2 = 135^2 + 210^2 - 2 \times 135 \times 210 \cos 69$

$PR = 204.9525...$

Distance between ports is 205 km (to nrst km)

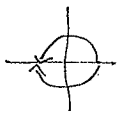
---

3 a) $\tan^2\theta - \tan\theta = 0$

$\tan\theta(\tan\theta - 1) = 0$

$\tan\theta = 0 \qquad \tan\theta = 1$

$\theta = 0° \qquad \theta = 45°, -135°$



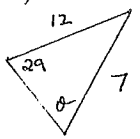
b) $\cos(180-\theta)\cot(90-\theta)$

$= -\cos\theta \times \tan\theta$

$= -\cos\theta \times \frac{\sin\theta}{\cos\theta}$

$= -\sin\theta$

c)



$\frac{\sin\theta}{12} = \frac{\sin 29}{7}$

$\sin\theta = \frac{12\sin 29}{7}$

$\theta = 56°13'$ or $123°47'$

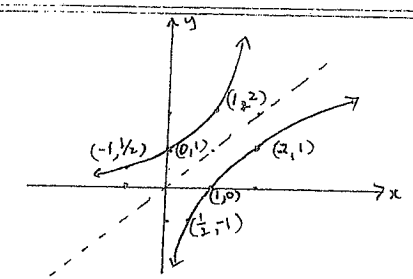
$56°13' + 29° < 180$

$123°47' + 29 < 180$

$\therefore \theta$ could be either $56°13'$ or $123°47'$

---

4 a)



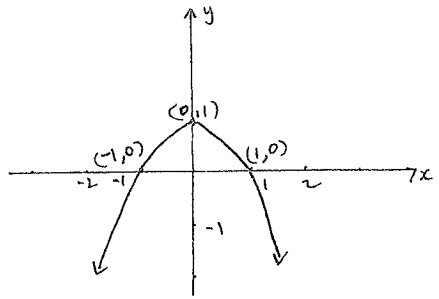
b) $f^{-1}(x)$: $x = \frac{1}{y} - 1$

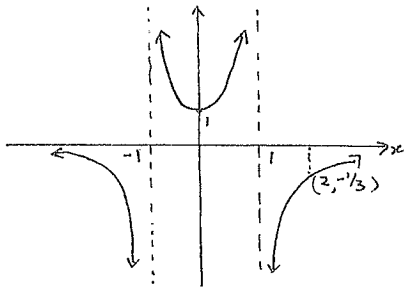
$xy = 1 - y$

$(1+x)y = 1$

$y = \frac{1}{1+x}$

4c)





$y = |x+1| \qquad y = 1-2x$

for $x \le 0 \qquad |x+1| \le 1-2x$

$\therefore |x+1| + 2x \le 1$

6. a) i) $\frac{2x}{7} = 1 - \frac{5}{7}y$

$2x = 7 - 5y$

$2x + 5y - 7 = 0$

ii) x intercept $y=0 \therefore x = \frac{7}{2}$

y intercept $x=0 \therefore y = \frac{7}{5}$

b) i) $AB = \sqrt{(5+3)^2 + (4-1)^2}$

$= \sqrt{73}$

ii) $m_{AB} = \frac{4-1}{5+3}$

$= \frac{3}{8}$

$y - 1 = \frac{3}{8}(x+3)$

$8y - 8 = 3x + 9$

$3x - 8y + 17 = 0.$

(iii) $d = \frac{|2\times 3 - 8\times -3 + 17|}{\sqrt{3^2 + (-8)^2}}$

$= \frac{47}{\sqrt{73}}$

(iv) $\therefore$ area of $\Delta ABC = \frac{1}{2} \times AB \times d$

$= \frac{1}{2} \times \sqrt{73} \times \frac{47}{\sqrt{73}}$

$= 23\frac{1}{2}$ sq. u.

5. a) $|5x+3| = 7$

$5x+3 = 7 \qquad 5x+3 = -7$

$5x = 4 \qquad 5x = -10$

$x = \frac{4}{5} \qquad x = -2$

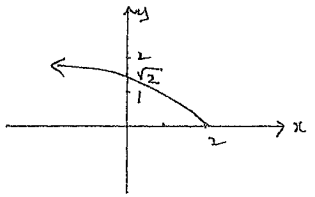
b) i) D: $2 - x \ge 0$

$2 \ge x$

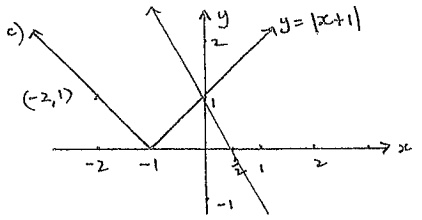
$x \le 2.$

R: $y \ge 0$

ii)



c)



7. a) $3x - 2y = 12$

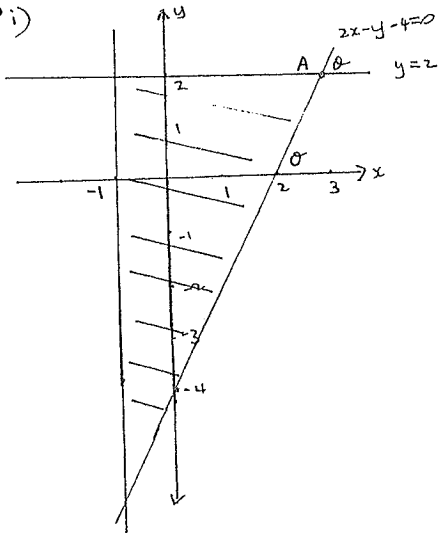
$m = \frac{-3}{-2} = \frac{3}{2}.$

$\therefore y + 3 = \frac{3}{2}(x+1)$

$2y + 6 = 3x + 3$

$3x - 2y - 3 = 0.$

b) i)



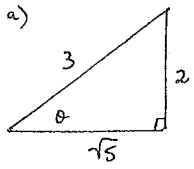
(ii) Acute angle at A $= \theta$

$\therefore \tan\theta =$ grad. of line $2x - y - 4 = 0$

$= 2$

$\therefore \theta = 63°26'$

8 a)



$\sin\theta = \frac{2}{3}$

$\tan\theta < 0$

$\therefore \theta$ is in Quad 2.

i) $\cos\theta = -\frac{\sqrt{5}}{3}$

ii) $\cot\theta = -\frac{\sqrt{5}}{2}$

iii) $\frac{\sin\theta}{\cos\theta} = \frac{\frac{2}{3}}{-\frac{\sqrt{5}}{3}} = \frac{-2}{\sqrt{5}}$

$= \tan\theta$

8b) $2x - 3y - 3 = 0$

$x + ay + b = 0$

If lines $\therefore \frac{-2}{-3} \times -\frac{1}{a} = -1$

$-\frac{1}{a} = -\frac{3}{2}$

$\therefore a = \frac{2}{3}$

$x = 1, \qquad 2 - 3y - 3 = 0$

$-3y = 1$

$y = -\frac{1}{3}.$

$x = 1, y = -\frac{1}{3}:$

$1 - \frac{1}{3} \times \frac{2}{3} + b = 0$

$\therefore b = -\frac{7}{9}.$