

Question 1

- a) Find the midpoint of the interval from $A(-3,6)$ to $B(4,0)$. 1
- b) Factorise fully $12a^3 - 4a^2 - a$ 2
- c) Find $\frac{d}{dx}(6x^2 - 3x - 1)$ 1
- d) Find the exact value of $\sin 240^\circ$ 1
- e) Solve $|x - 3| = |2x + 1|$ 2
- f) Lenny paid \$207 for a meal in a restaurant. This included a 15% tip.
What was the cost of the meal without the tip? 2
- g) Solve $6x^2 = x$ 2

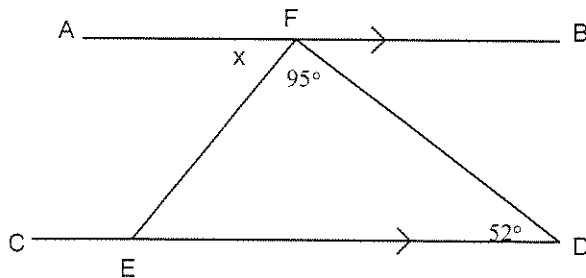
Question 2 (Start a new page)

- a) Evaluate the following correct to 3 significant figures. 2

$$\sqrt{\frac{4^2 + 18^2}{23^2 - 18^2}}$$

- b) Solve $x - \frac{x+4}{4} = 2$ 2

- c) Find x , giving reasons 2



- d) i) Sketch $y = \sqrt{9 - x^2}$ 2

- ii) State the range of $y = \sqrt{9 - x^2}$ 1

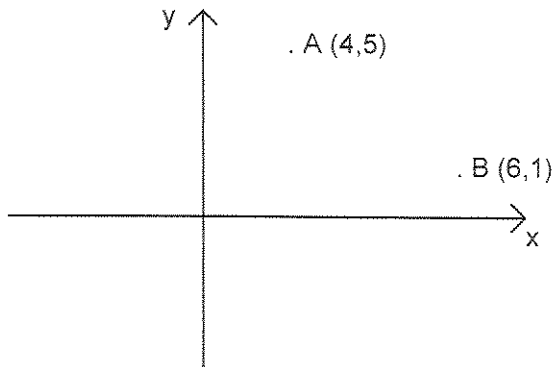
- e) Solve simultaneously for x and y 2

$$2x - y = 7$$

$$x - 2y = -1$$

Question 3 (Start a new page)

a)



- i) Show that the equation of the line AB is given by 2

$$2x + y - 13 = 0$$

- ii) The line AB is produced to meet the x axis at T. 1

Find the coordinates of T.

- iii) Find the distance AB 1

- iv) Find the perpendicular distance of the point $(-1, 5)$ 2

to the line AB.

- v) ABC is a right angled isosceles triangle with $\angle BAC = 90^\circ$ 2

Find all the possible coordinates of the point C.

- b) Sketch the region satisfied simultaneously by the inequalities 3

$$x + y + 2 \leq 0 \quad \text{and} \quad x - 2y \geq 0$$

(Your diagram must be neat and clearly labelled)

Question 4 (Start a new page)

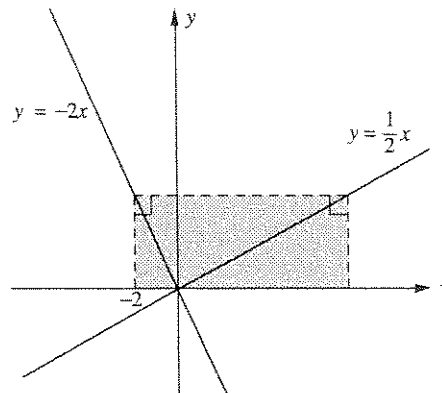
a) Find the equation of the line that passes through the point $(2, 4)$ and is parallel to the x axis. 1

b) If $\sin \theta = \frac{1}{5}$ and θ is obtuse, find the exact value of $\cos \theta$. 2

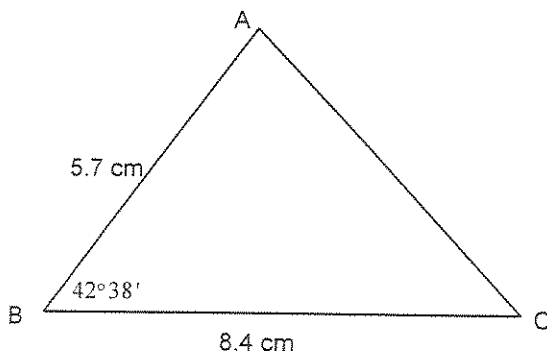
c) Evaluate $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x - 2}$ 1

d) If $\frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}} = x - y\sqrt{6}$ find the values of x and y . 3

e) Calculate the shaded area 2



f) Find the area of triangle ABC correct to 1 decimal place. 2



Question 5 (Start a new page)

- a) Simplify $\sqrt{98} - \sqrt{8} + \sqrt{12}$ 1
- b) Simplify $\frac{x+1}{x+2} \div \frac{x^2-9}{x^2+5x+6}$ 2
- c) Find the equation of the line which passes through the point (1,4) 3
and is perpendicular to the line $3x - 2y - 4 = 0$, giving your
answer in general form.
- d) From a point 26 metres from the base of a tower the angle of elevation 2
of the top of the tower is 68° . Find the height of the tower,
giving your answer correct to the nearest centimetre.
- e) Solve $5 \sin \theta + 2 = 0$ for $0^\circ \leq \theta \leq 360^\circ$ 3
(Give answers correct to the nearest degree)

Question 6 (Start a new page)

- a) Find, correct to the nearest degree, the acute angle the line 2

$$3x - y - 12 = 0 \text{ makes with the } x \text{ axis.}$$

- b) Use the formula $\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ 2

to differentiate $y = 3x^2 + x$ by first principles.

- c) A function is defined by

$$f(x) = \begin{cases} x^2 - 1 & \text{for } x \geq 1 \\ |x - 1| & \text{for } x < 1 \end{cases}$$

- i) evaluate $f(4) + f(0)$ 1

- ii) Sketch the function $y = f(x)$ 2

- d) A geologist drives 30 kilometres from point A on a bearing of

324° to point B. He then drives 50 kilometres from point B,

on a bearing of 040° to point C.

- i) Draw a diagram showing this information 1

- ii) Find the size of angle ABC. 1

- iii) Find the distance of point C from point A. 2

(Give answer correct to the nearest kilometre)

Question 7 (Start a new page)

- a) How many sides has a regular polygon with interior angles equal to 165° ? 2
- b) Differentiate with respect to x
- i) $y = (2x+1)(x-3)$ 1
- ii) $y = \frac{1}{x} + x$ 2
- c) Find the equation of the tangent to $y = x^3 - 2x^2 - 3$ at the point $(2, -3)$. 3
- d) If $f(x) = (x+2)(x-1)^4$ evaluate $f'(2)$ 3

Question 8 (Start a new page)

a) Evaluate $\lim_{x \rightarrow \infty} \frac{3x^2 - 2x - 1}{x^2 - 1}$ 1

b) Prove the identity $\tan \theta + \cot \theta = \frac{1}{\sin \theta \cos \theta}$ 3

c) Differentiate with respect to x .

i) $y = \sqrt{x^2 - 5}$ 1

ii) $y = \frac{3x}{x-3}$ 2

d) i) Find the gradient of the line $2x + 5y + 6 = 0$ 1

ii) The line $ax + by + 11 = 0$ passes through the point $(1, 4)$ 3

and is parallel to the line $2x + 5y + 6 = 0$.

Find the values of a and b .

End of Paper

SOLUTIONS

QUESTION 1

a) $(\frac{1}{2}, 3)$

b) $a(12a^2 - 4a - 1)$
 $= a(6a+1)(2a-1)$

c) $12x - 3$

d) $\sin 240^\circ = -\sin 60^\circ$
 $= -\frac{\sqrt{3}}{2}$

e) $x - 3 = 2x + 1$ or $x - 3 = -(2x + 1)$
 $x = -4$ or $x - 3 = -2x - 1$
 $3x = 2$
 $x = \frac{2}{3}$

f) $115\% \equiv \$207$
 $\therefore 100\% \equiv \180

g) $6x^2 - x = 0$
 $x(6x - 1) = 0$
 $x = 0, \frac{1}{6}$

QUESTION 2

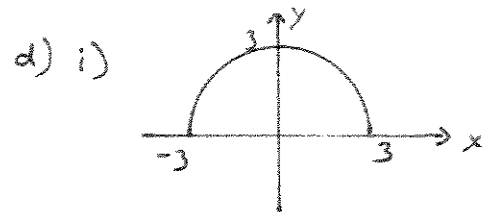
a) 1.29

b) $x - \frac{x+4}{4} = 2$
 $4x - (x+4) = 8$
 $3x - 4 = 8$
 $3x = 12$
 $x = 4$

c) $\angle BFD = 52^\circ$ (alternate angles, $AB \parallel CD$)

$\therefore x + 95 + 52 = 180$ (straight angle)

$\therefore x = 33^\circ$



ii) Range: $0 \leq y \leq 3$

e) $2x - y = 7$
 $x - 2y = -1$
 $4x - 2y = 14$
 $x - 2y = -1$

 $3x = 15$
 $x = 5$
 $\therefore y = 3$

~~subtract~~
subtract

QUESTION 3

a) i) $m_{AB} = \frac{5-1}{4-6}$
 $= -2$

$\therefore y - 5 = -2(x - 4)$
 $y - 5 = -2x + 8$
 $2x + y - 13 = 0$

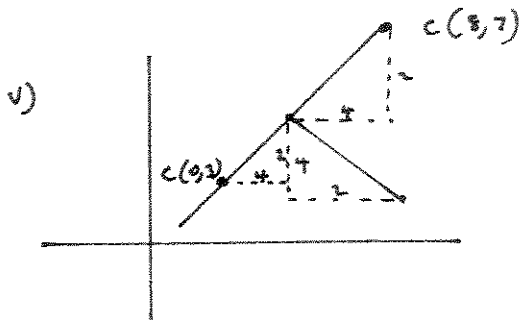
ii) when $y = 0$ $x = 6\frac{1}{2}$
 $\therefore T(6\frac{1}{2}, 0)$

$$\text{iii) } d = \sqrt{(4-6)^2 + (5-1)^2}$$

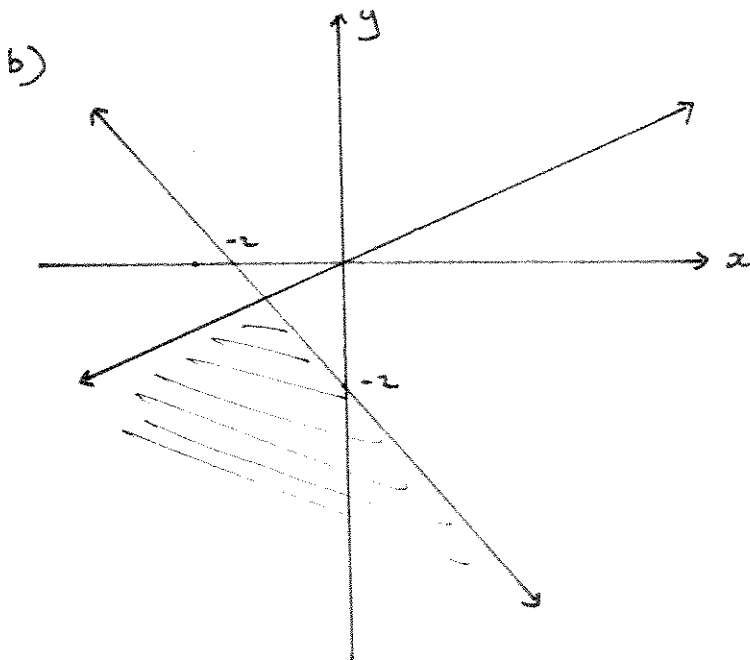
$$= \sqrt{20} \text{ units}$$

$$\text{iv) } d = \frac{|-1 \times 2 + 5 \times 1 - 13|}{\sqrt{2^2 + 1^2}}$$

$$= \frac{10}{\sqrt{5}} \text{ units}$$



$c(8,7)$ or $c(0,3)$

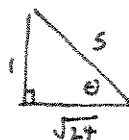


QUESTION 4

a) $y = 4$

b) $\sin \theta = \frac{1}{5}$

$\cos \theta = -\frac{\sqrt{24}}{5}$



(2nd quad)

c) $\lim_{x \rightarrow 2} \frac{(x-2)(x+1)}{x-2}$

$$= 3$$

d) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$$= \frac{(\sqrt{3}-\sqrt{2})^2}{1}$$

$$= 3 - 2\sqrt{6} + 2$$

$$= 5 - 2\sqrt{6}$$

$\therefore x = 5, y = 2$

e) $A = 10 \times 4$

$$= 40 \text{ sq units}$$

f) $A = \frac{1}{2} \times 8.4 \times 5.7 \times \sin 42^\circ 38'$

$$= 16.2 \text{ cm}^2$$

QUESTION 5

a) $7\sqrt{2} - 2\sqrt{2} + 2\sqrt{3}$

$$= 5\sqrt{2} + 2\sqrt{3}$$

b) $\frac{x+1}{x+2} \times \frac{(x+3)(x+2)}{(x-3)(x+3)}$

$$= \frac{x+1}{x-3}$$

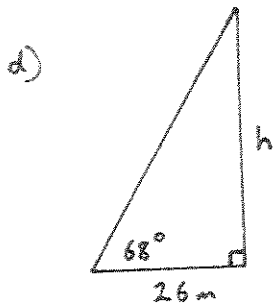
c) $3x - 2y - 4 = 0$

$$m = \frac{3}{2}$$

$\therefore m_{\perp} = -\frac{2}{3}$

$\therefore y - 4 = -\frac{2}{3}(x - 1)$

$2x + 3y - 14 = 0$



$$\tan 68^\circ = \frac{h}{26}$$

$$h = 26 \times \tan 68^\circ$$

$$= 64.35 \text{ m}$$

e) $5 \sin \theta + 2 = 0$

$$\sin \theta = -\frac{2}{5}$$

(3rd/4th $\Rightarrow \theta = 24^\circ$)

$$\therefore \theta = 204^\circ, 336^\circ$$

QUESTION 6

a) $m = 3$

$$\therefore \tan \theta = 3$$

$$\therefore \theta = 72^\circ$$

b) $y' = \lim_{h \rightarrow 0} \frac{3(x+h)^2 + (x+h) - [3x^2 + x]}{h}$

$$= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 + x + h - 3x^2 - x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{6xh + 3h^2 + h}{h}$$

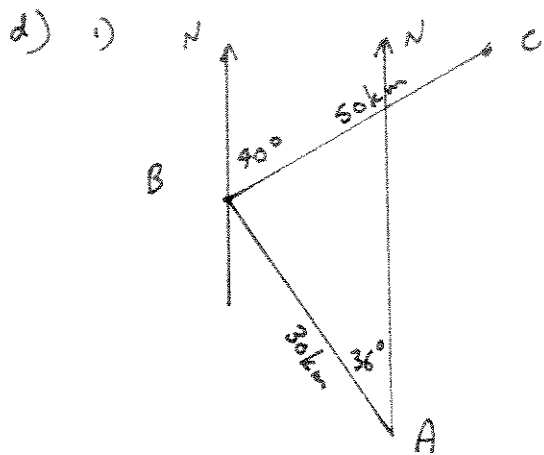
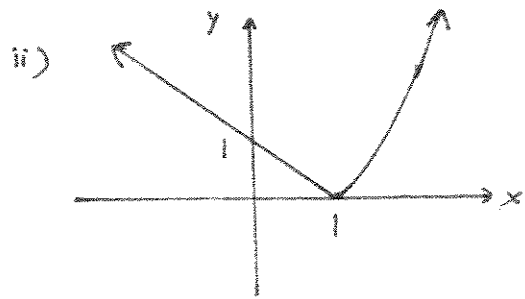
$$= \lim_{h \rightarrow 0} 6x + 3h + 1$$

$$= 6x + 1$$

c) i) $f(4) + f(0)$

$$= 15 + 1$$

$$= 16$$



ii) $\angle ABC = 180^\circ - 36^\circ - 40^\circ$

$$= 104^\circ$$

iii) $AC^2 = 30^2 + 50^2 - 2 \times 30 \times 50 \times \cos 104^\circ$

$$\therefore AC = 64 \text{ km}$$

QUESTION 7

a) ext angle = 15°

$$\therefore \text{no. of sides} = \frac{360}{15}$$

$$= 24$$

b) i) $y = 2x^2 - 5x - 3$

$$\therefore y' = 4x - 5$$

ii) $y = x^{-1} + x$

$$\therefore y' = -x^{-2} + 1$$

$$c) \quad y' = 3x^2 - 4x$$

when $x=2$

$$m_T = 3 \times 2^2 - 4 \times 2$$

$$= 4$$

$$\therefore y+3 = 4(x-2)$$

$$y+3 = 4x-8$$

$$y = 4x-11$$

$$d) \quad f(x) = (x+2)(x-1)^4$$

$$f'(x) = (1)(x-1)^4 + (x+2)4(x-1)^3$$

$$\therefore f'(2) = 1^4 + 4 \times 2 \times 1^3$$

$$= 17$$

QUESTION 8

a) 3

$$b) \quad \text{LHS} = \tan \theta + \cot \theta$$

$$= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}$$

$$= \frac{1}{\cos \theta \sin \theta}$$

$$= \text{RHS}$$

$$c) \quad i) \quad y = (x^2 - 5)^{\frac{1}{2}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2} (x^2 - 5)^{-\frac{1}{2}} \cdot 2x$$

$$= x (x^2 - 5)^{-\frac{1}{2}}$$

$$= \frac{x}{\sqrt{x^2 - 5}}$$

$$ii) \quad \frac{dy}{dx} = \frac{(x-3)(3) - (3x)(1)}{(x-3)^2}$$

$$= \frac{-9}{(x-3)^2}$$

$$d) \quad i) \quad m = -\frac{2}{3}$$

$$ii) \quad \text{gradient of } ax+by+11=0$$

$$m = -\frac{a}{b}$$

as parallel $-\frac{a}{b} = -\frac{2}{3}$

$$\therefore 3a = 2b \quad *$$

sub (1,4) into $ax+by+11=0$

$$a + 4b + 11 = 0 \quad *$$

Solve simultaneously (*)

$$3a = 2b$$

$$a + 4b + 11 = 0$$

$$\therefore a + 2 \times 3a + 11 = 0$$

$$11a + 11 = 0$$

$$a = -1$$

$$\therefore b = -\frac{5}{2}$$

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