Angelina
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11M5

St George Girls High School

Year 11

Common Test – 2

June 2004

Mathematics

Time Allowed: 75 minutes

Instructions

1. All questions should be attempted.
2. Show all working.
3. START EACH QUESTION ON A NEW PAGE.
4. Marks will be deducted for careless work or poorly presented solutions.
5. On the cover sheet of the answer booklet clearly show:
   a) your name
   b) your mathematics class and teacher

Question 1 (10 marks) – **Start a New Page** **Marks**

a) Solve the following inequations:

(i) $-5 \leq 3x-2 \leq 7$ **2**

(ii) $x^2+x-12>0$ **3**

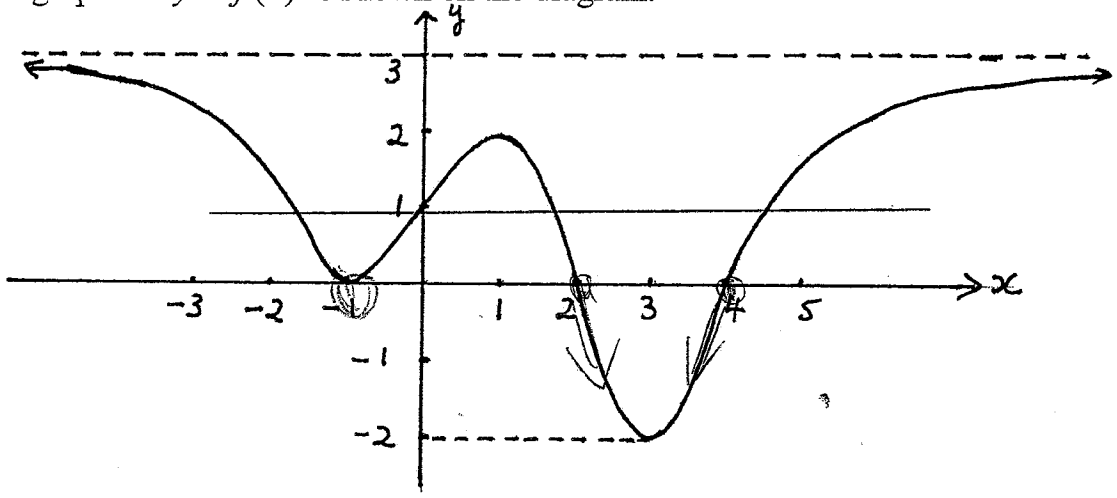
(iii) $3^x < \frac{1}{27}$ **2**

b) Show that $g(x)=\frac{x^2+3}{x^3+5x}$ is an odd function. **2**

c) What is the natural domain of the function $h(x)=\frac{x}{\sqrt{x+1}}$? **1**

Question 2 (10 marks) – **Start a New Page** **Marks**

a) The graph of $y = f(x)$ is shown on the diagram.



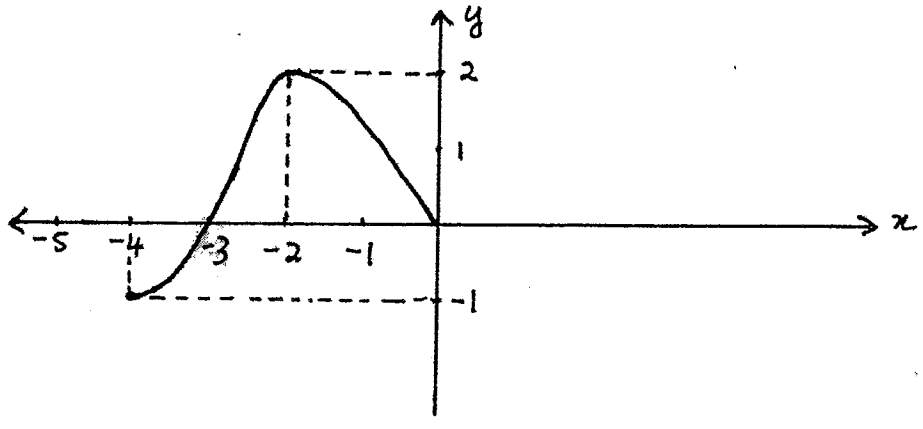
(i) For what values of x is $f(x) = 0$? **1**

(ii) For what values of x is $f(x) \leq 0$? **2**

(iii) What is the range of $y = f(x)$? **1**

(iv) How many solutions does the equation $f(x) = 1$ have? **1**

b) Copy and complete the graph of $y = g(x)$ given that $g(x)$ is an even function. **2**



c) Solve $|3x - 7| = 14$ **3**

Question 3 (10 marks) – **Start a New Page** **Marks**

a) (i) On the same diagram draw neat sketches of the graphs of **3**

$y = |x+2|$ and $y = 1-2x$

(ii) Solve the equation $|x+2| = 1-2x$ **3**

(iii) Hence solve $|x+2| > 1-2x$ **1**

b) Show that $\dfrac{\sin 225^\circ + \cos 300^\circ}{\sin 225^\circ - \cos 300^\circ} = 3 - 2\sqrt{2}$ **3**

Question 4 (10 marks) – **Start a New Page** **Marks**

a) Find the value of $\dfrac{\sec 40^\circ}{\tan 37^\circ 12'}$ correct to 3 decimal places. **1**

b) If θ is obtuse and $\sin\theta = \dfrac{2}{7}$ find the exact value of $\tan\theta$. **3**

c) From a point at the top of a vertical cliff the angle of depression of a swimmer who is 350m from the base of the cliff is $16^\circ 27'$. Find the height of the cliff to the nearest metre. **3**

d) Solve $\tan 2\theta = \sqrt{3}$ for $0^\circ \le \theta \le 360^\circ$ **3**

Question 5 (10 marks) – **Start a New Page** **Marks**

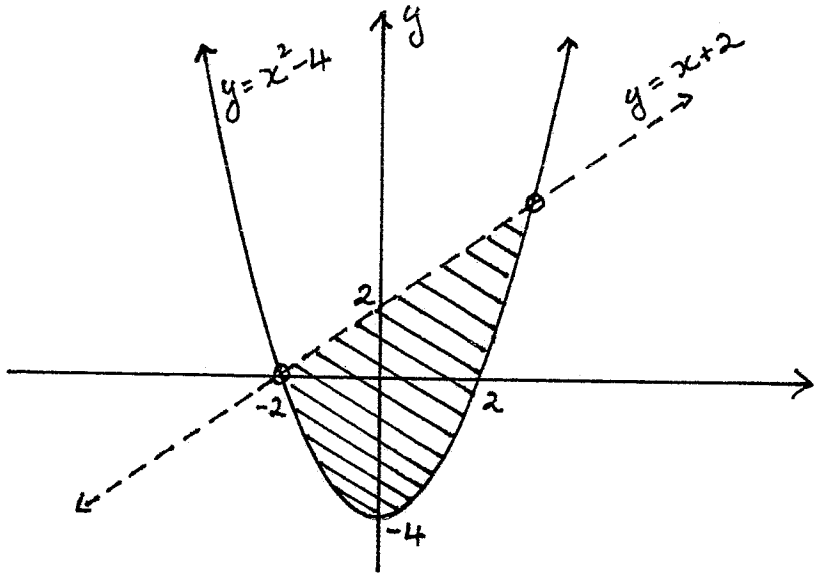
a) On separate diagrams sketch the following regions.

(i) $y < 2x+1$ **2**

(ii) $x^2 + y^2 \leq 4$ **2**

(iii) $y \leq \sqrt{9-x^2}$ **2**

b) Write down the inequations which describe the shaded region. **2**



c) Sketch the graph of $y = \left| 4 - x^2 \right|$ **2**

Question 6 (10 marks) – **Start a New Page** **Marks**

a) Find the value of θ given that $\cos\theta = \sin 72^\circ$ and θ is acute. **1**

b) Solve for $0^\circ \le \theta \le 360^\circ$ **3**

$$3\sin^2\theta - 2\sin\theta - 1 = 0$$

c) Prove that

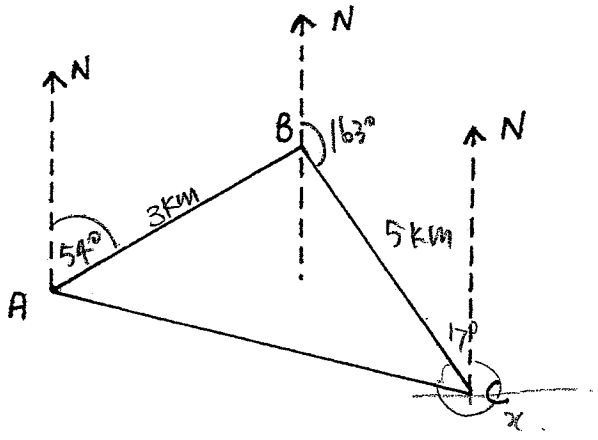
(i) $\dfrac{\sin(90^\circ - \theta)}{\cos(90^\circ - \theta)} \equiv \cot\theta$ **2**

(ii) $\dfrac{\cos\theta}{1-\sin\theta} - \dfrac{\cos\theta}{1+\sin\theta} \equiv 2\tan\theta$ **2**

d) If $x = r\cos\theta$ and $y = r\sin\theta$ show that $x^2 + y^2 = r^2$ **2**

Question 7 (10 marks) – **Start a New Page** **Marks**

a)



$a^2 = b^2 + c^2 - 2bc\cos a$

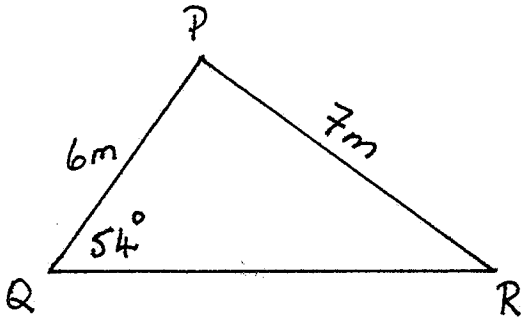
From a point A, Joyce walks 3km on a bearing of 054° to a point B. From there she walks for 5km on a bearing of 163° to a point C.

(i) Copy the diagram and mark on it all the given information. **1**

(ii) Find the distance from C to A. **2**

(iii) What is the bearing of A from C? **3**

b)



(i) Find the size of $P\hat{R}Q$ to nearest degree. **2**

(ii) Find the area of ΔPQR correct to 1 decimal place. **2**

St. George Girls' High School

MATHEMATICS

ANSWER BOOKLET

STUDENT NAME: Angelina Tjokrowidjaja.

MATHEMATICS CLASS: 11 M 5

MATHEMATICS TEACHER: Ms O'dea

QUESTION	MARKS						TOTALS
1	(a)	(b)	(c)	(d)	(e)	(f)	10/
2	(a)	(b)	(c)	(d)	(e)	(f)	10/10
3	(a)	(b)	(c)	(d)	(e)	(f)	10/10
4	(a)	(b)	(c)	(d)	(e)	(f)	10/10
5	(a)	(b)	(c)	(d)	(e)	(f)	10/10
6	(a)	(b)	(c)	(d)	(e)	(f)	10/
7	(a)	(b)	(c)	(d)	(e)	(f)	10/10
8	(a)	(b)	(c)	(d)	(e)	(f)	/
9	(a)	(b)	(c)	(d)	(e)	(f)	/
10	(a)	(b)	(c)	(d)	(e)	(f)	/

TOTAL MARK: 70/70 = 100 % wow! Congratulations...

Question 1.

(a)(i) $-5 \leq 3x-2 \leq 7$

$-3 \leq 3x \leq 9$ ✓

$-1 \leq x \leq 3$. ✓ 2

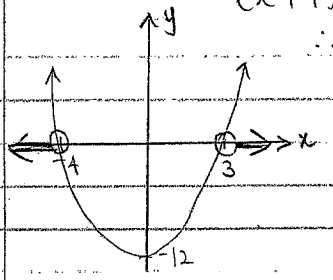
(ii) $x^2+x-12>0$

$(x+4)(x-3)>0$

$\therefore$ $x+4>0$ or $x-3>0$

$x>-4$ $x>3$

$\therefore x<-4$ or $x>3$. ✓ 3



(iii) $3^x < 3^{-3}$

$\therefore x < -3$ ✓ 2

(b) $g(x) = \frac{x^2+3}{x(x^2+5)}$.

$g(-x) = \frac{(-x)^2+3}{(-x)((-x)^2+5)} = \frac{x^2+3}{-x(x^2+5)}$.

$\therefore g(-x) = \frac{x^2+3}{x(x^2+5)} \times \frac{1}{-1} = \frac{x^2+3}{-x(x^2+5)}$. ✓

$\therefore g(-x) = -g(x)$

$\therefore g(x)$ is an odd function. ✓ 2

(c) $D = \{x > -1\}$ ✓ 1

A.T

10/10

Question 2.

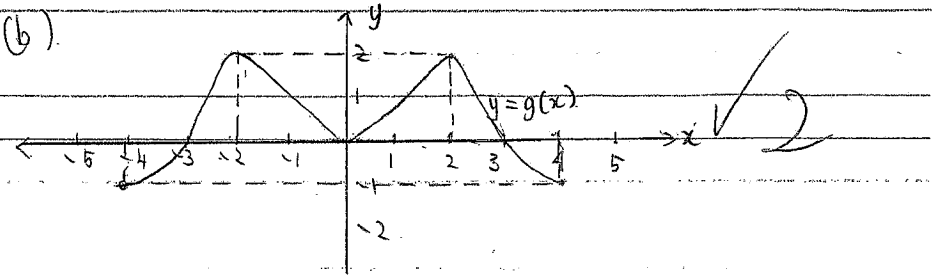
(a) (i) $f(x) \geq 0$
when $x \geq -1, 2, 4$ ✓ 1

(ii) $f(x) \leq 0$
when $x \geq -1$, $2 \leq x \leq 4$ ✓ 2

(iii) $R: \{-2 \leq y < 3\}$ ✓ 1

(iv) 4 solutions ✓ 1

(b)



✓ 2

(c) $|3x-7| \geq 14$

$3x-7 \geq 14$ or $3x-7 \leq -14$

$3x \geq 21$ $3x \geq -7$

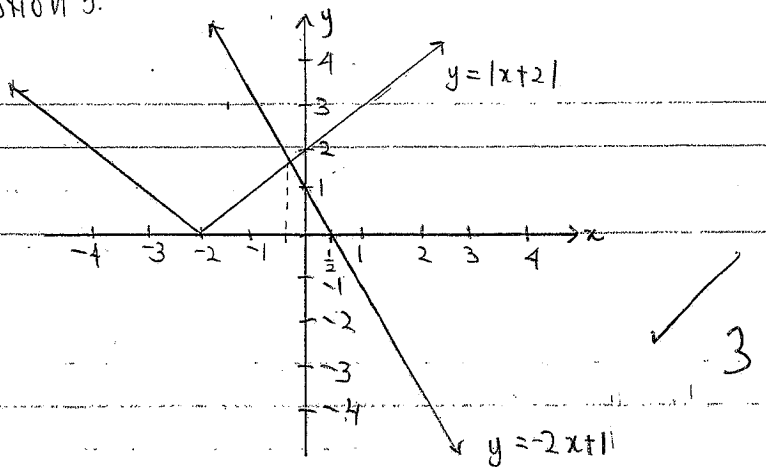
$x \geq 7$ $x \geq -\frac{7}{3}$

$\therefore x \geq 7, -\frac{7}{3}$ ✓ 3

A.T.

Question 3.

(a)(i)



✓ 3

(ii) $|x+2| \geq 1-2x$.

Case $x+2 \geq 1-2x$ or $x+2 \geq -(1+2x)$

$3x \geq -1$ $x \geq -3$

$x \geq -\frac{1}{3}$ $x \geq 3$

$\therefore$ solution. but when $x=3$ sub in. $1-2x$, answer is negative and so not possible for an absolute value to be negative

$\therefore x \geq -\frac{1}{3}$ ✓ 3

(iii) $x > -\frac{1}{3}$ ✓ 1

(b) $\frac{\sin 225 + \cos 300}{\sin 225 - \cos 300} = 3 - 2\sqrt{2}$

LHS $= \frac{-\sin 45 + \cos 60}{-\sin 45 - \cos 60} = \frac{-\frac{1}{\sqrt{2}} + \frac{1}{2}}{-\frac{1}{\sqrt{2}} - \frac{1}{2}} = \frac{\frac{-\sqrt{2}+1}{2}}{\frac{-\sqrt{2}-1}{2}}$

$= \frac{-\sqrt{2}+1}{2} \times \frac{2}{-\sqrt{2}-1} = \frac{-\sqrt{2}+1}{-1-\sqrt{2}} \times \frac{-1+\sqrt{2}}{-1+\sqrt{2}}$

$= \frac{\sqrt{2}-2-1+\sqrt{2}}{1-2}$

$= \frac{2\sqrt{2}-3}{-1} = 3-2\sqrt{2}$

$= RNS$

$\therefore LNS = RNS$

$\therefore \frac{\sin 225 + \cos 300}{\sin 225 - \cos 300} = 3-2\sqrt{2}$ ✓

3

10

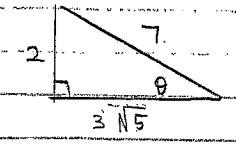
Question 4.

(a) $\frac{1}{\cos 40} = \tan 37°12' = 1.720$ (to 3 dp) ✓

(b) $\sin\theta = \frac{3}{7}$

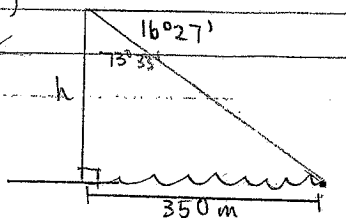
$\theta = 163°24'$

$\tan\theta = -\frac{2}{3\sqrt{5}}$ ✓



10

(c)



Let h be the height of the cliff.

$\therefore \tan 73°33 = \frac{350}{h}$ (since complementary $\angle$s $90 - 16°27'$)

$\therefore h = \frac{350}{\tan 73°33'}$

$= 103.34257...$ ✓

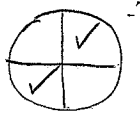
$= 103$ m

∴ height of cliff is 103 m (to nearest m)

(d) $\tan 2\theta = \sqrt{3}$ for $0° \leq \theta \leq 360°$.

let u be 2θ for $0° \leq \theta \leq 720°$.

$\therefore \tan u = \sqrt{3}$



$u = 60°, 240°, 420°, 600°$

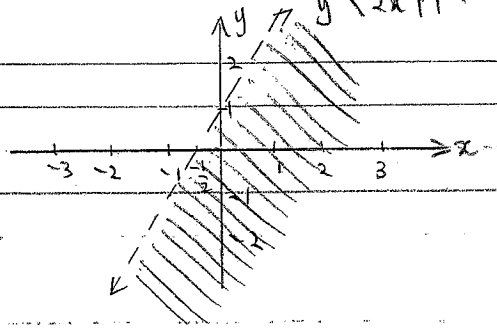
$\therefore 2\theta = 60°, 240°, 420°, 600°$ ✓

$\therefore \theta = 30°, 120°, 210°, 300°$

Question 5.

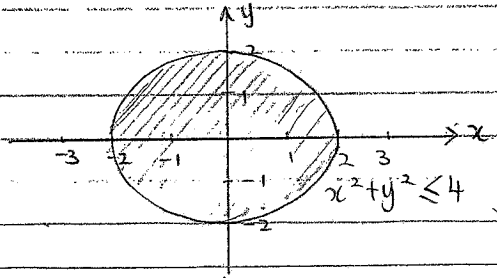
(a)

(i)



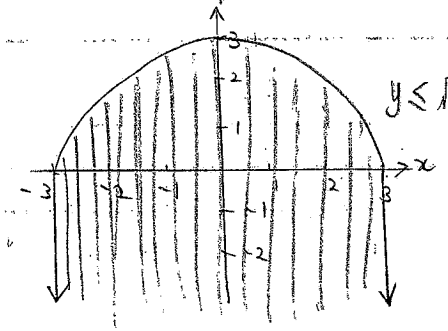
√2

(ii)



√2

(iii)



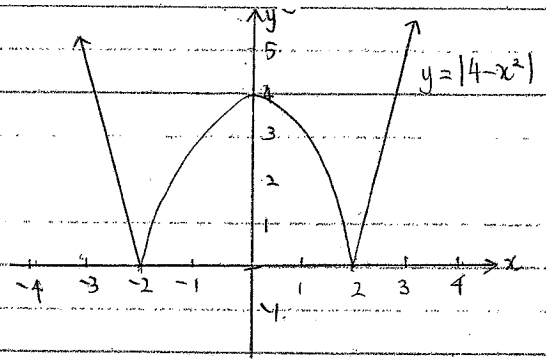
√2

(b) $y \geqslant x^2 - 4$ ∩ and $y < x + 2$. ✓ ✓

2

(c) $y = |4 - x^2|$.

or



√2

10/10

Question 6.

(a) $\cos\theta = \sin 72$.

$\cos 18 = \sin 72$ (Complementary $\angle$s)

$\therefore \theta = 18°$ ✓1

(b) $(3\sin\theta + 1)(\sin\theta - 1) = 0$

$\therefore 3\sin\theta = -1$

$\sin\theta = -\frac{1}{3}$ for $0° \le \theta \le 360°$

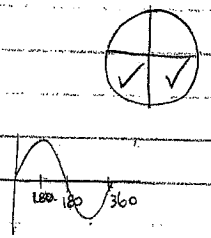
$\theta = 199°28', 340°32'$

or/

$\sin\theta = 1$

$\therefore \theta = 90°$

$\therefore \theta = 90°, 199°28', 340°32'$ ✓3



(c) $\frac{\sin(90° - \theta)}{\cos(90° - \theta)} \equiv \cot\theta$

RHS: $\frac{\cos\theta}{\sin\theta}$

LHS: $\frac{\sin(90° - \theta)}{\cos(90° - \theta)} = \frac{\cos\theta}{\sin\theta}$ (both are complementary $\angle$s of each other) ✓2

$= RHS$

$\therefore LHS = RHS$

$\therefore \frac{\sin(90° - \theta)}{\cos(90° - \theta)} \equiv \cot\theta$.

(i) $\frac{\cos\theta}{1 - \sin\theta} + \frac{\cos\theta}{1 + \sin\theta} \equiv 2\tan\theta$.

LHS: $\frac{\cos\theta(1 + \sin\theta) + \cos\theta(1 - \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)}$

$= \frac{\cos\theta + \cos\theta\sin\theta + \cos\theta - \cos\theta\sin\theta}{1 - \sin^2\theta}$

$= \frac{2\cos\theta}{\cos^2\theta}$ using $\sin^2\theta + \cos^2\theta = 1$.

$= \frac{2\sin\theta}{\cos\theta} = 2\tan\theta$ ✓2

$= RHS$

$\therefore LHS = RHS$

(d) $x = r\cos\theta$ $\therefore \cos\theta = \frac{x}{r}$

$y = r\sin\theta$ $\sin\theta = \frac{y}{r}$

$\therefore \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2 = 1$. since $\sin^2\theta + \cos^2\theta = 1$.

$\frac{y^2}{r^2} + \frac{x^2}{r^2} = 1$. multiply throughout by r^2

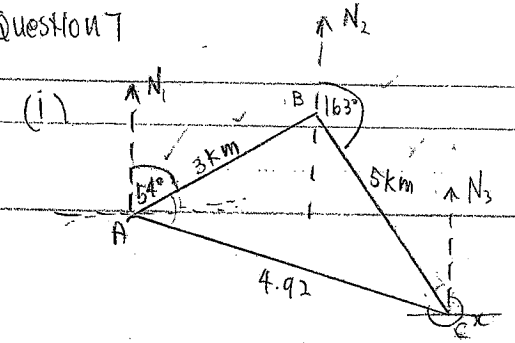
$y^2 + x^2 = r^2$ ✓2

$\therefore x^2 + y^2 = r^2$

10/10

Question 7

(i)



(ii) $\angle N_2BA = 126°$

$\therefore \angle ABC = 360° - 163 - 126$ ($\angle$ about a point = 360°)

$= 71°$

$\therefore CA^2 = 3^2 + 5^2 - 2(5)(3)\cos 71.$

$= 24.232955.\text{ km}$

$\therefore AC = 4.92\text{ km}$ (to 2 dp)

2

(iii) $\cos C = \dfrac{(4.92)^2 + 25 - 9}{2(5)(4.92)}$

$= 0.817742995$ (to 9 dp)

$\therefore C = 35°8'$ (nearest minute).

$\therefore N_3\hat{C}B = 180 - 163$ (co-interior $\angle$ of parallel N's)

$= 17°$

$\therefore 90° - 17° - 35°8' = 37°52°$

$\therefore 270 + 37°52' = 307°52'$

$\therefore$ Bearing of A from C is 308°T (to nearest degree).

3

AT

(b)(i) $\dfrac{\sin PRQ}{6} = \dfrac{\sin 54}{7}$

$\therefore \sin PRQ = \dfrac{6\sin 54}{7}$

$\therefore PRQ = 43°54'$

$\approx 44°$ (to nearest degree)

2

(ii) $\angle QPR = 180 - 54 - 43°54'$

$= 82°6'$ (nearest minute).

$\therefore$ A of $\triangle PQR = \frac{1}{2} \times 6 \times 7 \times \sin 82°6'$

$= 20.8\text{ m}^2$ (to 1 dp)

2

(10)