

NAME:.....

CLASS: .....

SYDNEY TECHNICAL HIGH SCHOOL

YEAR 12

HSC ASSESSMENT TASK 2

MARCH 2007

MATHEMATICS

Time Allowed : 70 minutes

Instructions:

- Write your name and class at the top of each page
- All necessary working must be shown. Marks may be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- Start each question on a new page.

Question 1	Question 2	Question 3	Question 4	Question 5	Total
/12	/12	/12	/12	/10	/58

QUESTION 1

Marks

a) Differentiate the following:

i)  $y = x^3 + 4x^2 + 2$

1

ii)  $y = \frac{3x}{x+2}$

2

iii)  $y = (2x+1)^4$

2

b) Find the gradient of the tangent to the curve  $y=4x^3 + x$  at the point ( 1, 5)

2

c) Find:

i)  $\int x^4 + 3x^2 dx$

1

ii)  $\int (x-5)(x+4) dx$

2

iii)  $\int \frac{x^3 - 3x^4}{x^2} dx$

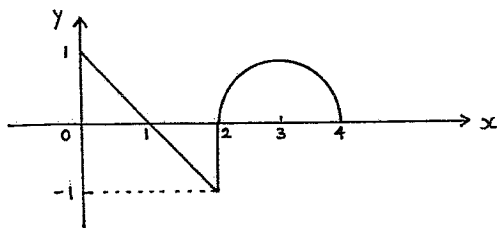
2

QUESTION 2

Marks

- a) Find the exact value of  $\int_0^4 f(x) dx$  given

2

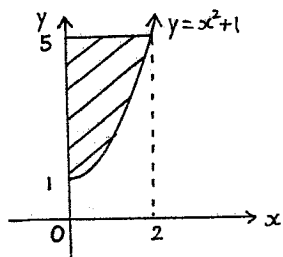


- b) Find  $\int (2x-1)^5 dx$

2

- c) The sketch shows an arc of the curve  $y = x^2 + 1$ .

3



Calculate the shaded area.

- d) The gradient of a curve is given by

$$\frac{dy}{dx} = 3x^2 - 12$$

- i) Find  $\frac{d^2y}{dx^2}$  1  
 ii) Find the values of  $x$  for which the curve both increases **and** is concave downwards. 2  
 iii) If the curve passes through  $(1, -2)$  find the equation of the curve. 2

QUESTION 3

Marks

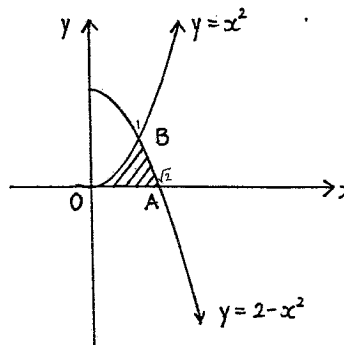
- a) Find the primitive function of  $\sqrt{x}$

2

- b) Melanie joined a Superannuation Fund, investing \$P at the beginning of every year at 8% p.a. compound interest (compounded yearly).

- i) Write an expression for the amount of her investment at the end of the first year. 1  
 ii) Write an expression for the amount of her investment at the end of the second year. 1  
 iii) Write an expression for the amount of her investment at the end of twenty five years 1  
 iv) If at the end of twenty five years, she wishes to collect \$500,000 calculate the value of \$P to the nearest dollar. 2

- c)



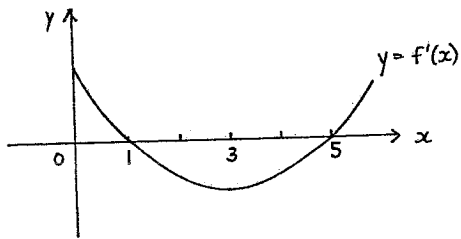
The shaded region OAB is bounded by the parabolas  $y = x^2$  and  $y = 2 - x^2$  and the  $x$  axis from  $x = 0$  to  $x = \sqrt{2}$ .

- i) B is the point of intersection of the two parabolas in the first quadrant. Find the co-ordinates of B. 2  
 ii) Calculate the area of the shaded region OAB (2 dp). 3

QUESTION 4

Marks

- a) A function is defined by  $y=3x^2-2x^3$
- i) Find the co-ordinates of any turning points and determine their nature 3
  - ii) Given that there is a point of inflexion, find its co-ordinates.  $y''=0$  1
  - iii) Sketch the function from  $x=-1$  to  $x=2$  2
- Note** Your sketch must be neat  
Use a ruler to draw the axes  
Label all important points
- iv) Find the area bounded by the curve  $y=3x^2-2x^3$  and the  $x$  axis from  $x=0$  to  $x=2$  4
- b) 2



The diagram shows the graph of the gradient function of the curve  $y=f(x)$ .  
For what value of  $x$  does  $f(x)$  have a local minimum?

Test continues over page ..

QUESTION 5

Marks

- a) A cylindrical container closed at both ends is made from a sheet of thin plastic.  
The surface area of the cylinder is  $600\pi$  centimetres<sup>2</sup>.
- i) Show that the height  $h$  of the cylinder is given by the expression: 2
- $$h = \frac{300}{r} - r, \text{ where } r \text{ is the radius.}$$
- ii) Find an expression for the volume  $V$  in terms of  $r$ . 1
  - iii) Find the height of the container if the volume is to be a maximum. 3
- b) i) Differentiate  $y=x^3(1+x)^3$  2
- ii) Hence, solve  $\frac{dy}{dx}=0$  2

End of Test

QUESTION 1	QUESTION 2
a) i. $y = x^3 + 4x^2 + 2$ $\frac{dy}{dx} = \underline{\underline{3x^2 + 8x}}$	a) $\int_0^4 f(x) dx = \frac{1}{2} \times \pi \times 1^2$ $= \underline{\underline{\frac{\pi}{2}}}$
ii. $y = \frac{3x}{x+2}$ $u = 3x \quad v = x+2$ $u' = 3 \quad v' = 1$ $\frac{dy}{dx} = \frac{3(x+2) - 3x(1)}{(x+2)^2}$ $= \frac{3x+6-3x}{(x+2)^2}$ $= \underline{\underline{\frac{6}{(x+2)^2}}}$	b) $\int (2x-1)^5 dx = \frac{(2x-1)^6}{6 \times 2} + c$ $= \underline{\underline{\frac{(2x-1)^6}{12} + c}}$
iii. $y = (2x+1)^4$ $\frac{dy}{dx} = 4(2x+1)^3 \times 2$ $= \underline{\underline{8(2x+1)^3}}$	c) $A = 5 \times 2 - \int_0^2 x^2 + 1 dx$ $= 10 - \left[ \frac{x^3}{3} + x \right]_0^2$ $= 10 - \left[ \frac{2^3}{3} + 2 - 0 \right]$ $= 10 - 4\frac{2}{3}$ $= \underline{\underline{5\frac{1}{3}}}$
b) $y = 4x^3 + x$ $\frac{dy}{dx} = 12x^2 + 1$ when $x=1$ , $m_{\text{tangent}} = 12 \times 1^2 + 1$ $= \underline{\underline{13}}$	d) $\frac{dy}{dx} = 3x^2 - 12$ i. $\frac{d^2y}{dx^2} = 6x$ ii. Increasing: $\frac{dy}{dx} > 0$ $3x^2 - 12 > 0$ $3(x+2)(x-2) > 0$ $x < -2, x > 2$ Concave down: $\frac{d^2y}{dx^2} < 0$ $6x < 0$ $x < 0$
c) i. $\int x^4 + 3x^2 dx$ $= \underline{\underline{\frac{x^5}{5} + x^3 + c}}$	$\therefore$ both increasing and concave down $\underline{\underline{x < -2}}$
ii. $\int (x-5)(x+4) dx$ $= \int x^2 - x - 20 dx$ $= \underline{\underline{\frac{x^3}{3} - \frac{x^2}{2} - 20x + c}}$	iii. $y = \int 3x^2 - 12 dx$ $y = x^3 - 12x + c$ when $x=1, y=-2$ $-2 = 1^3 - 12 \cdot 1 + c$ $c = 9$ $\therefore \underline{\underline{y = x^3 - 12x + 9}}$
iii. $\int \frac{x^3 - 3x^4}{x^2} dx$ $= \int x - 3x^2 dx$ $= \underline{\underline{\frac{x^2}{2} - x^3 + c}}$	

QUESTION 3	QUESTION 4
a) $\sqrt{x} = x^{\frac{1}{2}}$ primitive $= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$ $= \underline{\underline{\frac{2}{3} x^{\frac{3}{2}} + c}}$	a) $y = 3x^2 - 2x^3$ $\frac{dy}{dx} = 6x - 6x^2$ $\frac{d^2y}{dx^2} = 6 - 12x$
b) i. $A_1 = P(1.08)$ ii. $A_2 = P(1.08)^2 + P(1.08)$ $= P(1.08^2 + 1.08)$ iii. $A_{25} = P(1.08^{25} + 1.08^{24} + \dots + 1.08)$ iv. $A_{25} = \frac{P \times 1.08(1.08^{25} - 1)}{1.08 - 1}$ $500\,000 = \frac{P \times 1.08(1.08^{25} - 1)}{0.08}$ $P = \underline{\underline{\$6333}}$ (nearest dollar)	i. Stat pts: $\frac{dy}{dx} = 0$ $6x - 6x^2 = 0$ $6x(1-x) = 0$ $x = 0, 1$ When $x=0, y=0$ $\frac{d^2y}{dx^2} > 0$ $\therefore \underline{\underline{\text{min at } (0,0)}}$ When $x=1, y=1$ $\frac{d^2y}{dx^2} < 0$ $\therefore \underline{\underline{\text{max at } (1,1)}}$
c) i. $x^2 = 2 - x^2$ $2x^2 = 2$ $x^2 = 1$ $x = 1$ (1st quadrant) $y = 1^2$ $\therefore \underline{\underline{B(1,1)}}$	ii. Inflexion: $\frac{d^2y}{dx^2} = 0$ $6 - 12x = 0$ $12x = 6$ $x = \frac{1}{2}$ $\therefore \underline{\underline{\text{inflexion at } (\frac{1}{2}, \frac{1}{2})}}$
ii. $A = \int_0^1 x^2 dx + \int_1^{\sqrt{2}} 2 - x^2 dx$ $= \left[ \frac{x^3}{3} \right]_0^1 + \left[ 2x - \frac{x^3}{3} \right]_1^{\sqrt{2}}$ $= \left[ \frac{1^3}{3} - 0 \right] + \left[ 2\sqrt{2} - \frac{\sqrt{2}^3}{3} - \left( 2 \times 1 - \frac{1^3}{3} \right) \right]$ $= \frac{1}{3} + \left[ 2\sqrt{2} - \frac{2\sqrt{2}}{3} - \left( 2 - \frac{1}{3} \right) \right]$ $\therefore A = \underline{\underline{0.55}}$ (2 dp)	iii.

$$\text{iv. } A = \int_0^{\frac{3}{2}} 3x^2 - 2x^3 dx + \left| \int_{\frac{3}{2}}^2 3x^2 - 2x^3 dx \right|$$

$$= \left[ x^3 - \frac{x^4}{2} \right]_0^{\frac{3}{2}} + \left[ x^3 - \frac{x^4}{2} \right]_{\frac{3}{2}}^2$$

$$= \left[ \left(\frac{3}{2}\right)^3 - \frac{1}{2} \times \left(\frac{3}{2}\right)^4 - 0 \right] + \left[ \left[2^3 - \frac{2^4}{2}\right] - \left[\left(\frac{3}{2}\right)^3 - \frac{1}{2} \times \left(\frac{3}{2}\right)^4\right] \right]$$

$$= \frac{27}{32} + \left| -\frac{27}{32} \right|$$

$$\therefore A = \underline{\underline{1\frac{11}{16} u^2}} \quad (1.6875)$$

b)  $f'(x) = 0$  at  $x = 1$  and  $5$   
 i.e. stationary points on  $f(x)$

when  $x < 1$ ,  $f'(x) > 0$   
 i.e. increasing

when  $1 < x < 5$ ,  $f'(x) < 0$   
 i.e. decreasing

when  $x > 5$ ,  $f'(x) > 0$   
 i.e. increasing

$\therefore$  min when  $x = 5$

### QUESTION 5

a) i.  $SA = 2\pi r^2 + 2\pi rh$   
 $600\pi = 2\pi r^2 + 2\pi rh$   
 $300 = r^2 + rh$   
 $rh = 300 - r^2$   
 $h = \frac{300 - r^2}{r}$

$$\therefore h = \frac{300 - r}{r}$$

ii.  $V = \pi r^2 h$   
 $= \pi r^2 \left( \frac{300 - r}{r} \right)$

$$\therefore V = 300\pi r - \pi r^3$$

iii.  $\frac{dV}{dr} = 300\pi - 3\pi r^2$   
 $\frac{d^2V}{dr^2} = -6\pi r$   
 Max  $V$ :  $\frac{dV}{dr} = 0$   
 $300\pi - 3\pi r^2 = 0$   
 $3\pi(100 - r^2) = 0$   
 $r = 10$  only ( $r > 0$ )

when  $r = 10$ ,  $\frac{d^2V}{dr^2} < 0$   
 $\therefore$  max  $V$  when  $r = 10$

$$\therefore \text{height} = \frac{300 - 10}{10}$$

$$\therefore h = \underline{\underline{20 \text{ cm}}}$$

b) i.  $y = x^3(1+x)^3$

$$u = x^3 \quad v = (1+x)^3$$

$$u' = 3x^2 \quad v' = 3(1+x)^2$$

$$\frac{dy}{dx} = 3x^2(1+x)^3 + 3x^3(1+x)^2$$

$$= 3x^2(1+x)^2[(1+x) + x]$$

$$= 3x^2(1+x)^2(1+2x)$$

ii.  $0 = 3x^2(1+x)^2(1+2x)$

$$\therefore \underline{\underline{x = 0, -1, -\frac{1}{2}}}$$