NAME: ____________________

CLASS: ……………………………

SYDNEY TECHNICAL HIGH SCHOOL

YEAR 12

HSC ASSESSMENT TASK 2

MARCH 2007

MATHEMATICS

Time Allowed : **70 minutes**

Instructions:

- Write your name and class at the top of each page
- All necessary working must be shown. Marks may be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- Start each question on a new page.

Question 1	Question 2	Question 3	Question 4	Question 5	Total
/12	/12	/12	/12	/10	/58

QUESTION 1 **Marks**

a) Differentiate the following:

i) $y = x^3 + 4x^2 + 2$ **1**

ii) $y = \dfrac{3x}{x+2}$ **2**

iii) $y = (2x+1)^4$ **2**

b) Find the gradient of the tangent to the curve $y = 4x^3 + x$ at the point (1, 5) **2**

c) Find:

i) $\int x^4 + 3x^2 \, dx$ **1**

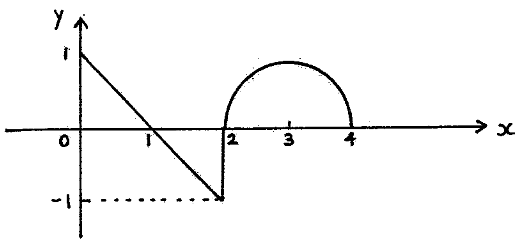
ii) $\int (x-5)(x+4) \, dx$ **2**

iii) $\int \dfrac{x^3 - 3x^4}{x^2} \, dx$ **2**

QUESTION 2

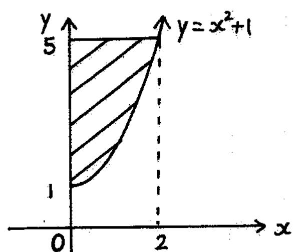
Marks

a) Find the exact value of $\int_0^4 f(x)\,dx$ given **2**



b) Find $\int (2x-1)^5\,dx$ **2**

c) The sketch shows an arc of the curve $y = x^2 + 1$. **3**



Calculate the shaded area.

d) The gradient of a curve is given by

$$\frac{dy}{dx} = 3x^2 - 12$$

i) Find $\dfrac{d^2y}{dx^2}$ **1**

ii) Find the values of x for which the curve both increases **and** is concave downwards. **2**

iii) If the curve passes through (1, - 2) find the equation of the curve. **2**

QUESTION 3

Marks

a) Find the primitive function of $\sqrt{x}$ **2**

b) Melanie joined a Superannuation Fund, investing \$P at the beginning of every year at 8% p.a. compound interest (compounded yearly).

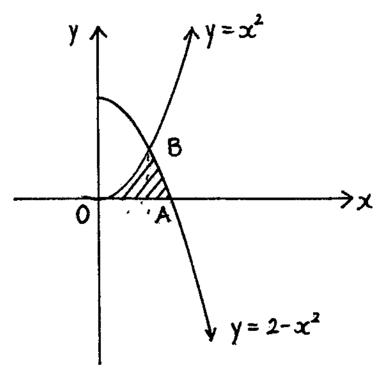
i) Write an expression for the amount of her investment at the end of the first year. **1**

ii) Write an expression for the amount of her investment at the end of the second year. **1**

iii) Write an expression for the amount of her investment at the end of twenty five years **1**

iv) If at the end of twenty five years, she wishes to collect \$500,000 calculate the value of \$P to the nearest dollar. **2**

c)



The shaded region OAB is bounded by the parabolas $y = x^2$ and $y = 2 - x^2$ and the x axis from $x = 0$ to $x = \sqrt{2}$.

i) B is the point of intersection of the two parabolas in the first quadrant. Find the co-ordinates of B. **2**

ii) Calculate the area of the shaded region OAB (2 dp). **3**

QUESTION 4 **Marks**

a) A function is defined by $y=3x^2-2x^3$

i) Find the co-ordinates of any turning points and determine their nature **3**

ii) Given that there is a point of inflexion, find its co-ordinates. **1**

iii) Sketch the function from $x=-1$ to $x=2$ **2**

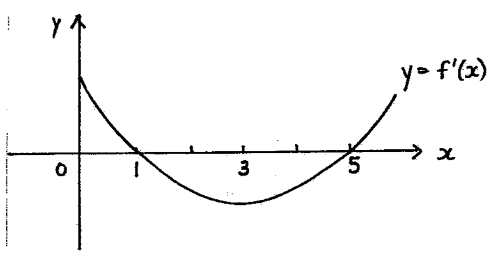
Note Your sketch must be neat

Use a ruler to draw the axes

Label all important points

iv) Find the area bounded by the curve $y=3x^2-2x^3$ and the x axis from $x=0$ to $x=2$ **4**

b) **2**



The diagram shows the graph of the gradient function of the curve $y=f(x)$.

For what value of x does $f(x)$ have a local minimum?

Test continues over page . .

QUESTION 5 **Marks**

a) A cylindrical container closed at both ends is made from a sheet of thin plastic.

The surface area of the cylinder is 600π centimetres^2.

i) Show that the height h of the cylinder is given by the expression: **2**

$$h=\frac{300}{r}-r, \text{ where } r \text{ is the radius.}$$

ii) Find an expression for the volume V in terms of r. **1**

iii) Find the height of the container if the volume is to be a maximum. **3**

b) i) Differentiate $y=x^3(1+x)^3$ **2**

ii) Hence, solve $\frac{dy}{dx}=0$ **2**

End of Test

QUESTION 1

a) i. $y = x^3 + 4x^2 + 2$

$\frac{dy}{dx} = 3x^2 + 8x$

ii. $y = \frac{3x}{x+2}$

$u = 3x \quad v = x+2$

$u' = 3 \quad v' = 1$

$\frac{dy}{dx} = \frac{3(x+2) - 3x(1)}{(x+2)^2}$

$= \frac{3x + 6 - 3x}{(x+2)^2}$

$= \frac{6}{(x+2)^2}$

iii. $y = (2x+1)^4$

$\frac{dy}{dx} = 4(2x+1)^3 \times 2$

$= 8(2x+1)^3$

b) $y = 4x^3 + x$

$\frac{dy}{dx} = 12x^2 + 1$

when $x = 1$, $m_{tangent} = 12 \times 1^2 + 1$

$= 13$

c) i. $\int x^4 + 3x^2 \, dx$

$= \frac{x^5}{5} + x^3 + c$

ii. $\int (x-5)(x+4) \, dx$

$= \int x^2 - x - 20 \, dx$

$= \frac{x^3}{3} - \frac{x^2}{2} - 20x + c$

iii. $\int \frac{x^3 - 3x^4}{x^2} \, dx$

$= \int x - 3x^2 \, dx$

$= \frac{x^2}{2} - x^3 + c$

QUESTION 2

a) $\int_0^4 f(x) \, dx = \frac{1}{2} \times \pi \times 1^2$

$= \frac{\pi}{2}$

b) $\int (2x-1)^5 \, dx = \frac{(2x-1)^6}{6 \times 2} + c$

$= \frac{(2x-1)^6}{12} + c$

c) $A = 5 \times 2 - \int_0^2 x^2 + 1 \, dx$

$= 10 - \left[\frac{x^3}{3} + x\right]_0^2$

$= 10 - \left[\frac{2^3}{3} + 2 - 0\right]$

$= 10 - 4\frac{2}{3}$

$= 5\frac{1}{3} \; u^2$

d) $\frac{dy}{dx} = 3x^2 - 12$

i. $\frac{d^2y}{dx^2} = 6x$

ii. Increasing : $\frac{dy}{dx} > 0$

$3x^2 - 12 > 0$

$3(x+2)(x-2) > 0$

$x < -2, \; x > 2$

Concave down : $\frac{d^2y}{dx^2} < 0$

$6x < 0$

$x < 0$

$\therefore$ both increasing and concave down $x < -2$

iii. $y = \int 3x^2 - 12 \, dx$

$y = x^3 - 12x + c$

when $x = 1$, $y = -2$

$-2 = 1^3 - 12.1 + c$

$c = 9$

$\therefore y = x^3 - 12x + 9$

QUESTION 3

a) $\sqrt{x} = x^{\frac{1}{2}}$

primitive $= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \frac{2}{3} x^{\frac{3}{2}} + c$

b) i. $A_1 = P(1\cdot08)$

ii. $A_2 = P(1\cdot08)^2 + P(1\cdot08)$

$= P(1\cdot08^2 + 1\cdot08)$

iii. $A_{25} = P(1\cdot08^{25} + 1\cdot08^{24} + \ldots + 1\cdot08)$

iv. $A_{25} = \frac{P \times 1\cdot08(1\cdot08^{25} - 1)}{1\cdot08 - 1}$

$500\,000 = \frac{P \times 1\cdot08(1\cdot08^{25} - 1)}{0\cdot08}$

$P = \$6333$ (nearest dollar)

c) i. $x^2 = 2 - x^2$

$2x^2 = 2$

$x^2 = 1$

$x = 1$ (1st quadrant)

$y = 1^2$

$\therefore B(1,1)$

ii. $A = \int_0^1 x^2 \, dx + \int_1^{\sqrt{2}} 2 - x^2 \, dx$

$= \left[\frac{x^3}{3}\right]_0^1 + \left[2x - \frac{x^3}{3}\right]_1^{\sqrt{2}}$

$= \left[\frac{1^3}{3} - 0\right] + \left[\left(2\sqrt{2} - \frac{\sqrt{2}^3}{3}\right) - \left(2 \times 1 - \frac{1^3}{3}\right)\right]$

$= \frac{1}{3} + \left[2\sqrt{2} - \frac{2\sqrt{2}}{3} - \left(2 - \frac{1}{3}\right)\right]$

$\therefore A = 0\cdot55$ (2 dp)

QUESTION 4

a) $y = 3x^2 - 2x^3$

$\frac{dy}{dx} = 6x - 6x^2$

$\frac{d^2y}{dx^2} = 6 - 12x$

i. Stat pts : $\frac{dy}{dx} = 0$

$6x - 6x^2 = 0$

$6x(1-x) = 0$

$x = 0, 1$

When $x = 0$, $y = 0$

$\frac{d^2y}{dx^2} > 0$

$\therefore$ min at $(0,0)$

When $x = 1$, $y = 1$

$\frac{d^2y}{dx^2} < 0$

$\therefore$ max at $(1,1)$

ii. Inflexion : $\frac{d^2y}{dx^2} = 0$

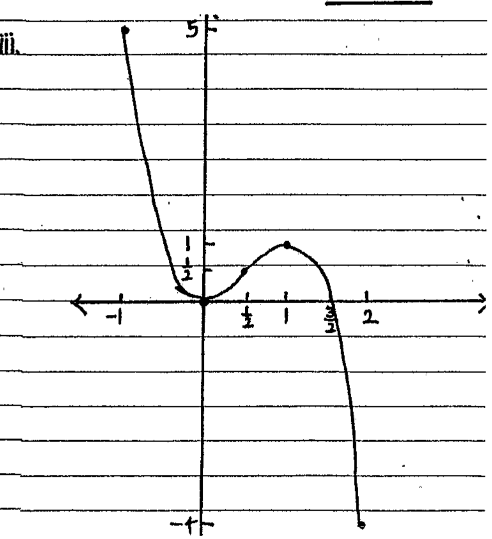
$6 - 12x = 0$

$12x = 6$

$x = \frac{1}{2}$

$\therefore$ inflexion at $\left(\frac{1}{2}, \frac{1}{2}\right)$

iii.



iv. $A = \int_0^{\frac{3}{2}} 3x^2 - 2x^3\,dx + \left|\int_{\frac{3}{2}}^{2} 3x^2 - 2x^3\,dx\right|$

$= \left[x^3 - \frac{x^4}{2}\right]_0^{\frac{3}{2}} + \left|\left[x^3 - \frac{x^4}{2}\right]_{\frac{3}{2}}^{2}\right|$

$= \left[\left(\frac{3}{2}\right)^3 - \frac{1}{2}\times\left(\frac{3}{2}\right)^4 - 0\right] + \left|\left[2^3 - \frac{2^4}{2}\right] - \left[\left(\frac{3}{2}\right)^3 - \frac{1}{2}\times\left(\frac{3}{2}\right)^4\right]\right|$

$= \frac{27}{32} + \left|-\frac{27}{32}\right|$

$\therefore A = 1\frac{11}{16}\ u^2 \qquad (1 \cdot 6875)$

b) $f'(x) = 0$ at $x = 1$ and 5

ie stationary points on $f(x)$

when $x < 1$, $f'(x) > 0$

ie increasing

when $1 < x < 5$, $f'(x) < 0$

ie decreasing

when $x > 5$, $f'(x) > 0$

ie increasing

$\therefore$ min when $x = 5$

QUESTION 5

a) i. $SA = 2\pi r^2 + 2\pi rh$

$600\pi = 2\pi r^2 + 2\pi rh$

$300 = r^2 + rh$

$rh = 300 - r^2$

$h = \frac{300 - r^2}{r}$

$\therefore h = \frac{300}{r} - r$

ii. $V = \pi r^2 h$

$= \pi r^2\left(\frac{300}{r} - r\right)$

$\therefore V = 300\pi r - \pi r^3$

iii. $\frac{dV}{dr} = 300\pi - 3\pi r^2$

$\frac{d^2V}{dr^2} = -6\pi r$

max V : $\frac{dV}{dr} = 0$

$300\pi - 3\pi r^2 = 0$

$3\pi(100 - r^2) = 0$

$r = 10$ only $(r > 0)$

when $r = 10$, $\frac{d^2V}{dr^2} < 0$

$\therefore$ max V when $r = 10$

$\therefore$ height $= \frac{300}{10} - 10$

$\therefore h = 20$ cm

b) i. $y = x^3(1+x)^3$

$u = x^3 \qquad v = (1+x)^3$

$u' = 3x^2 \qquad v' = 3(1+x)^2$

$\frac{dy}{dx} = 3x^2(1+x)^3 + 3x^3(1+x)^2$

$= 3x^2(1+x)^2[(1+x) + x]$

$= 3x^2(1+x)^2(1+2x)$

ii. $0 = 3x^2(1+x)^2(1+2x)$

$\therefore x = 0, -1, -\frac{1}{2}$