

2008



Mathematics Extension 1

Total Marks – 84

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Attempt ALL questions.
- Begin each question on a new booklet
- Write your student number on each page
- All necessary working must be shown.
- Diagrams are not to scale.
- Board-approved calculators may be used.
- The mark allocated for each question is listed at the side of the question.

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

TABLE OF STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 - a^2}), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2})$$

Note $\ln x = \log_e x, \quad x > 0$

Question 1 – (12 marks)

Marks

- a) Find $\frac{d}{dx}(e^x \sin^{-1} x)$ 2
- b) Evaluate $\int_0^2 \frac{dx}{(4+x^2)}$ 2
- c) Solve for $x: \frac{x+3}{x-2} \geq 2$ 2
- d) Find the general solution of $2 \sin \theta + 1 = 0$ 2
- e) Use the substitution $u = 1 + x$ to evaluate $\int_0^1 \frac{x}{\sqrt{(1+x)^3}} dx$ 4

Question 2 – (12 marks)

Marks

- a) The curves $y = x^2 - 4x + 2$ and $y = e^x + 1$ intersect at the point $(0, 2)$. Find the acute angle between the two curves at this point. 3
- b) If $\log_a 2 = 0.75$, find the value of $\log_a 3$ correct to two decimal places. 2
- c) Solve for $x: \ln(\ln x) = 0$ 2
- d) (i) Sketch the graph of $y = 3 \cos^{-1}\left(\frac{x}{2}\right)$ clearly indicating the domain, the range and any intercepts. 2
- (ii) The region bounded by this curve, the x-axis and the y-axis, is rotated about the y-axis. Show that the volume of the solid so formed is given by $\pi \int_0^{\frac{3\pi}{2}} 4 \cos^2\left(\frac{y}{3}\right) dy$ 1
- (iii) Hence find the volume of this solid. 2

Question 3 – (12 marks)

Marks

- a) (i) Find the value of b if $x - 2$ is a factor of $P(x) = 2x^3 + x^2 - bx + 6$ 1
- (ii) Hence solve for x : $P(x) = 0$ 2
- b) A particle is moving along the x -axis such that its velocity, v m/s, at displacement x metres, is given by $v = \sqrt{(5x - x^2)}$. Find the acceleration of the particle when $x = 4$ 2
- c) Use mathematical induction to prove that
 $(1 \times 1!) + (2 \times 2!) + (3 \times 3!) + \dots + (n \times n!) = (n+1)! - 1$ for all positive integers n . 3
- d) (i) Write down the co-efficient of x^7 in the expansion of $(5 + 2x)^{12}$ in simplest terms. 1
- (ii) Hence find the greatest co-efficient in the expansion of $(5 + 2x)^{12}$ 3

Question 4 – (12 marks)

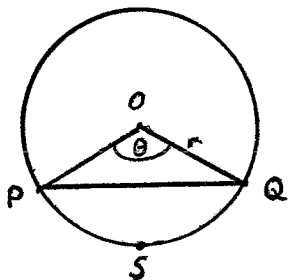
Marks

- a) (i) Find the largest possible domain of positive values for which the function $f(x) = x^2 - 4x + 9$ has an inverse which is a function. 1
- (ii) Find $f^{-1}(x)$ clearly stating the domain and range. 2
- (iii) Find $f^{-1}[f(2 - a^2)]$ 1
- b) Jenny borrowed \$400 000 over 30 years at 6.6%pa reducible monthly. If the outstanding balance at the end of n months is B_n and the monthly repayment is R
- (i) Show that $B_n = 400\,000(1.0055)^n - \frac{R(1.0055^n - 1)}{0.0055}$ 2
- (ii) Find the value of R required to repay the loan and interest over 30 years. 1
- (iii) Before making the first repayment, Jenny decides to increase her monthly repayments to \$2 800. What time period is required to pay out the loan? 3
- c) The temperature of a cooling body, $T^\circ\text{C}$, at time t minutes, is given by $T = 20 + 40 e^{-0.04t}$. At what rate is the temperature changing when the temperature is 30°C ? 2

Question 5 – (12 marks)

Marks

- a) In the diagram below, the points P and Q lie on the circle centre O of radius r . The chord PQ divides the sector $OPSQ$ into two regions of equal area.



- (i) Show that $\theta = 2\sin \theta$ 2
- (ii) The first approximation to the solution of the equation $\theta - 2\sin \theta = 0$ is $\theta = 1.91$ radians. Use one application of Newton's method to find a better approximation correct to 4 decimal places. 2
- b) A ladder, 12 metres long, leans against a vertical wall with its lower end on horizontal ground. The lower end is slipping away from the wall at 3m/s. Find the rate at which the upper end is slipping down the wall when the lower end is 7.2 m from the wall. 5
- c) The velocity, v m/s, of a particle moving in a straight line is given by $v = \frac{e^{-2x}}{2}$. Initially the particle was at the origin. Find its displacement after 2 seconds. 3

Question 6 – (12 marks)

Marks

- a) A particle moves along a straight line such that its displacement, x metres, at time t seconds, is given by $x = 5 \sin 2t + 5 \cos 2t$
- (i) Show that this motion is simple harmonic by showing that $\ddot{x} = -4x$ 2
- (ii) Find the period of the motion. 1
- (iii) Show that the velocity function can be written in the form $x = R \cos(2t + \alpha)$ where $R > 0$ and $0 < \alpha < \pi$ 2
- (iv) Find the first occasion when the velocity is $5\sqrt{2}$ m/s 1
- b) The point $P(2at, at^2)$ is a variable point on the parabola $x^2 = 4ay$. The normal at P cuts the y -axis at Q . PQ is then produced to R such that $PQ = QR$.
- (i) Show that the equation of the normal at P is $x + ty = at^3 + 2at$ 2
- (ii) Find the co-ordinates of Q and R . 2
- (iii) Deduce the equation of the locus of R 2

Marks

Question 7 - (12 marks)

- a) The top of a tower is viewed from two points, A and B . A is due East of the tower and B is due South of A . The angles of elevation of the top from A and B are 40° and 20° respectively. If the distance from A to B is 100m, find the height of the tower.
- b) A ball is kicked with velocity V m/s at an angle of 45° to the ground towards a person who will catch it 2 metres above ground level. At the instant the ball is kicked, the person is 20m from the kicker and is running away at a speed of 2m/s. The person continues to run away at this speed. Using $g = 10 \text{ m/s}^2$
- (i) Derive the six equations of motion for the ball.
- (ii) Find the maximum height reached by the ball in terms of V .
- (iii) Find V correct to one decimal place.

4

2

2

4

Q1 (a) $\frac{d}{dx} (e^x \sin^{-1} x) = e^x \sin^{-1} x + \frac{1}{\sqrt{1-x^2}} \cdot e^x$

$= e^x \left(\sin^{-1} x + \frac{1}{\sqrt{1-x^2}} \right)$

(b) $\int_0^2 \frac{dx}{4+x^2} = \left[\frac{1}{2} \tan^{-1} \left(\frac{x}{2} \right) \right]_0^2$

$= \frac{1}{2} (\tan^{-1} 1 - 0)$

$= \frac{1}{2} \times \frac{\pi}{4}$

$= \frac{\pi}{8}$

(c) $\frac{x+3}{x-2} \geq 2$ or $x+3-2 \geq 0$

$\frac{(x+3)(x-2)}{(x-2)^2} \geq 2$

$\frac{x^2+x-6}{(x-2)^2} \geq \frac{2x^2-8x+8}{(x-2)^2}$

$x^2+x-6 \geq 2x^2-8x+8$

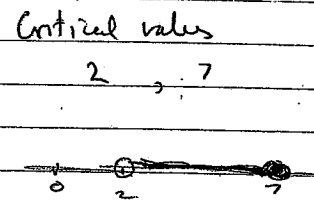
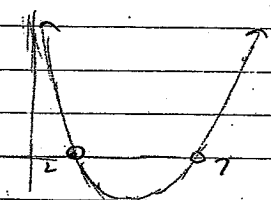
$\therefore x^2-9x+14 \leq 0$

$(x-7)(x-2) \leq 0$

or $x+3-2 \geq 0$

$x+1 \geq 0$

$x \geq -1$



$x \neq 2 \therefore 2 < x \leq 7$

Test $x=3$

$4 \geq 0 \checkmark$

$2 < x \leq 7$

(d) $2\sin\theta + 1 = 0$

$\sin\theta = -\frac{1}{2}$

$\theta = (-1)^n \sin^{-1}\left(-\frac{1}{2}\right) + n\pi$

$\theta = \frac{7\pi}{6} + 2n\pi$ (n an integer) $\theta = (-1)^n \times \left(\frac{4\pi}{6}\right) + n\pi$

or $\theta = \left(-\frac{\pi}{6}\right) + 2n\pi$

(e) let $u = 1+x$

then $x = u-1$

$1 = \frac{du}{dx}$

when $x=0$, $u=1$

$x=1$, $u=2$

$\int_0^1 \frac{x dx}{\sqrt{(1+x)^3}} = \int_1^2 \frac{(u-1) \frac{du}{dx}}{u^{\frac{3}{2}}}$

$= \int_1^2 \frac{(u-1) du}{u^{\frac{3}{2}}}$

$= \int_1^2 \left(u^{-\frac{1}{2}} - u^{\frac{3}{2}} \right) du$

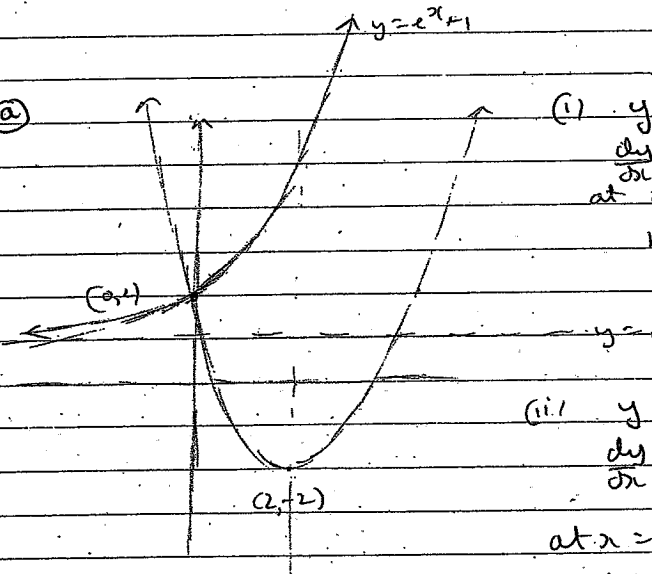
$= \left[2u^{\frac{1}{2}} - \frac{2}{5}u^{\frac{5}{2}} \right]_1^2$

$= \left(2\sqrt{2} + \frac{2}{5}\sqrt{2} \right) - \left(2 + \frac{2}{5} \right)$

$= 3\sqrt{2} - 4$

Q2

(a)



(i) $y = x^2 - 4x + 2$

$\frac{dy}{dx} = 2x - 4$

at $x=0$

$m_1 = -4$

(ii) $y = e^x + 1$

$\frac{dy}{dx} = e^x$

at $x=0$

$m_2 = 1$

then $\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{-4 - 1}{1 + (-4)(1)} = \frac{5}{3}$

acute angle $\theta = 59^\circ 2'$

(b) Given $\log_a 2 = 0.75$

then $\frac{\log_e 2}{\log_e a} = \frac{3}{4} \Rightarrow \log_e a = \frac{4}{3} \log_e 2 \dots (1)$

also $\log_a 3 = \frac{\log_e 3}{\log_e a} \Rightarrow \frac{\log_e 3}{\frac{4}{3} \log_e 2}$ from (1)
 $= 1.19$ (2 dec. places)

(c) $\ln(\ln a) = 0$

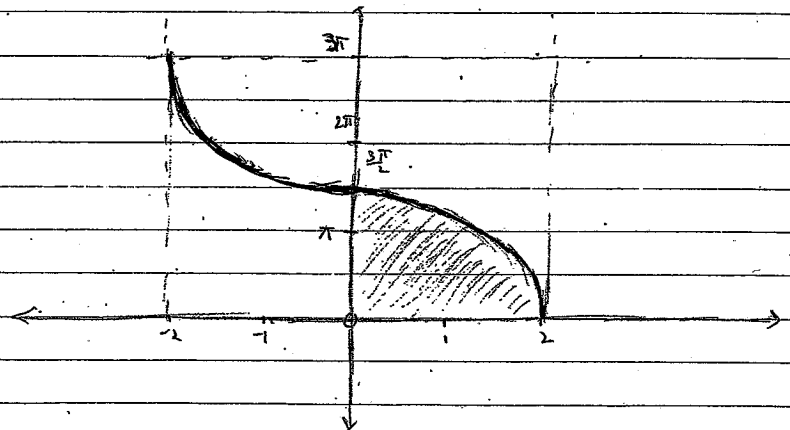
then $\ln a = 1$

$x = e^1$

(d) Domain: $-1 \leq \frac{x}{2} \leq 1$

(i) $-2 \leq x \leq 2$

Range: $0 \leq 3\cos^{-1}\left(\frac{x}{2}\right) \leq 3\pi$



$$\begin{aligned} \text{(ii)} \quad V &= \pi \int_0^{\frac{3\pi}{2}} (f(y)) dy & y &= 3 \cos^{-1}\left(\frac{x}{3}\right) \\ & & \frac{y}{3} &= \cos^{-1}\left(\frac{x}{3}\right) \\ &= \pi \int_0^{\frac{3\pi}{2}} \left[2 \cos\left(\frac{y}{3}\right)\right]^2 dy & \cos\left(\frac{y}{3}\right) &= \frac{x}{3} \\ &= \pi \int_0^{\frac{3\pi}{2}} 4 \cos^2\left(\frac{y}{3}\right) dy & \therefore 2 \cos\left(\frac{y}{3}\right) &= x \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad V &= 2\pi \int_0^{\frac{3\pi}{2}} \left[\cos\left(\frac{2y}{3}\right) + 1\right] dy & 2 \cos^2\left(\frac{y}{3}\right) &= \cos\left(\frac{2y}{3}\right) + 1 \\ &= 2\pi \left[\frac{3}{2} \sin\left(\frac{2y}{3}\right) + y\right]_0^{\frac{3\pi}{2}} \\ &= 2\pi \left[\left(\frac{3}{2} \sin \pi + \frac{3\pi}{2}\right) - 0\right] \\ &= 3\pi^2 \text{ cubic units.} \end{aligned}$$

Q3

$$\begin{aligned} \text{(a) (i)} \quad P(2) &= 0 \\ \therefore 2 \times 2^3 + 2^2 - 2b + 6 &= 0 \\ 26 - 2b &= 0 \\ \therefore b &= 13 \end{aligned}$$

$$\begin{array}{r} \text{(ii)} \quad \begin{array}{r} 2x^2 + 5x - 3 \\ x-2 \overline{) 2x^3 + x^2 - 13x + 6} \\ \underline{2x^3 - 4x^2} \\ 5x^2 - 13x + 6 \\ \underline{5x^2 - 10x} \\ -3x + 6 \\ \underline{-3x + 6} \\ 0 \end{array} \end{array}$$

$$\begin{aligned} \therefore P(x) &= (x-2)(2x^2 + 5x - 3) \\ &= (x-2)(2x-1)(x+3) \end{aligned}$$

let $P(x) = 0$

then $x = 2, \frac{1}{2}$ and -3 .

$$\begin{aligned} \text{(b)} \quad v &= (5x - x^2)^{\frac{1}{2}} & a &= \frac{d}{dx} \left[\frac{1}{2} v^2\right] \\ & & a &= \frac{d}{dx} \left[\frac{1}{2} (5x - x^2)\right] \end{aligned}$$

$$a = \frac{1}{2} (5 - 2x)$$

When $x = 4$

$$\begin{aligned} a &= \frac{1}{2} (5 - 8) \\ &= -\frac{3}{2} \text{ m/sec}^2 \end{aligned}$$

(c) (i) let $n = 1$

then L.H.S.

and R.H.S.

$$(1 \times 1)! = 1$$

$$(1+1)! - 1 = 2! - 1$$

$$= 2 - 1$$

$$= 1$$

True for $n = 1$

(ii) Assume result is true for $n = k$, k a positive integer.

$$\text{Then } (1 \times 1!) + (2 \times 2!) + (3 \times 3!) + \dots + (k \times k!) = (k+1)!$$

(iii) Next value of n , $n = k+1$

has L.H.S.

$$(1 \times 1!) + (2 \times 2!) + (3 \times 3!) + \dots + (k \times k!) + [(k+1) \times (k+1)!]$$

$$= (k+1)! - 1 + [(k+1) \times (k+1)!]$$

[From (ii)]

$$= (k+1)! [(k+1) + 1] - 1$$

$$= (k+1)! \times (k+2) - 1$$

$$= (k+2)! - 1$$

$$= [(k+1) + 1]! - 1$$

as required for R.H.S.

(iv) from above, if result is true for $n=k$ then it is true for next n , $n=k+1$.
 Since true for $n=1$ then it is true for $n=2$ and by induction true for all n .

(2) (i) General term $\binom{12}{r} 5^{12-r} (2x)^r$
 \therefore coefficient on x^r is $\binom{12}{r} 2^r \cdot 5^{12-r}$

(ii) require $\frac{t_{k+1}}{t_k} > 1$

then $\frac{\binom{12}{k+1} 2^{k+1} \cdot 5^{12-(k+1)}}{\binom{12}{k} 2^k \cdot 5^{12-k}} > 1$

$\frac{12!}{(k+1)! [12-(k+1)]!} \times \frac{2}{5} > 1$

$\frac{k! (12-k)!}{(k+1)! [11-(k+1)]!} > \frac{5}{2}$

$\frac{12-k}{k+1} > \frac{5}{2}$

$24-2k > 5k+5$

$19 > 7k$

gives $k < 2\frac{5}{7}$

so $t_{k+1} > t_k$ for $k=0, 1$ and 2 } $t_0 < t_1 < t_2$

and $t_{k+1} < t_k$ for $k=3, 4$ } $t_3 > t_4 > \dots$

$t_0 < t_1 < t_2 < t_3 > t_4 > \dots$

Greatest coeff: $t_3 = \binom{12}{3} 5^9 2^3$

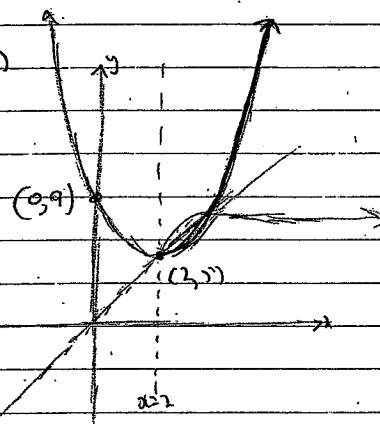
$= \frac{12!}{3! 9!} \times 5^9 \times 2^3$

$= \frac{12 \times 11 \times 10}{3 \times 2 \times 1} \times 5^9 \times 2^3$

$= 11 \cdot 5^{10} \cdot 2^5$

Q4

(i)



axis $x = -\frac{-4}{2 \times 1}$

$x = 2$

largest possible domain $x \geq 2$

(ii)

$y = x^2 - 4x + 4 + 5$
 $y = (x-2)^2 + 5$

$\begin{cases} x \geq 2 \\ y \geq 5 \end{cases}$

so $y-5 = (x-2)^2$

$\sqrt{y-5} = x-2$

$x = 2 + \sqrt{y-5}$

Domain

$f^{-1}(x) = 2 + \sqrt{x-5}$

$\begin{cases} x \geq 5 & \text{Domain} \\ y \geq 2 & \text{Range} \end{cases}$

(iii)

$f^{-1}[f(2+a^2)] = 2+a^2$

Since $(2+a^2) \geq 2$ for all a .

$$(b) (i) B_1 = 400000 + \frac{400000 \times .066}{12} - R$$

$$= 400000 (1 + .0055) - R$$

$$B_1 = 400000 (1.0055) - R$$

$$\text{Then } B_2 = B_1 + B_1 \times .066 - R$$

$$= B_1 [1.0055] - R$$

$$B_2 = 400000 (1.0055)^2 - R \times 1.0055 - R$$

$$\text{and } B_3 = B_2 + B_2 \times .066 - R$$

$$= B_2 (1.0055) - R$$

$$B_3 = 400000 (1.0055)^3 - R \times 1.0055^2 - R \times 1.0055 - R$$

$$= 400000 (1.0055)^3 - R \left[\frac{1(1.0055^3 - 1)}{1.0055 - 1} \right]$$

$$B_3 = 400000 (1.0055)^3 - R \left[\frac{1.0055^3 - 1}{.0055} \right]$$

Sequence gives.

$$B_n = 400000 (1.0055)^n - R \left[\frac{1.0055^n - 1}{.0055} \right]$$

$$(ii) \text{ Require } B_{360} = 0$$

$$\Rightarrow 400000 (1.0055)^{360} - R \left[\frac{1.0055^{360} - 1}{.0055} \right]$$

$$R = \frac{400000 (1.0055)^{360} \times .0055}{1.0055^{360} - 1}$$

$$= \$2554.64$$

Repay \$2554.64 per month

$$(iii) \text{ let } R = 2800 \Rightarrow 400000 (1.0055)^n - \frac{2800 (1.0055^n - 1)}{.0055} = 0$$

$$400000 (1.0055)^n - \frac{2800 (1.0055)^n}{.0055} + \frac{2800}{.0055} = 0$$

$$(1.0055)^n \left[\frac{400000 - 2800}{.0055} \right] = \frac{-2800}{.0055}$$

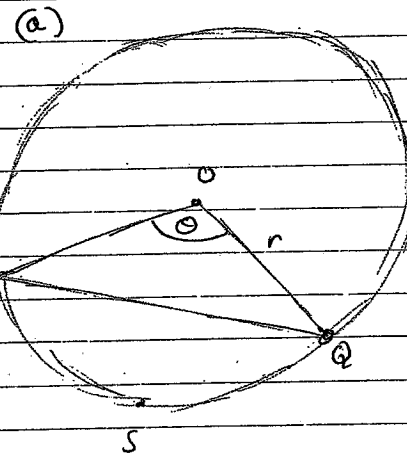
$$(1.0055)^n = 4.66 \dots$$

$$n = \frac{\log_e (4.66 \dots)}{\log_e (1.0055)}$$

$$= 280.85 \dots$$

Repaid in 281 months [just over 23 years]

Q5



(i) Area triangle POQ

$$= \frac{1}{2} r^2 \sin \theta$$

and area of segment PQS

$$= \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin \theta$$

Since areas are equal

$$\frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin \theta = \frac{1}{2} r^2 \sin \theta$$

$$\frac{1}{2} r^2 \theta = r^2 \sin \theta$$

$$\theta = 2 \sin \theta$$

$$(ii) f(\theta) = \theta - 2 \sin \theta$$

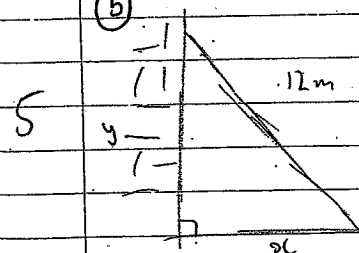
$$f'(\theta) = 1 - 2 \cos \theta$$

$$\text{Given } x_1 = 1.91 \text{ then } x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\Rightarrow x_2 = 1.91 - \frac{f(1.91)}{f'(1.91)}$$

$$x_2 = 1.8956 \text{ [4 dec. pt]}$$

(b)



$$\text{Given } \frac{dx}{dt} = 3 \text{ m/s}$$

$$\text{and } x^2 + y^2 = 12^2$$

$$y = \sqrt{12^2 - x^2}$$

$$\frac{dy}{dx} = \frac{1}{2} (12^2 - x^2)^{-\frac{1}{2}} \times -2x$$

$$\frac{dy}{dx} = \frac{-x}{\sqrt{12-x^2}}$$

Reqn. $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$$= \frac{-x}{\sqrt{12-x^2}} \times 3$$

at $x = 7.2$ then $\frac{dy}{dt} = \frac{-7.2 \times 3}{\sqrt{12-7.2^2}}$
 $= -2.25 \text{ m/s}$

Slipping down at rate
at 2.25 m/s .

(c) $v = \frac{1}{2} e^{-2x}$ at $t=0$, $x=0$.

$$\frac{dx}{dt} = \frac{1}{2} e^{-2x} \Rightarrow dt = 2e^{2x}$$

$$t = e^{2x} + C$$

at $t=0$, $x=0$

$$0 = 1 + C$$

$$C = -1$$

$$\therefore t = e^{2x} - 1$$

$$t+1 = e^{2x}$$

$$\log_e(t+1) = 2x \Rightarrow x = \frac{1}{2} \log_e(t+1)$$

at $t=2$; $x = \frac{1}{2} \log_e 3$

Q6

(a) (i) $x = 5 \sin 2t + 5 \cos 2t$

$$v = \dot{x} = 10 \cos 2t - 10 \sin 2t$$

$$a = \ddot{x} = -20 \sin 2t - 20 \cos 2t$$

$$= -4 [5 \sin 2t + 5 \cos 2t]$$

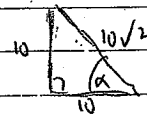
$$\ddot{x} = -4x$$

satisfies acceleration proportional to displacement, S.H.M.
 $\ddot{x} = -n^2 x$

(ii) $T = \frac{2\pi}{n} \therefore T = \pi$ period.

(iii) $\dot{x} = 10 \cos 2t - 10 \sin 2t$

$$\dot{x} = 10\sqrt{2} \left[\frac{1}{\sqrt{2}} \cos 2t - \frac{1}{\sqrt{2}} \sin 2t \right]$$



$$= 10\sqrt{2} [\cos 2t \cos \frac{\pi}{4} - \sin 2t \sin \frac{\pi}{4}]$$

$$\dot{x} = 10\sqrt{2} \cos(2t + \frac{\pi}{4})$$

$$\tan \alpha = 1$$

$$\alpha = \frac{\pi}{4}$$

$$0 < \alpha < \pi$$

(iv) let $\dot{x} = 5\sqrt{2} = 10\sqrt{2} \cos(2t + \frac{\pi}{4})$

$$\cos(2t + \frac{\pi}{4}) = \frac{1}{2}$$

$$t > 0$$

Then $2t + \frac{\pi}{4} = \frac{\pi}{3}, \frac{5\pi}{3}$

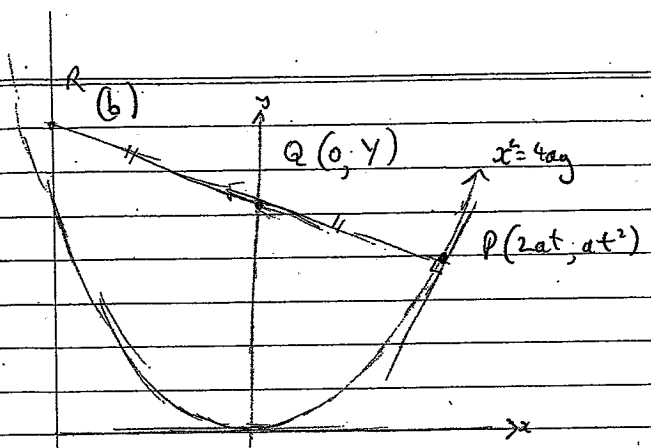
$$2t = \frac{\pi}{12}$$

$$t = \frac{\pi}{24}$$

$$\dot{x} = 5\sqrt{2} \text{ m/s}$$

first when $t = \frac{\pi}{24}$ s.

Q7 (a)



(i) $y = \frac{x^2}{4a}$ } Equation of normal with $m = -\frac{1}{t}$

$\frac{dy}{dx} = \frac{x}{2a}$ }
 at $x = 2at$
 $m = t$

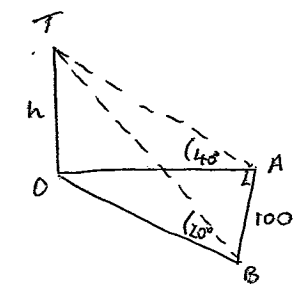
$y - at^2 = -\frac{1}{t}(x - 2at)$
 $ty - at^3 = -x + 2at$
 $x + ty = at^3 + 2at$

(ii) let $x=0$
 then $0 + ty = at^3 + 2at \Rightarrow Q(0, a(t^2 + 2))$
 $y = at^2 + 2a$

A(x, y) then $0 = \frac{x + 2at}{2}, at^2 + 2a = \frac{y + a}{2}$
 [midpoint of PR] $x = -2at, 2at^2 + 2a = \frac{y + a}{2}$
 $y = 4a + at$
 $\therefore R(-2at, 4a + at^2)$

(iii) $x = -2at \rightarrow t = -\frac{x}{2a}$
 $y = a[4 + t^2]$

Sb $y = a[4 + \frac{x^2}{4a^2}]$ Equation
 $y = 4a + \frac{x^2}{4a} \Rightarrow x^2 = 4a(y - 4a)$



$\tan 40^\circ = \frac{h}{OA}$
 $OA = \frac{h}{\tan 40^\circ}$
 $\tan 20^\circ = \frac{h}{OB}$
 $OB = \frac{h}{\tan 20^\circ}$

$OB^2 = OA^2 + AB^2$
 $AB^2 = OB^2 - OA^2$
 $100^2 = \frac{h^2}{\tan^2 20^\circ} - \frac{h^2}{\tan^2 40^\circ}$
 $= h^2 \left(\frac{1}{\tan^2 20^\circ} - \frac{1}{\tan^2 40^\circ} \right)$
 $= h^2 \left(\frac{\tan^2 40^\circ - \tan^2 20^\circ}{\tan^2 20^\circ \tan^2 40^\circ} \right)$

$h^2 = \frac{100^2 \tan^2 20^\circ \tan^2 40^\circ}{\tan^2 40^\circ - \tan^2 20^\circ}$
 $h = 40.395 \text{ m correct to 3 dec. pl.}$

(b) (i) Vertical	Horizontal
$\ddot{y} = -10$	$\ddot{x} = 0$
$y = -10t + C_1$	$\dot{x} = C_3$
when $t=0, y = \frac{V}{\sqrt{2}}$	when $t=0, \dot{x} = \frac{V}{\sqrt{2}}$
$y = -10t + \frac{V}{\sqrt{2}}$	$\dot{x} = \frac{V}{\sqrt{2}}$
$y = \frac{-10t^2}{2} + \frac{Vt}{\sqrt{2}} + C_2$	$x = \frac{Vt}{\sqrt{2}} + C_4$
when $t=0, y=0$	when $t=0, x=0$
$y = \frac{-10t^2}{2} + \frac{Vt}{\sqrt{2}}$	$x = \frac{Vt}{\sqrt{2}}$
$= -5t^2 + \frac{Vt}{\sqrt{2}}$	

(ii) Maximum height when $y=0$

$$-10t + \frac{V}{\sqrt{2}} = 0$$

$$t = \frac{V}{10\sqrt{2}}$$

Find y when $t = \frac{V}{10\sqrt{2}}$ for maximum height

$$y = \frac{V}{\sqrt{2}} \cdot \frac{V}{10\sqrt{2}} - 5 \left(\frac{V}{10\sqrt{2}} \right)^2$$

$$= \frac{V^2}{20} - \frac{5V^2}{200}$$

$$= \frac{V^2}{40}$$

(iii) Require ball and catcher at the same x value when $y=2$

Let time T elapse for catch to be taken

horizontal distance for ball is $x = \frac{VT}{\sqrt{2}}$

distance from kicker for catcher is $x = 20 + 2T$

$$\therefore \frac{VT}{\sqrt{2}} = 20 + 2T$$

$$\frac{VT}{\sqrt{2}} - 2T = 20$$

$$T \left(\frac{V}{\sqrt{2}} - 2 \right) = 20$$

$$T = \frac{20\sqrt{2}}{V - 2\sqrt{2}}$$

At this time $y=2$

$$y = \frac{V}{\sqrt{2}} t - 5t^2$$

$$2 = \frac{V}{\sqrt{2}} T - 5T^2$$

$$2 = \frac{V}{\sqrt{2}} \left(\frac{20\sqrt{2}}{V - 2\sqrt{2}} \right) - 5 \left(\frac{20\sqrt{2}}{V - 2\sqrt{2}} \right)^2$$

$$2(V - 2\sqrt{2})^2 = 20V(V - 2\sqrt{2}) - 5(20\sqrt{2})^2$$

$$2(V^2 - 4\sqrt{2}V + 8) = 20V^2 - 40\sqrt{2}V - 4000$$

$$V^2 - 4\sqrt{2}V + 8 = 10V^2 - 20\sqrt{2}V - 2000$$

$$0 = 9V^2 - 16\sqrt{2}V - 2008$$

$$V = \frac{16\sqrt{2} \pm \sqrt{(16\sqrt{2})^2 + 4 \times 9 \times 2008}}{18}$$

$$V > 0$$

$$V = 16.2 \text{ m/s} \quad \text{correct to 1 dec. pl.}$$