SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS

HSC ASSESSMENT TASK 3

JUNE 2008

Time Allowed: 70 minutes

Instructions:

- Write using blue or black pen
- Approved calculators may be used
- Attempt all questions
- All necessary working must be shown. Mark may not be awarded for careless or badly arranged work
- Marks indicated are a guide only and may be varied if necessary
- Start each question on a new side of a page
- A table of standard integrals is supplied

Name:

Q1	Q2	Q3	Q4	Q5	Total

Question 1 **(11 marks)** **Marks**

a) Write $100°$ in radians in terms of π 1

b) Evaluate $log_{10}5$ correct to 3 significant figures 1

c) Find $lim \lim_{x\to 0} \frac{sin2x}{x}$ 2

d) Solve $cosx = \frac{\sqrt{3}}{2}$ for $0 \le x \le 2\pi$ 2

e) Sketch $y = 2\sin(\pi x)$ over the domain $0 \le x \le 2$ 2

f) If $\log_4 Y = 3.22$ evaluate $\log_4 4Y$ 2

g) Find the exact value of $sin\frac{7\pi}{4}$ 1

Question 2 **(11 marks)**

a) Differentiate with respect to x:

(i) $y = e^{3x}$ 1

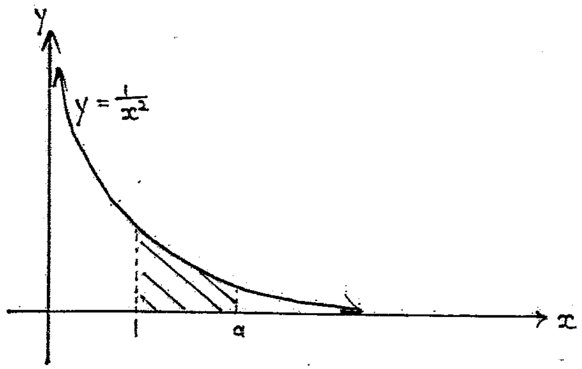
(ii) $y = \cos(1 - x^2)$ 2

(iii) $y = log_e \frac{x^2+1}{x}$ 2

(iv) $y = e^x sinx$ 2

(v) $y = 10^x$ 1

b) 3



The shaded area above is equal to $\frac{2}{3}\,unit^2$. Find a

Question 3 (11 marks)

a) Find

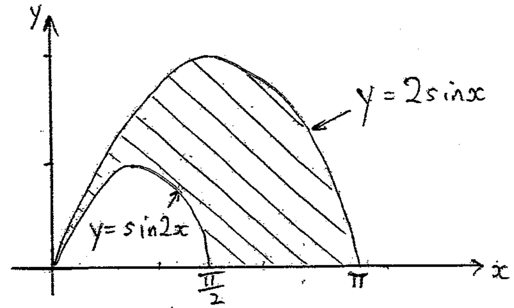
(i) $\int 2+\frac{3}{x}\ dx$ 1

(ii) $\int sec^2(6x+1)dx$ 1

(iii) $\int 3e^{2x}\,dx$ 1

(iv) $\int_{\frac{\pi}{2}}^{\pi} cos\frac{x}{2}dx$ (exact value) 3

b) Calculate the area of the shaded region below. 3



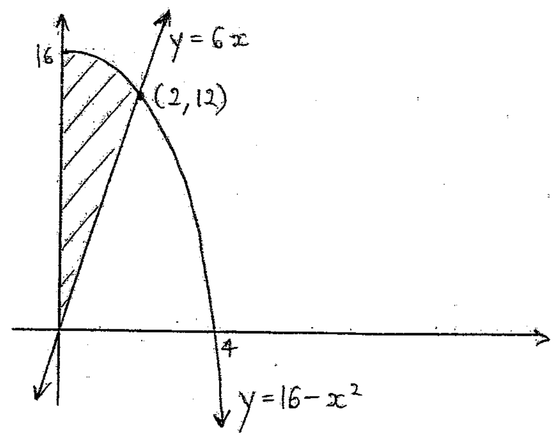
c) By writing $cosecx$ as $(\,sinx)^{-1}$.

Show that $\frac{d}{dx}(cosecx) = -cosecx\ cotx$ 2

Question 4 (11 marks)

a) Find $\int \sin\left(\frac{\pi}{4}-x\right)dx$ 2

b) 3



The region above is rotated around the y axis. Find the volume of the solid formed to the nearest whole number.

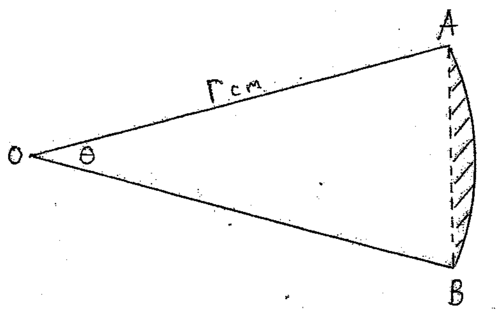
c) Evaluate $\int_0^{\frac{\pi}{3}} \frac{1+cos^3x}{cos^2x}dx$ 3

d) (i) Show that $\frac{d}{dx}(x\log_e x) = 1 + \log_e x$ 1

(ii) Hence find $\int \log_e x dx$ 2

Question 5 (11 marks) **Marks**

a) The sector OAB below has an area of $2\pi cm^2$. The arc has length $\frac{\pi}{2}\,cm$.

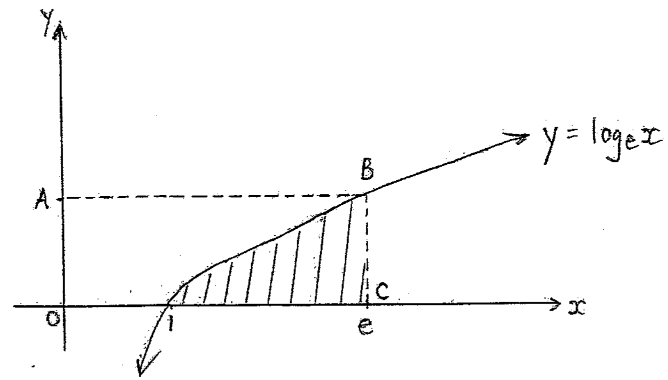


(i) Use this information to form 2 equations. 2

(ii) Hence solve these equations simultaneously to find r and θ 2

(iii) Now find the area of the minor segment shaded above correct to 2 decimal places 2

b)



(i) Using the graph above find the y value at point B 1

(ii) Hence find the area of rectangle ABCO. 1

(iii) Hence or otherwise find the shaded area. 3

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Solutions to 2008 Yr 12 2 Unit Ass. Task 3

Question 1

a) $100^\circ = 100 \times \frac{\pi}{180}$

$= \frac{5\pi}{9}$ radians (1)

b) $\log_{10} 5 = 0.6987$

$= 0.699$ to 3 sig. fig (1)

c) $\lim_{x \to 0} \frac{\sin 2x}{x}$

$= 2 \lim_{x \to 0} \frac{\sin 2x}{2x}$

$= 2$ since $\lim_{x \to 0} \frac{\sin x}{x} = 1$ (2)

d) $\cos x = \frac{\sqrt{3}}{2}$

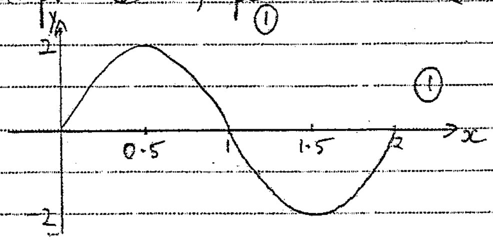
$x = \frac{\pi}{6}$ working angle

S | A ✓
T | C ✓

$x = \frac{\pi}{6}, \frac{11\pi}{6}$ (2)

e) $y = 2\sin \pi x$

amplitude $= 2$, period $= \frac{2\pi}{\pi} = 2$ (1)



(1)

f) $\log_4 4Y$

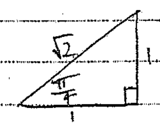
$= \log_4 4 + \log_4 Y$ (1)

$= 1 + 3.22$

$= 4.22$ (1)

g) $\sin \frac{7\pi}{4}$

$= -\sin \frac{\pi}{4}$ (1)



$= \frac{-1}{\sqrt{2}}$ (1)

Question 2

a) (i) $y = e^{3x}$

$y' = 3e^{3x}$ (1)

(ii) $y = \cos(1 - x^2)$

$y' = -2x \sin(1 - x^2)$ (2)

(iii) $y = \log_e \frac{x^2+1}{x}$

$y = \log_e(x^2+1) - \log_e x$

$y' = \frac{2x}{x^2+1} - \frac{1}{x}$ (2)

(iv) $y = e^x \sin x$

$y' = e^x \cos x + \sin x \, e^x$

$y' = e^x(\sin x + \cos x)$ (2)

(v) $y = 10^x$

$y' = 10^x \log_e 10$ (1)

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b) $\int_1^a \frac{1}{x^2}\, dx = \frac{2}{3}$ (1)

$\left[\frac{-1}{x}\right]_1^a = \frac{2}{3}$ (1)

$\frac{-1}{a} - \left(\frac{-1}{1}\right) = \frac{2}{3}$

$\frac{-1}{a} = \frac{-1}{3}$

$a = 3$ (1)

Question 3

a) (i) $\int 2 + \frac{3}{x}\, dx$

$= 2x + 3\log_e x + c$ (1)

(ii) $\int \sec^2(6x+1)\, dx$

$= \frac{1}{6}\tan(6x+1) + c$ (1)

(iii) $\int 3e^{2x}\, dx$

$= \frac{3}{2} e^{2x} + c$ (1)

(iv) $\int_{\frac{\pi}{2}}^{\pi} \cos \frac{x}{2}\, dx$

$= \int_{\frac{\pi}{2}}^{\pi} \cos \frac{1}{2}x\, dx$ (1)

$\left[2\sin \frac{1}{2}x\right]_{\frac{\pi}{2}}^{\pi}$ (1)

$2\left[\sin \frac{\pi}{2} - \sin \frac{\pi}{4}\right]$

$2\left(1 - \frac{1}{\sqrt{2}}\right)$ (1)

b) Area $= \int_0^{\pi} 2\sin x\, dx - \int_0^{\frac{\pi}{2}} \sin 2x\, dx$ (1)

$= \left[-2\cos x\right]_0^{\pi} - \left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}}$ (1)

$= (-2\cos \pi - -2\cos 0) - \left(\frac{-1}{2}\cos \pi - -\frac{1}{2}\cos 0\right)$

$= (2 + 2) - \left(\frac{1}{2} + \frac{1}{2}\right)$

$= 3$ (1)

c) $\frac{d}{dx}(\operatorname{cosec} x)$

$= \frac{d}{dx}(\sin x)^{-1}$

$= -(\sin x)^{-2} \times \cos x$ (1)

$= \frac{-\cos x}{\sin^2 x}$

$= \frac{-\cos x}{\sin x} \times \frac{1}{\sin x}$

$= -\operatorname{cosec} x \cot x$ (1)

Question 4

a) $\int \sin\left(\frac{\pi}{4} - x\right) dx$

$= -\cos\left(\frac{\pi}{4} - x\right) + c$ (1)

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b) $V = \pi\int_0^{12} x^2\,dy + \pi\int_{12}^{16} x^2\,dy$

$= \pi\int_0^{12} \left(\frac{y}{6}\right)^2 dy + \pi\int_{12}^{16} 16 - y\,dy$ ①

$= \frac{\pi}{36}\left[\frac{y^3}{3}\right]_0^{12} + \pi\left[16y - \frac{y^2}{2}\right]_{12}^{16}$ ①

$= \frac{\pi}{36}\left(\frac{1728}{3}\right) + \pi\left[256 - 128 - (192 - 72)\right]$ ①

$= 75 \text{ units}^3$ ①

c) $\int_0^{\frac{\pi}{3}} \frac{1 + \cos^3 x}{\cos^2 x}\,dx$

$\int_0^{\frac{\pi}{3}} \sec^2 x + \cos x\,dx$ ①

$\left[\tan x + \sin x\right]_0^{\frac{\pi}{3}}$ ①

$\tan\frac{\pi}{3} + \sin\frac{\pi}{3}$

$\sqrt{3} + \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$ ①

or $2\cdot6/2\cdot59$

d) (i) $\frac{d}{dx}(x\log_e x) = x \times \frac{1}{x} + \log_e x$

$= 1 + \log_e x$ ①

(ii) $\log_e x = \frac{d}{dx}(x\log_e x) - 1$

from part (i)

$\int \log_e x\,dx = \int \frac{d}{dx}(x\log_e x)\,dx - \int dx$ ①

$= x\log_e x - x + c$ ①

Question 5

(i) $A = \frac{1}{2}r^2\theta$

$2\pi = \frac{1}{2}r^2\theta$ ①

$l = r\theta$

$\frac{\pi}{2} = r\theta$ ①

(ii) $\theta = \frac{4\pi}{r^2}$ sub. into

$\frac{\pi}{2} = r\theta$

$\frac{\pi}{2} = r \times \frac{4\pi}{r^2}$

$r = 8\text{ cm}$ ① $\therefore \theta = \frac{\pi}{16}$ ①

(iii) $A = \frac{1}{2}r^2(\theta - \sin\theta)$ ①

$= \frac{1}{2} \times 8^2\left(\frac{\pi}{16} - \sin\frac{\pi}{16}\right)$

$= 0\cdot04\text{ cm}^2$ ①

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b) At B, $x = e$

$\therefore y = \log_e e$

$= 1$ ①

(ii) Area $= 1 \times e$

$= e$ ①

(iii) Area = Rectangle $- \int x\,dy$

$= e - \int_0^1 e^y\,dy$ ①

$= e - \left[e^y\right]_0^1$ ①

$= e - \left[e - e^0\right]$

$= e - e + 1$

$= 1 \text{ unit}^2$ ①