Name : _____

Teacher/Class: _____

SYDNEY TECHNICAL HIGH SCHOOL

HSC ASSESSMENT TASK 1

DECEMBER 2006

MATHEMATICS

Time Allowed: **70 minutes**

Instructions:

- Write your name and class at the top of each page
- All necessary working must be shown. Marks may be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- Start **each** question on a **new** page.
- Diagrams unless otherwise stated are not to scale.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	TOTAL
/7	/7	/7	/7	/7	/7	/7	/7	/7	/56

QUESTION 1 (7 Marks) **(MARKS)**

a) Factorise $2x^2+5x-3$ (1)

b) Solve $x^2-4x>0$ (2)

c) Evaluate $\sum_{r=2}^{5}\frac{1}{r+1}$ (1)

d) Let α and β be the roots of $x^2-3x-7=0$ find

i) $\alpha+\beta$ (1)

ii) $\alpha\beta$ (1)

e) Form a quadratic equation whose roots are -2 and 3 (1)

QUESTION 2 (7 Marks) (start a new page) **(MARKS)**

a) If $x=-2$ is a root of $4x^2+x+k=0$, find k (1)

b) Find the limiting sum of the geometric series

$$\frac{13}{5}+\frac{13}{25}+\frac{13}{125}+\ldots\ldots\ldots$$ (2)

c) For the sequence -8 , -1 , 6 ……

i) Find the 29th term (1)

ii) Find the sum of the first 29 terms (1)

iii) Which term of the sequence has a valve of 167? (2)

QUESTION 3 (7 Marks) (start a new page) **(MARKS)**

a) i) Draw a neat sketch (using a ruler for the axes) of the parabola $x^2=8y$ (1)

ii) Find the co-ordinates of the focus (1)

iii) Find the equation of the directrix (1)

iv) Find the equation of the tangent to the parabola $x^2=8y$ that passes through the point (-8 , 8) (2)

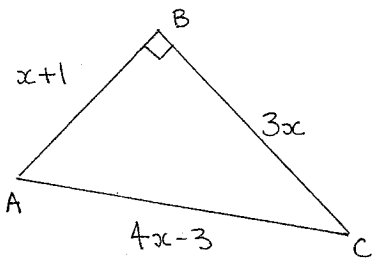
b) In solving a quadratic equation a student wrote his solution as

$$x = \frac{4 \pm \sqrt{16+96}}{6}$$ (2)

What was the original equation?

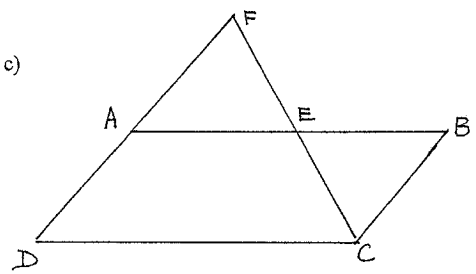
QUESTION 4 (7 Marks) (start a new page) **(MARKS)**

a) Triangle ABC below has angle ABC equal to 90°. Find all possible value(s) for x. (2)



b) How much will \$500 grow to at 12% p.a. if compounded quarterly for 5 years (1)

c)



$ABCD$ is a parallelogram $AE=EB$
DA produced intersects
CE produced at F

i) Copy the diagram onto your answers sheet

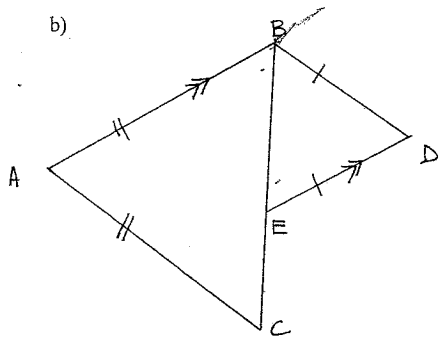
ii) Prove $\Delta AFE \equiv \Delta BCE$ (2)

iii) Hence explain why $DA = AF$ (2)

QUESTION 5 (7 Marks) (start a new page) **(MARKS)**

a) For what values of k does the equation $x^2 + kx + 3 - k = 0$ have real, different roots? (3)

b)



Triangles ABC and BDE are isosceles
$AB=AC$ and $BD=ED$ and AB ll ED

i) Copy the diagram onto your answer sheet.

ii) Prove ΔABC is similar to ΔBDE (3)

iii) If $BD = 5cm$, $BE = 4cm$ and $AC = 6cm$ find the length of EC (1)

QUESTION 6 (7 Marks) (start a new page) **(MARKS)**

a) Insert three numbers between 5 and 80 so as to form five numbers in a geometric sequence . (3)

b) The first n terms of an arithmetic sequence have a sum given by $S_n = 25n - 2n^2$

i) Find the first term and the second term (2)

ii) Find the common difference (1)

iii) Find the expression for the nth term (1)

QUESTION 7 (7 Marks) (start a new page) **(MARKS)**

a) A man places \$1500 at the beginning of each year into a superannuation fund, for 30 years. Interest on investments in the fund compounds at 12 % p.a.

i) Find the amount he has in the fund at the end of 30 years (3)

ii) If this amount in part i) is taken as a lump sum and taxed at the rate of 30 % for each dollar over \$50,000 , how much will he receive after tax. (2)

b) For the parabola $y = x^2 + 4x + 5$ find

i) the co-ordinates of the vertex (1)

ii) the co-ordinates of the focus (1)

QUESTION 8 (7 Marks) (start a new page) **(MARKS)**

a) A is the point $(8,0)$ and 0 is the origin. P is the variable point (x, y)

i) If P moves so that $PO = 3PA$, show that the locus of P is given by

$$x^2 + y^2 = 9[(x-8)^2 + y^2]$$ (2)

ii) Show that this locus is a circle by finding its centre and radius (2)

b) i) Find the sum of the geometric series

$$x^4 + x^3y + x^2y^2 + xy^3 + y^4$$ (2)

ii) Hence factorise $x^5 - y^5$ (1)

Teacher's Name: Parrish **Student's Name/N°:** Vincent Tay

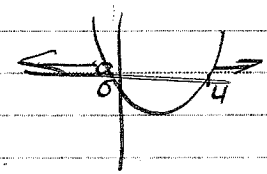
Question 1

a $2x^2 + 5x - 3$ -6 5

$(2x^2 - 6x) + (-x - 3)$ $-6, -1$

$= 2x(x-3) + -1(x-3)$

$(2x-1)(x-3)$ ✓ (1)



b $x(x-4) > 0$

$x - 4 > 0$

$x > 4$ (1) $x > 4$ and $x < 0$

c $a = 1/3$ $l = 1/6$ $n = 4$ $1/3, 1/4, 1/5, 1/6$

$4/2 (1/3 + 1/6)$ ✗ [19/20] ✓ (1)

d i 3 ✓ (1)

ii -7 ✓ (1)

e $\alpha + \beta = -b/a = -2 + 3 = 1$

$\alpha\beta = -2 \times 3 = -6 = c/a$

$x^2 - x - 6 = 0$ ✓ (1)

6/7

Teacher's Name: Student's Name/N°:

Question 2

a $h = -20$ ✗

b $a = 13/5 \quad r = 1/5$

$S_\infty = \frac{13/5}{1 - 1/5} = 13/5 \times 5/4 = 13/4 = 3\,1/4$ ✓ 2

c i $a = -8 \quad d = 7 \quad n = 29$

$A_{29} = -8 + (28)7 = 188$ ✓ (1)

ii $S_{29} = 29/2\,(-8 + 188)$

$= 2610$ ✓ (1)

iii $A_n = -8 + (n-1)7 = 167$

$7n - 7 = 175$

$7n = 182$

$n = 26$ ✓ (2)

6/7

2a When something is a root, it means replace

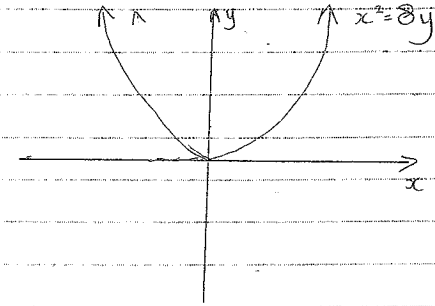
ie $x = -2 \quad \therefore 4(-2)^2 + (-2) + k = 0$

$k = -14$

Teacher's Name: Parmish Student's Name/N°: Vincent Tey

Q. 3

a i



1

ii $a = 2 \quad (0, 2)$

iii $y = -2$

1

iv $y = x^2/8$

$y = x/4 \leftarrow x = -8$

$-2 = m$

$y - 8 = -2(x + 8)$

$y - 8 = -2x - 16$

$2x + y + 8 = 0$

2

b $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = \frac{4 \pm \sqrt{16 + 96}}{6}$

$a = 3 \quad b = -4 \quad c = -8$

$3x^2 - 4x - 8 = 0$

2

(7)

Teacher's Name: _____ Student's Name/N°: _____

Q4

a $(x+1)^2 + (3x)^2 = (4x-3)^2$

$x^2+2x+1 + 9x^2 = 16x^2-24x+9$

$10x^2+2x+1 = 16x^2-24x+9$ 48 -26

$0 = 6x^2-26x+8$ -24 ; 2

$(6x^2-2x)+(-24x+8)$

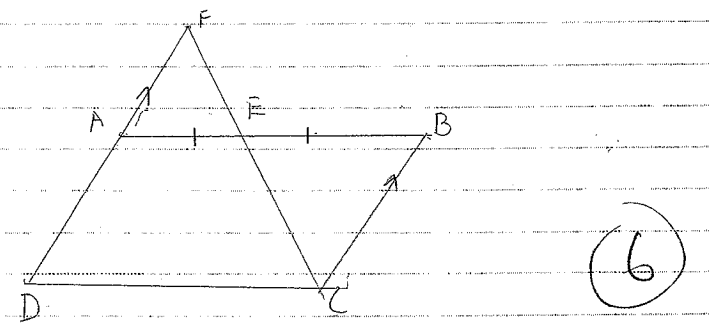
$2x(3x-1) -8(3x-1)$

$(2x-8)(3x-1)$ 1

$\therefore x = 4, \frac{1}{3}$ ✓

b $500(1.03)^{20} = \$903.06$ ✓ 1

c i



6

ii for ΔAFE and ΔBCE

$\angle AEF = \angle CEB$ (vertically opposite angles) ✓

$\angle FAE = \angle EBC$ (alternate angles equal on parallel lines) ✓

$AE = EB$ (given) ✓

$\therefore \Delta AFE \equiv \Delta BCE$ (AAS) 2

iii $BC = AF$ (corresponding sides on congruent triangles are equal)

$AD = BC$ (opposite sides on parallelogram are equal)

$\therefore AD = AF$

✓

Teacher's Name: Parrish Student's Name/N°: Vincent Teoy

Q5

a $\Delta > 0$ -6, 2

$\Delta = (k)^2 - 4 \times 1 \times (3-k)$

$= k^2 - 12 + 4k$

$k^2+4k-12$ 2

$(k+6)(k-2)$

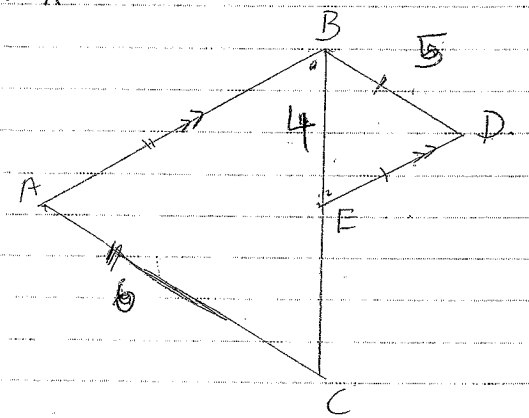
$\Delta = -6, 2$

$0 > -6, 2$ ← ~~$-6 > 0$~~ $-6 \not> 0$

$\therefore k > 2$

$k < -6, \quad k > 2$

b i



$\angle ABE = \angle DEB$ (alternate angles on parallel lines)

~~$\angle ACB = \angle ABE$ (isosceles Δ)~~

$\angle ABE = \angle ACB$ (base angles of isosceles triangle are equal)

$\angle DBE = \angle DEB$ ( ) (otherwise $\rightarrow 0°$) ($\angle ACB = \angle EBD$) 2

$\therefore \Delta ABC \parallel\parallel \Delta BDE$ (equiangular)

ii $BC = x$

$\frac{6}{5} / \frac{x}{x-4}$ $5x = 6x - 24$

$-x = -24$

$x = 24$

$24 - 4 = 20$

$EC = 20$

$\frac{5}{4} = \frac{6}{4+EC}$

$20 + 5(EC) = 24$ 1

$5(EC) = 4$

$EC = 4/5$

Teacher's Name: _____ Student's Name/N°: _____

Q6

a $a = 5$ $l = 80$

$A_5 = 5r^4 = 80$

$r^4 = 16$

$r = \pm 2$

5, 10, 20, 40, 80 or −10, 20, −40

5, −10, 20, −40, 80

3

b i 1st term = 23

2nd term = $[25 \times 2 - 2(2)^2] - 23$

$= 42 - 23$

$= 19$ ✓

ii −4 ✓

iii $A_n = 23 + (n-1) - 4$

$= 23 - 4n + 4$

$= 27 - 4n$ ✓

7

Teacher's Name: Parrish Student's Name/N°: Vikrant Tery

Q7

a i $T_1 = 1500(1.02)^{30}$

$T_2 = 1500(1.12)^{29}$

$\vdots$

$T_{30} = 1500(1.12)^1$

$S_{30} = 1500(1.12)^{30} + 1500(1.12)^{29} \ldots + 1500(1.12)^1$

$= 1500(1.12)\dfrac{(1.12^{30} - 1)}{0.12} = 1680 \times 241.33$

$= \$405\,438.91$ ✓ 3

ii $405\,438.91 - 50\,000 = \$355\,438.91$

$355\,438.91(0.3) = 106\,631.687$

$405\,438.91 - 106\,631.67 = \$298\,807.24$ ✓ 2

b i $\dfrac{-b}{2a} = -2$ $(-2, 1)$ ✓

ii (0, 1) ✗ 1

$4a = y$ $4a = 1$

$a = 1/4$

$(-2, 1\frac{1}{4})$

6

Teacher's Name: Student's Name/N°:

Q 8

a: P (x, y) A (8,0) O (0,0)

$PO = 3PA$

$\sqrt{x^2+y^2} = 3\sqrt{(x-8)^2+(y)^2}$

$x^2+y^2 = 3[x^2-16x+64+y^2]$

$= 3x^2+3y^2-48x+192$

$0 = 2x^2+2y^2-48x+192$

$x^2+y^2 = 9[(x-8)^2+y^2]$ ✓ (2)

ii $x^2+y^2 = 9[x^2-16x+64+y^2)$

$x^2+y^2 = 9x^2-144x+576+9y^2$

$8x^2-144x+8y^2+576 = 0$

$x^2-18x+y^2+72 = 0$ $\quad x^2-18x$

$x^2-18x+y^2 = +81+72$

$(x-9)^2+y^2 = 9$ ✓ (2)

centre (9,0) radius = 3

(b) (i) $a = x^4$, $r = \frac{y}{x}$, $n = 5$

$\therefore S_n = \frac{a(r^n-1)}{r-1}$

$\therefore S_5 = \frac{x^4\left(\left(\frac{y}{x}\right)^5-1\right)}{\frac{y}{x}-1}$

$= x^4\left(\frac{y^5-x^5}{x^5}\right) \div \left(\frac{y-x}{x}\right)$

$= \frac{y^5-x^5}{x} \times \frac{x}{y-x}$

$= \frac{y^5-x^5}{y-x}$ or $\frac{x^5-y^5}{x-y}$

(ii) Hence $x^4+x^3y+x^2y^2+xy^3+y^4 = \frac{x^5-y^5}{x-y}$

$\therefore (x-y)(x^4+x^3y+x^2y^2+xy^3+y^4) = x^5-y^5$