Name: .... Maths Class: ....

SYDNEY TECHNICAL HIGH SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE
2006

Mathematics

TIME ALLOWED: 3 hours plus 5 mins reading time.

Instructions:

- Write your name and class at the top of this page, and at the top of each answer sheet.
- At the end of the examination this examination paper must be attached to the front of your answers.
- All questions are of equal value and may be attempted.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.

(FOR MARKERS USE ONLY)

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	TOTAL

QUESTION 1:

(a) Completely factorise $2-8x^2$ 2

(b) Solve for x: $1-2(3x-5)=4$ 2

(c) Solve the following inequality and plot the solution on a number line. 3

$$|1-3x|>4$$

(d) Find the value of $e^{\frac{3\pi}{4}}$ correct to 4 decimal places 1

(e) Find the value of a if $\sqrt{75}+\sqrt{a} = 8\sqrt{3}$ 2

(f) Simplify $e^{4\ln x}$ 1

(g) Simplify $\dfrac{\frac{1}{m}+\frac{1}{n}}{m+n}$ 1

QUESTION 2: (Begin on a new page)

(a) Differentiate with respect to x: (i) $\dfrac{1}{\sqrt{x}}$ 1

(ii) $2\cos^3 x$ 1

(iii) $\log\left(\dfrac{x}{x-1}\right)$ 2

(b) Find indefinite integrals of: (i) $\dfrac{x^2+2}{x}$ 1

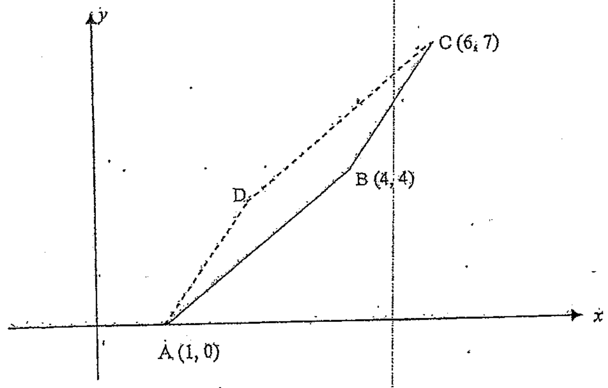
(ii) $\sin\dfrac{2x}{3}$ 2

(iii) e^{1-4x} 2

(c) Find the exact value of $\displaystyle\int_0^{\frac{\pi}{2}} 3\cos\frac{3x}{2}\,dx$ 3

QUESTION 3: (Begin on a new page)

(a) The points $A(1, 0)$, $B(4, 4)$ and $C(6, 7)$ are shown below:



(i) Find the co-ordinates of the point D if ABCD is a parallelogram 1

(ii) Find the equation of the line AB 2

(iii) Find the length of the perpendicular from D to AB 2

(iv) Find the length of AB 1

(v) Find the area of the quadrilateral ABCD 1

(b) If $y = x\ln x - x$, (i) find $\frac{dy}{dx}$ 1

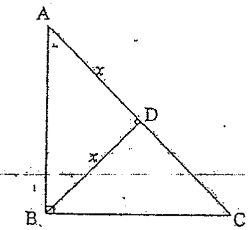
(ii) using part (i) find $\int_1^2 \ln x\,dx$ 2

(c) For a certain base, a, $\log_a 3 = 0.683$ and $\log_a 2 = 0.431$ 2

Use these values to find $\log_a 1.5$.

QUESTION 4: (Begin on a new page)

(a) Copy the following diagram neatly onto your answer sheet:



ΔABC is right-angled at B.

$AD = BD = x$ units

$BD \perp AC$

(i) Find the size of $\angle BAD$ and $\angle DCB$ 1

(ii) Prove that ΔABD is congruent to ΔCBD 2

(iii) Hence prove that D is the midpoint of AC. 1

(iv) Show that ΔABD is similar to ΔACB. 2

(v) Show that $AB^2 = AD.AC$ 1

(vi) Hence, or otherwise, find the length of BC 1

(b) The function $y = f(x)$ passes through the point (1, 1) and is defined by

$$f'(x) = \frac{1}{2x-1}$$

(i) Show that $y = \frac{1}{2}\ln(2x-1)+1$ 2

(ii) Hence show that $x = \frac{1}{2}(e^{2y-2}+1)$ 2

QUESTION 5 : (Begin on a new page)

(a) A function is defined by the following conditions:

$f(0) = f(2) = 2$

$f'(1) = f'(2) = 0$

$f'(x) > 0$ for $0 \le x < 1$

$f'(x) > 0$ for $2 < x \le 4$

$f''(2) > 0$

4

On your answer page, draw a possible graph of $y = f(x)$ for $0 \le x \le 4$ which incorporates all you may deduce about the function $f(x)$.

(b) Given that $y = 2x^3 - 21x^2 + 72x$,

(i) find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ 1

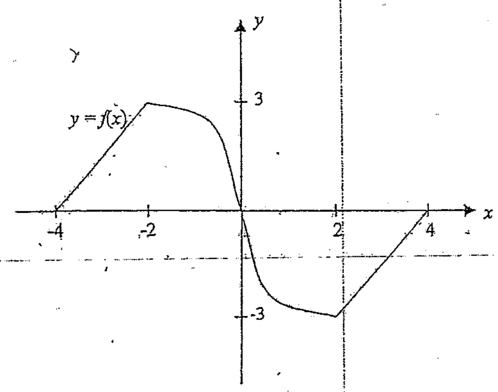
(ii) find all stationary points and determine their nature 4

(iii) find the x-value of the point of inflexion 1

(iv) sketch the curve, showing all important features (DO NOT DRAW TO SCALE) 2

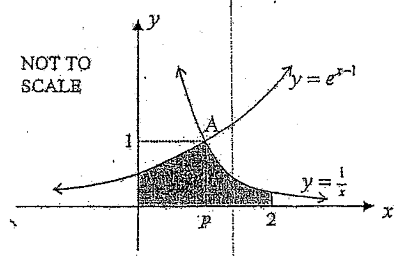
QUESTION 6: (Begin on a new page)

(a) The function drawn below is an odd function.



Find $\int_{-4}^{2} f(x)\,dx$ 2

(b) The curves $y = \frac{1}{x}$ and $y = e^{x-1}$ are shown below and intersect at the point A $(p, 1)$



(i) Find p, the x-coordinate of the point A. 1

(ii) Find the shaded area, correct to 2 decimal places. 4

(c) Find, by calculus, the volume of the solid formed when the curve $y = \sqrt{r^2 - x^2}$ is revolved about the x-axis. Leave your answer in exact form. 5

QUESTION 7: (Begin on a new page)

(a) Find the value(s) of k for which the equation **2**

$$x^2-(k+2)x+\frac{k}{2}+7=0 \quad \text{has real roots.}$$

(b) (i) You are given that the roots of $2x^2-4x+5=0$ are α and β.

Find the value of (I) $\alpha+\beta$ **1**

(II) $\alpha\beta$ **1**

(III) $\alpha^2+\beta^2$ **1**

(IV) $\frac{1}{\alpha}+\frac{1}{\beta}$ **1**

(ii) Form another quadratic equation with roots α^2 and β^2 where α and β are the roots referred to in part (i) above. **2**

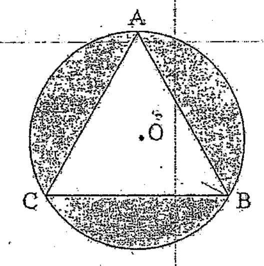
(c) (i) If A is the point (2, 1) and B is the point (-2, 1), find the algebraic equation giving the locus of a point $P(x, y)$ which moves so that $\angle APB = 90^\circ$. **3**

(ii) What geometric shape is this locus? **1**

QUESTION 8: (Begin on a new page)

(a) Find the value of $\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-$.............., **1**

(b) An equilateral triangle ABC is inscribed in a circle of radius 2 cm. Find the exact value of the area shaded. **4**



(c) A tower is formed by laying 1 metre cubes. The layers are in the form of a square and the first layer has 99 cubes in each side. The second layer has 97 cubes in each side while the third layer has 95 cubes, and so on. The final layer has only 1 cube.

(i) How many layers are there? **2**

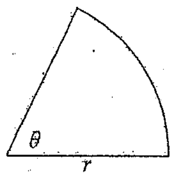
(ii) How many cubes make up the 18th layer? **1**

(d) Solve the equation $2\cos^2\theta-5\sin\theta+1=0$ for $0\le\theta\le 2\pi$. **4**

QUESTION 9: (Begin on a new page)

(a) Prove that $(1+\tan^2 A)\cos A = \sec A$. 2

(b) A landscape gardener wants to build a garden bed in the form of a sector as follows:



He wants the area of the garden to be exactly $16\,m^2$, but wants to use as little concrete as possible to build the edge around the garden.

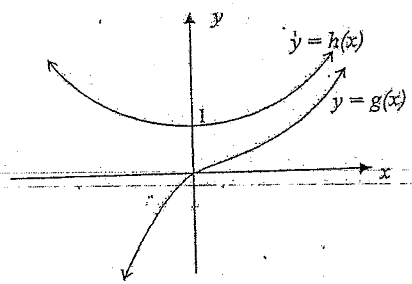
(i) Show that the perimeter, P, of the garden is given by 3

$$P = 2r + \frac{32}{r}$$

(ii) Find the value of r which will give the minimum perimeter and the angle, θ, needed to give this minimum. 3

(iii) What is this minimum perimeter? 1

(c) The graphs of $h(x) = \dfrac{e^x + e^{-x}}{2}$ and $g(x) = \dfrac{e^x - e^{-x}}{2}$ are shown below:

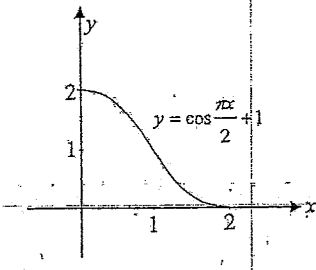


(i) Prove algebraically that the two curves do not intersect. 1

(ii) Hence, or otherwise, find the value of $\lim\limits_{x\to 0}[h(x) - g(x)]$. 2

QUESTION 10: (Begin on a new page)

(a) The curve $y = \cos\dfrac{\pi x}{2} + 1$ is shown below for $0 \le x \le 2$.



(i) By using two applications of the Trapezoidal Rule (ie. 3 function values) find an approximation for the area between the curve $y = \cos\dfrac{\pi x}{2} + 1$, the x-axis, and the lines $x = 0$ and $x = 1$. (LEAVE YOUR ANSWER AS A SURD) 3

(ii) By using integration, find an expression for the exact area in part (i) above. 2

(iii) Using your answers to parts (i) and (ii) above, find an approximation for π correct to 3 decimal places. 2

(b) A function $f(x)$ is defined in such a way that $\sin f(x) = x$, for $0 \le f(x) \le \dfrac{\pi}{2}$

(i) By differentiating both sides of $\sin f(x) = x$ with respect to x, show that 2

$$f'(x) = \frac{1}{\cos f(x)} = \frac{1}{\sqrt{1-x^2}}$$

(ii) A second function $g(x)$, is defined similarly, so that $\cos g(x) = x$, for $0 \le g(x) \le \dfrac{\pi}{2}$

leading to the fact that $g'(x) = \dfrac{-1}{\sqrt{1-x^2}}$. 2

Show by integration, or otherwise, that $f(x) + g(x)$ is a constant.

(iii) Hence find the value of $f(x) + g(x)$. 1

End of paper.

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SOLUTIONS

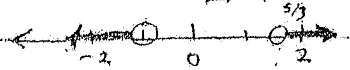
① (a) $2(1-4x^2)$ ① MARK

$= 2(1-2x)(1+2x)$ ① MARK

(b) $1-6x+10 = 4$; $6x = 7$; $x = 7/6$ ② 1 OFF EACH ERROR

(c) $1-3x > 4$ OR $1-3x < -4$

$x < -1$ OR $x > 5/3$ ②



①

(d) ① MARK

(e) $5\sqrt{3} + \sqrt{a} = 8\sqrt{3}$ 1 for this line

$\sqrt{a} = 3\sqrt{3}$

$\sqrt{a} = \sqrt{27}$

$a = 27$ ②

(f) $e^{\ln x^4} = x^4$ ①

(g) $\dfrac{\frac{n+m}{mn}}{m+n} = \dfrac{m+n}{mn(m+n)}$

$= \frac{1}{mn}$ ①

② (a)(i) $-\frac{1}{2}x^{-1/2}$ = ① MARK

$= -\frac{1}{2\sqrt{x}}$ (not necessary)

(ii) $6\cos^2 x(-\sin x)$

$= -6\sin x\cos^2 x$ } ① MARK for either

(iii) $\frac{d}{dx}[\log x - \log(x-1)]$ ①

$= \frac{1}{x} - \frac{1}{x-1}$ ①

(b) (i) $\int x + \frac{2}{x}\,dx = \frac{1}{2}x^2 + 2\ln x$ + ①

(ii) $-\frac{3}{2}\cos\frac{2x}{3} + k$ ① ①

(iii) $-\frac{1}{4}e^{1-4x} + k$ ②

Subtract ① if k omitted in any part.

(c) $\left[2\sin\frac{3x}{2}\right]_0^{\pi/2}$ ①

$= 2\sin\frac{3\pi}{4} - 2\sin 0$

$= \frac{2}{\sqrt{2}}$ ②

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③ (a) D (3, 3) ①

(ii) $m_{AB} = 4/3$ ①

$y - 0 = 4/3(x-1)$

$3y = 4x - 4$ } ① for either

$4x - 3y - 4 = 0$

(iii) $p = \left|\dfrac{4.3 - 3.3 - 4}{\sqrt{4^2+3^2}}\right|$

$= |-1/5|$

$= 1/5$ ② { Part marks for first line

(iv) $d_{AC} = \sqrt{(4-1)^2 + (4-0)^2}$

$= \sqrt{9+16}$

$= 5$ ①

(v) $A = \frac{1}{2} \times 5 \times \frac{1}{5}$

$= \frac{1}{2}$ units ①

(b) (i) $y = x\ln x - x$

$\frac{dy}{dx} = \ln x \cdot 1 + x\cdot\frac{1}{x} - 1$

$= \ln x$ ①

(ii) $\int_1^2 \ln x\,dx = [x\ln x - x]_1^2$ ①

$= (2\ln 2 - 2) + 1$

$= 2\ln 2 - 1$ ①

(c) $\log_a 1.5 = \log_a 3 - \log_a 2$

$= 0.683 - 0.431$

$= 0.252$ ①

④ (a) (i) $\angle BAD = 45°$, $\angle DCB = 45°$ ①

(ii) In $\triangle ABD$ and $\triangle CBD$,

$\angle ADB = \angle BDC = 90°$ (given)

$\angle BAD = \angle DCB = 45°$ (proven above) ①

BD is common ①

$\therefore \triangle ABD \equiv \triangle CBD$ (AAS)

(iii) $AD = DC$ (corresponding sides in congruent triangles)

D is midpoint ①

(iv) In $\triangle ABD$ and $\triangle ACB$,

$\angle ADB = \angle ABC = 90°$ (given)

$\angle BAD$ is common (= 45°) (above) ②

$\therefore \triangle ABD ||| \triangle ACB$ (equiangular)

(v) Because of similarity, sides are in the same ratio,

$\frac{AD}{AB} = \frac{AB}{AC}$

(b) $f'(x) = \frac{1}{2x-1}$ ①

(i) $f(x) = \frac{1}{2}\ln(2x-1)$

Passes through (1, 1)

$\therefore 1 = \frac{1}{2}\ln(1) + k$

$\therefore k = 1$ ①

$\therefore y = \frac{1}{2}\ln(2x-1) + 1$

(ii) $2y - 2 = \ln(2x-1)$

$\therefore 2x - 1 = e^{2y-2}$

$\therefore 2x = e^{2y-2} + 1$

$\therefore x = \frac{1}{2}(e^{2y-2} + 1)$

② MARKS 1 for each error

(MUST SHOW ST

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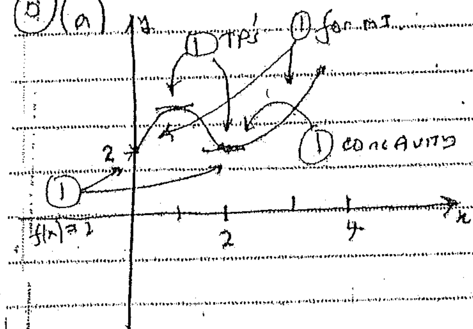
5 (a)



(b) $y = 2x^3 - 21x^2 + 72x$

(i) $\frac{dy}{dx} = 6x^2 - 42x + 72$

$\frac{d^2y}{dx^2} = 12x - 42$ } (1) for both

(ii) At S.P.'s $\frac{dy}{dx} = 0$

$\therefore x^2 - 7x + 12 = 0$ } (1)

$(x-4)(x-3) = 0$

$\therefore x = 4$ or $x = 3$ } (1) don't worry too much about y-values

$y = 80$ $y = 81$

$y'' = 6 > 0$ $y'' = -6 < 0$

$\therefore$ MIN at $(4, 80)$ (1) MAX at $(3, 81)$ (1)

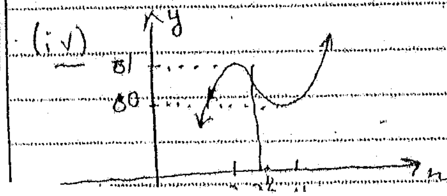
(iii) At I.P.'s $y'' = 0$

$\therefore x = \frac{42}{12}$ (1)

$(7/2)$

y'': + 0 − at 3, $\frac{42}{12}$, 4: y'' changes sign

(iv)



6 (a) The areas in the curved section cancel each other (because of odd function)

THEY MUST MENTION SOMETHING LIKE THIS FOR (1) MARK

$\therefore$ Area under triangle

$= \frac{1}{2} \times 2 \times 3$

$= 3\ u^2$

$\therefore \int_{-4}^{2} f(x) = 3$ (1)

(b) (i) P is $(1, 1)$

$\therefore p = 1$ (1)

(ii) $A_1 = \int_0^1 e^{x-1}\,dx$

$= \left[e^{x-1}\right]_0^1$

$= e^0 - e^{-1} = 1 - \frac{1}{e}$

$A_2 = \int_1^2 \frac{1}{x}\,dx$

$= \left[\ln x\right]_1^2$

$= \ln 2$

$\therefore A = 1 - \frac{1}{e} + \ln 2$

(c) $V = \pi \int_{-r}^{r} y^2\,dx$ (1) for limits

$= \pi \int_{-r}^{r} r^2 - x^2\,dx$ (1) for

$= \pi\left[r^2 x - \frac{1}{3}x^3\right]_{-r}^{r}$ (1)

$= \pi\left[\left(r^3 - \frac{1}{3}r^3\right) - \left(-r^3 + \frac{1}{3}\ldots\right)\right]$ (1

$\frac{4}{3}\pi r^3$ (1)

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7 (a) Real roots means $\Delta \geq 0$

$\therefore (k+2)^2 - 4\left(\frac{k}{2} + 7\right) \geq 0$ (1)

$k^2 + 4k + 4 - 2k - 28 \geq 0$

$\therefore k^2 + 2k - 24 \geq 0$

$(k+6)(k-4) \geq 0$

$\therefore k \leq -6$ or $k \geq 4$ (1)

(subtract 1 if no end sign)

(b) (i) (I) $\alpha + \beta = \frac{4}{2} = 2$ (1)

(II) $\alpha\beta = \frac{5}{2}$ (1)

(III) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 4 - 5$

$= -1$ (1)

(IV) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$

$= 4/5$ (1)

(ii) $(x - \alpha^2)(x - \beta^2) = 0$

$\therefore x^2 - (\alpha^2 + \beta^2)x + \alpha^2\beta^2 = 0$ (1)

$\therefore x^2 + x + \frac{25}{4} = 0$ } (1)

$4x^2 + 4x + 25 = 0$ } either

REMEMBER SUBSEQUENT ERRORS

(c) (i) $m_{PA} = \frac{y-1}{x-2}$ } (1)

$m_{PB} = \frac{y-1}{x+2}$

$m_{PA} \cdot m_{PB} = -1$

$\therefore \frac{y-1}{x-2} \cdot \frac{y-1}{x+2} = -1$ } (1)

$\therefore -1(x^2 - 4) = (y-1)^2$

$\therefore x^2 + (y-1)^2 = 4$ } EITHER

$x^2 + y^2 - 2y - 3 = 0$ } (1)

(ii) CIRCLE (1)

(nothing more required)

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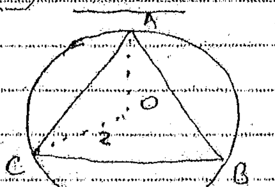
(8) (a) G.P. $a = \frac{1}{2}$

$r = -\frac{1}{2}$

$S_\infty = \frac{1/2}{1 + 1/2}$

$= \frac{1}{3}$ (1) RIGHT or WRONG ONLY

(b) METHOD 1:



$\angle COA = 120°$ $\left(\frac{2\pi}{3}\right)$ (1)

Area sector $AOB = \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(4)\left(\frac{2\pi}{3}\right)$

$= \frac{4\pi}{3}$ (1)

Area $\triangle AOC = \frac{1}{2}(r)(r)\sin\theta$

$= 2 \sin 120°$

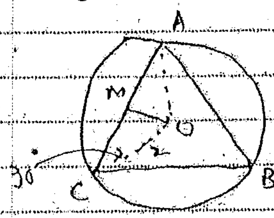
$= \sqrt{3}$ (1)

Shaded area $= 3\left(\frac{4\pi}{3} - \sqrt{3}\right)$

$= 4\pi - 3\sqrt{3}$ (1)

OR

METHOD 2:



$\angle ACO = 30°$ (angle bisector of $\angle ACB$)

$OM = 1$ $\left(\sin 30° = \frac{OM}{2}\right)$

$CM = \sqrt{3}$ (Pythagoras)

Area $(AOC) = \frac{1}{2} \times 1 \times 2\sqrt{3}$

$= \sqrt{3}$ (1)

Area $\triangle ACB = 3\sqrt{3}$

Area circle $= \pi(2)^2$ (1)

$\therefore$ shaded area $= 4\pi - 3\sqrt{3}$ (1)

(c) (i) $99, 97, 95, \ldots, 1$

$a = 99$, $d = -2$, $T_n = 1$

$\therefore 99 + (n-1)(-2) = 1$ (1)

$99 - 2n + 2 = 1$

$2n = 100$

$n = 50$ (1)

(ii) $T_{18} = a + 17d$

$= 99 - 34$

$= 65$

$\therefore$ cube $= (65)^2$

$= 4225$ (1)

(d) $2\cos^2\theta - 5\sin\theta + 1 = 0$

$\therefore 2(1 - \sin^2\theta) - 5\sin\theta + 1 = 0$ (1)

$2 - 2\sin^2\theta - 5\sin\theta + 1 = 0$

$2\sin^2\theta + 5\sin\theta - 3 = 0$

$(2\sin\theta - 1)(\sin\theta + 3) = 0$ (1)

$\therefore \sin\theta = \frac{1}{2}$ $\quad \sin\theta = -3$

$\theta = \frac{\pi}{6}$ OR $\frac{5\pi}{6}$ (1) (1) $\quad$ NO SOLUTION

(give only 1 mark for $30°, 150°$)

(9) (a) $(1 + \tan^2 A)\cos A$

$= \sec^2 A \cos A$ (1)

$= \sec A \cdot \frac{1}{\cos A} \cdot \cos A$

$= \sec A$ (1)

(b) (i) $l = r\theta$, $A = \frac{1}{2} r^2 \theta = 16$

$\therefore \theta = \frac{32}{r^2}$ (1)

$\therefore P = r\theta + 2r$

$= \frac{32}{r} + 2r$ (1)

(ii) $\frac{dP}{dr} = -\frac{32}{r^2} + 2$

$\frac{d^2P}{dr^2} = +\frac{64}{r^3}$ (1)

At min. $\frac{dP}{dr} = 0$

$\therefore -\frac{32}{r^2} + 2 = 0$

$r^2 = 16$

(1) $\rightarrow$ $\therefore r = 4$ OR $r = -4$

(1) $\rightarrow$ $P''(4) = +1 > 0$ $\quad$ NOT A SOL^N.

$\therefore$ MIN at $r = 4$

(1) $\rightarrow$ $\theta = 2$

(iii) $P = 16$ (1)

(c) (i) $\frac{e^x + e^{-x}}{2} = \frac{e^x - e^{-x}}{2}$

$\therefore 2e^{-x} = 0$ (1)

$e^{-x} = 0$ has no solution $\therefore$ does not intersect

(ii) $\lim_{x \to 0} [h(x) - g(x)]$

$= \lim_{x \to 0} \frac{2e^{-x}}{2}$

$= \lim_{x \to 0} e^{-x} = e^0$ (1)

$= 1$

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(10) (a) (i) $A_1 = \frac{1}{2}\cdot\frac{1}{2}\left[2 + \cos\frac{\pi}{2}\cdot\frac{1}{2}+1\right]$

$= \frac{1}{4}\left[3 + \cos\frac{\pi}{4}\right]$

$= \frac{3 + 1/\sqrt{2}}{4}$ (1) $= \frac{3\sqrt{2}+1}{4\sqrt{2}}$ (not necessary)

$A_2 = \frac{1}{2}\times\frac{1}{2}\left[\frac{1}{\sqrt{2}} + 1 + \cos\frac{\pi}{2} + 1\right]$

$= \frac{1}{4}\left[\frac{1}{\sqrt{2}} + 2\right]$

$= \frac{2 + 1/\sqrt{2}}{4}$ (1) $= \frac{2\sqrt{2}+1}{4\sqrt{2}}$ (not necessary)

$\therefore A = \frac{5 + 2/\sqrt{2}}{4}$ (1) $= \frac{5\sqrt{2}+2}{4\sqrt{2}}$ (not necessary)

(ii) $A = \int_0^1 \left(\cos\frac{\pi x}{2} + 1\right) dx$

$= \left[\frac{2}{\pi}\sin\frac{\pi x}{2} + x\right]_0^1$ (1)

$= \frac{2}{\pi}\sin\frac{\pi}{2} + 1 - \sin 0 + 0$

$= \frac{2}{\pi} + 1$ (1)

(iii) $\frac{4}{\pi} \approx \frac{5 + 2/\sqrt{2}}{4}$, $\pi \approx \frac{16}{5 + 2/\sqrt{2}}$ (1) or $\frac{5\sqrt{2}+2}{4\sqrt{2}} \approx \frac{4}{\pi}$, $\frac{16\sqrt{2}}{5\sqrt{2}+2} \approx \pi$

$\pi \approx 3.3137$ (1) $\pi \approx 3.3137$

(don't penalise for not correct to 3 dec. pl.)

(b) (i) $\sin f(x) = x$ —— (*)

$f'(x)\cos f(x) = 1$

$\therefore f'(x) = \frac{1}{\cos f(x)}$ (1)

$= \frac{1}{\sqrt{1-\sin^2 f(x)}}$ ← (1) subs (*)

$= \frac{1}{\sqrt{1-x^2}}$

(b)

(ii) $f'(x) + g'(x) = 0$

$\therefore \frac{d}{dx}[f(x) + g(x)] = 0$

$\therefore f(x) + g(x) = \text{const}.$

(iii) given $\sin f(x) = x$, $\cos g(x) = x$

(Subs. $x=0$) $\sin f(0) = 0$ ↓ $f(0) = 0$; $\cos g(0) = 0$ ↓ $g(0) = \frac{\pi}{2}$

$\therefore f(0) + g(0) = \frac{\pi}{2}$

$\therefore f(x) + g(x) = \frac{\pi}{2}$ for all x.

ie $k = \frac{\pi}{2}$