St Patrick's College Strathfield

MATHEMATICS & EXTENSION I MATHEMATICS

ASSESSMENT TASK 2

March 2008

TIME ALLOWED: 45 minutes

TOTAL POSSIBLE SCORE: 35 marks

INSTRUCTIONS:

- There are three questions in this paper
- Attempt all questions
- Diagrams are not necessarily drawn to scale
- All working must be shown
- The marks allocated for each question are in the right hand side column
- Marks may not be awarded for careless or badly arranged solutions
- A Table of Standard Integrals is provided.

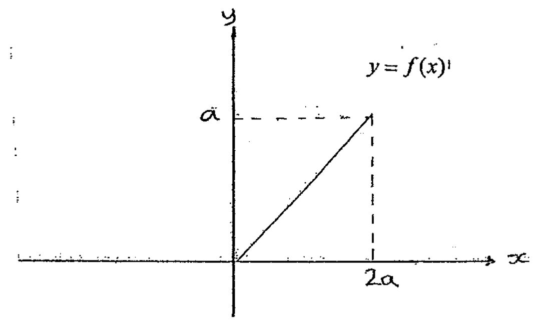
QUESTION 1 (13 marks) *Start a new page* **Marks**

(a) Find the primitive function of: $3x^2 - 3$ **1**

(b) It is given that: $f'(x) = 4x^3 + 4$ and $f(x) = 0$ when $x = 1$
Find an expression for $f(x)$ **2**

(c) The graph of the function $f(x)$ is defined and continuous in the interval $0 \le x \le 2a$.
This graph is represented in the diagram below.
Use an appropriate area formula to calculate the following integral.

$\int_0^{2a} f(x).dx$ Leave your answer in terms of a **2**



(d) (i) Complete this table for the function $y = \sqrt{x}$. Express your answers correct to one decimal place **2**

x	9	10	11	12	13	14	15	16
y	3		3.3		3.6			4

(ii) *Hence* use the Trapezoidal Rule with the *eight* function values above to estimate:
$\int_9^{16} \sqrt{x}.dx$ **2**

(e) Evaluate the following;

(i) $\int (x^{\frac{1}{3}} - 3x - \frac{1}{5\sqrt{x}}).dx$ **2**

(ii) $\int (\frac{4x^3 - 3x + 7}{x^3}).dx$ **2**

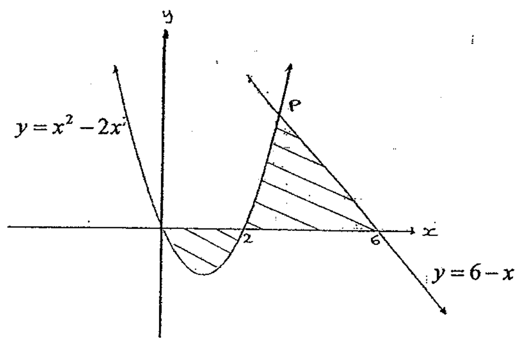
QUESTION 2 (12 marks) *Start a new page* **Marks**

(a) Evaluate the following;

$$\int_1^2 (x^{\frac{1}{2}} - x^{-\frac{1}{2}}).dx$$ 2

(b) Show $\int_0^5 \sqrt{3x+1}.dx = 14$ 3

(c) The diagram below shows the graph of the parabola $y = x^2 - 2x$ and the line $y = 6 - x$ The graphs intersect in the first quadrant at P.



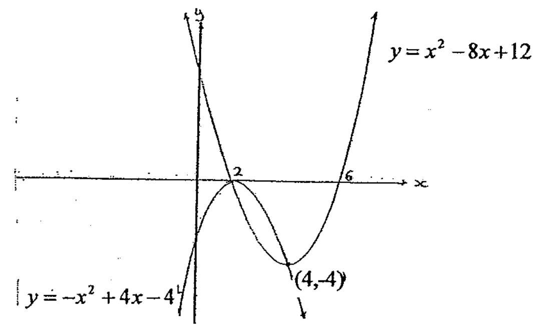
(i) Find the co-ordinates of P. 2

(ii) Find the magnitude (size) of this shaded area. 3

(d) The area bounded by the lines $y = 2x$, $x = 4$ and the co ordinate axes is rotated about the x axis.
Show that the exact volume of the solid of revolution is $\frac{256\pi}{3}$ *units*^{3} 2

QUESTION 3 (10 marks) *Start a new page* **Marks**

(a) The diagram below shows the curves $y = -x^2 + 4x - 4$ and $y = x^2 - 8x + 12$ meeting at the points (2, 0) and (4,-4).

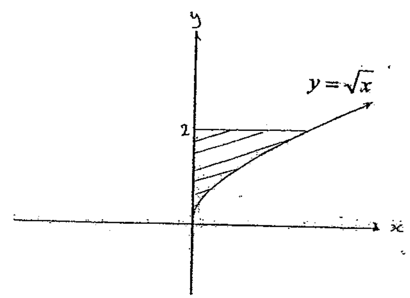


(i) Copy the above diagram on your answer sheet.

(ii) Shade the area bounded by the two curves. 1

(iii) Evaluate the magnitude (size) of this area. 3

(b) The diagram below shows the area bounded by the y axis, the curve $y = \sqrt{x}$ and the line $y = 2$



(i) Evaluate the shaded area. 3

(ii) Evaluate the volume when the same bounded area is rotated about the y axis. 3

END OF ASSESSMENT TASK

Q1. Solutions.

a) If $\frac{dy}{dx} = 3x^2 - 3$ H1 H9

$$y = \frac{3x^3}{3} - 3x + c$$

$$= x^3 - 3x + c \quad \checkmark$$

b) If $f'(x) = 4x^3 + 4$

$$f(x) = \frac{4x^4}{4} + 4x + c$$

$$= x^4 + 4x + c \quad \checkmark$$

$$f(1) = \frac{4(1)^4}{4} + 4(1) + c$$

$$0 = 1 + 4 + c$$

$$\therefore c = -5$$

$$f(x) = x^4 + 4x - 5 \quad \checkmark$$

c) $$\int_0^{2a} f(x).dx = \frac{1}{2} \times 2a \times a \quad \checkmark$$

$$= a^2 \quad \checkmark$$

d)

x	9	10	11	12	13	14	15	16
y	3	3.2	3.3	3.5	3.6	3.7	3.9	4

each correct $\frac{1}{2}$ mark H1 H4

ii) Use the trapezoidal rule with the eight function values above to estimate

$$\int_9^{16} \sqrt{x}.dx$$

2

$$\int_9^{16} \sqrt{x}.dx \approx \frac{1}{2}\left(3 + 4 + 2(3.2 + 3.3 + 3.5 + 3.6 + 3.7 + 3.9)\right) \quad \checkmark$$

$$\approx 24.7 \text{ units}^2 \quad \checkmark$$

Q1 Solutions continued.

e) i) $\int \left(x^{\frac{1}{3}} - 3x - \frac{1}{5\sqrt{x}}\right) \cdot dx$

$\int x^{\frac{1}{3}} - 3x - \frac{1}{5} x^{-\frac{1}{2}} \cdot dx$

$= \frac{x^{\frac{4}{3}}}{\frac{4}{3}} - \frac{3x^2}{2} - \frac{1}{5} \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= \frac{3x^{4/3}}{4} - \frac{3x^2}{2} - \frac{2x^{\frac{1}{2}}}{5} + C$ ✓✓

or

$\frac{3\sqrt[3]{x^4}}{4} - \frac{3x^2}{2} - \frac{2\sqrt{x}}{5} + C$ ✓✓

H1

ii) $\int \left(\frac{4x^3 - 3x + 7}{x^3}\right) dx$

$\int \left(4 - \frac{3}{x^2} + \frac{7}{x^3}\right) dx$

$\int (4 - 3x^{-2} + 7x^{-3}) dx$

$= 4x + 3x^{-1} - 7x^{-2} + C$ ✓

$4x + \frac{3}{x} - \frac{7}{x^2} + C$ ✓

H1

Q2

a) $\int_1^2 \left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right) \cdot dx$

$\left[\frac{2x^{\frac{3}{2}}}{3} - 2x^{\frac{1}{2}}\right]_1^2$ ✓

$\left(\frac{2 \cdot 2^{\frac{3}{2}}}{3} - 2 \cdot 2^{\frac{1}{2}}\right) - \left(\frac{2 \cdot 1^{\frac{3}{2}}}{3} - 2(1)^{\frac{1}{2}}\right)$

H1

$\left(\frac{2 \cdot \sqrt{8}}{3} - 2\sqrt{2}\right) - \left(\frac{2}{3} - 2\right)$

$= \left(\frac{4\sqrt{2}}{3} - 2\sqrt{2}\right) - \left(-\frac{4}{3}\right)$

$\frac{4\sqrt{2}}{3} - \frac{6\sqrt{2}}{3} + \frac{4}{3}$

$-\frac{2\sqrt{2}}{3} + \frac{4}{3} = \frac{4 - 2\sqrt{2}}{3}$ ✓

b) $\int_0^5 \sqrt{3x+1} \cdot dx = 14$

H1

H2

LHS $\int_0^5 (3x+1)^{\frac{1}{2}} \cdot dx = \left[\frac{(3x+1)^{\frac{3}{2}}}{\frac{3}{2} \times 3}\right]_0^5$ ✓

$= \left[\frac{2(3x+1)^{\frac{3}{2}}}{9}\right]_0^5 = \frac{2}{9} \times (16)^{\frac{3}{2}} - \left(\frac{2}{9} \times (1)^{\frac{3}{2}}\right)$

$= 14\frac{2}{9} - \frac{2}{9} = 14$ ✓✓

$=$ RHS

Question 2 continued.

c). i) $y = x^2 - 2x$, $y = 6 - x$

$\therefore x^2 - 2x = 6 - x$ H1

$x^2 - 2x + x - 6 = 0$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3 \quad x = -2$

We are looking for the point of intersection in the first quadrant $\therefore x = 3$ ✓

$y = 6 - 3$
$= 3$ ✓

$P(3,3)$

ii) $\int_2^3 (x^2-2x)\,dx + \int_3^6 (6-x).dx + \left|\int_0^2 (x^2-2x).dx\right|$ H1

$\left[\frac{x^3}{3} - x^2\right]_2^3 + \left[6x - \frac{x^2}{2}\right]_3^6 + \left|\left[\frac{x^3}{3} - x^2\right]_0^2\right|$ ✓ H8

$\left(\frac{(3)^3}{3} - (3)^2\right) - \left(\frac{(2)^3}{3} - (2)^2\right) + \left(\left(6(6) - \frac{(6)^2}{2}\right) - \left(6(3) - \frac{(3)^2}{2}\right)\right) + \left|\left(\frac{8}{3} - 4\right) - 0\right|$ ✓

$(9 - 9) - \left(\frac{8}{3} - 4\right) + \left((36 - 18) - \left(18 - \frac{9}{2}\right)\right) + \left|-\frac{4}{3}\right|$

$\frac{4}{3} + \frac{9}{2} + \frac{4}{3} = \frac{43}{6}$ units^2 ✓

Question 2 continued

C ii) students could use the area of a triangle to equate H1 H4

$\int_3^6 (6-x).dx =$

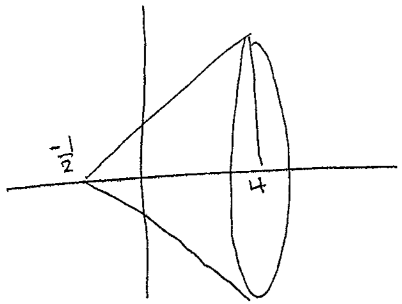
$6 - 3 = $ base

y co-ordinate $3 =$ height

Area $= \frac{1}{2} \times 3 \times 3 = 4\frac{1}{2}$

as per previous solution of $\frac{9}{2}$

Q2. d.



H1
H4
H8
H2

$y = 2x + 1$ When $y = 0$ $x = -\frac{1}{2}$

When $x = 0$ $y = -1$, $x = 4$ $y = 9$

$\therefore$ Vol of a cone with $\left.\begin{array}{l} r = 9 \\ h = 4\frac{1}{2} \end{array}\right\}$ ✓

$V = \frac{1}{3} \times \pi \times 9^2 \times 4\frac{1}{2}$

$= \frac{\pi}{3} \times 81 \times \frac{9}{2}$

$= \frac{729\pi}{6} = 121\frac{1}{2}\pi$

$= \frac{243\pi}{2}$ units^3 ✓

or

H1
H8
H2

$y = 2x+1$

$y^2 = 4x^2 + 4x + 1$

$\pi \int_{-\frac{1}{2}}^{4} y^2 \, dx = \pi \int_{-\frac{1}{2}}^{4} (4x^2 + 4x + 1) \, dx$

$= \pi \left[\frac{4x^3}{3} + \frac{4x^2}{2} + x \right]_{-\frac{1}{2}}^{4}$ ✓

$= \pi \left(\left(\frac{256}{3} + 32 + 4 \right) - \left(\frac{4}{3} \times -\frac{1}{8} + 2 - \frac{1}{2} \right) \right)$

$= \pi \times 121\frac{1}{2}$

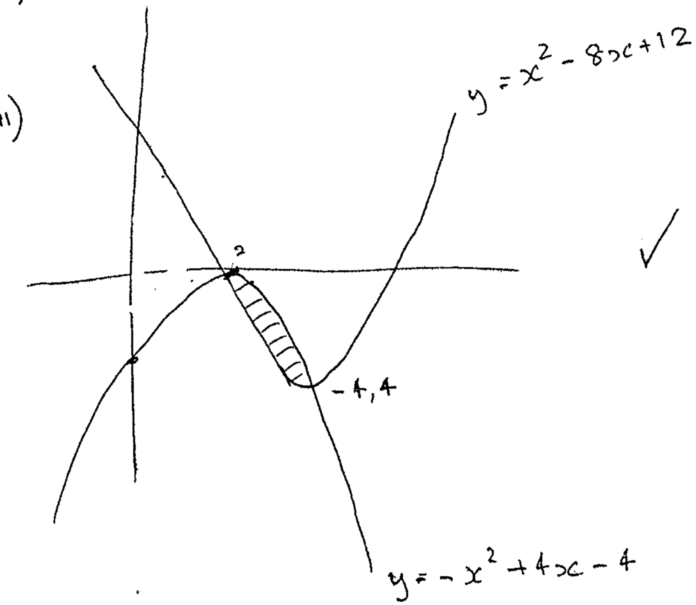
$= 243\pi$ units^3 ✓

Question 3

a)

H1
H8
H9

ii)



✓

iii) LHS

~~$\int_0^2 (x^2 - 8x + 12) \, dx - \int_0^2 (-x^2 + 4x - 4) \, dx$~~

~~can be~~

~~$\int_0^2 (x^2 - 8x + 12) - (-x^2 + 4x - 4) \, dx$~~

~~$\int_0^2 (2x^2 - 12x + 16) \, dx$~~

~~$\left[\frac{2x^3}{3} - 6x^2 + 16x \right]_0^2 = \left(\frac{2(2)^3}{3} - 6(2)^2 + 16(2) \right) - 0$~~

~~$= \frac{16}{3} - 24 + 32$~~

~~$= 13\frac{1}{3}$ units^2~~ ✓

Q3 continued

iii) RHS

$\int_2^4 (-x^2+4x-4)\,dx - \int_2^4 (x^2-8x+12).\,dx$

can be

$\int_2^4 \big((-x^2+4x-4)-(x^2-8x+12)\big)\,dx$

$\int_2^4 (-2x^2+12x-6).\,dx$

$\left[-\frac{2x^3}{3}+6x^2-16x\right]_2^4$ ✓

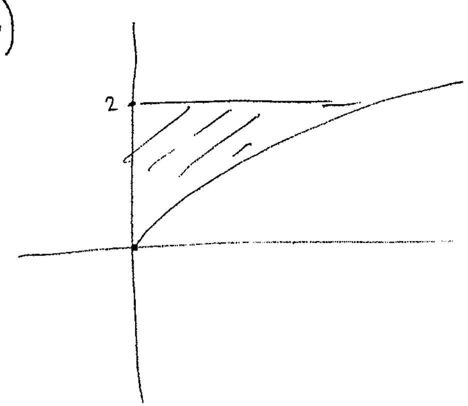
$= \left(-\frac{2(4)^3}{3}+6(4)^2-16(4)\right)-\left(-\frac{2(2)^3}{3}+6(2)^2-16(2)\right)$

$= \left(-\frac{128}{3}+96-64\right)-\left(-\frac{16}{3}+24-32\right)$

$-10\frac{2}{3}-\left(-13\frac{1}{3}\right) = 2\frac{2}{3}$ units^2 ✓

Question 5 continued.

b)



$\int_0^2 x \cdot dy$

$y = x^{1/2}$ by squaring both sides

$(y^2) = (x^{1/2})^2 \Longrightarrow x = y^2$

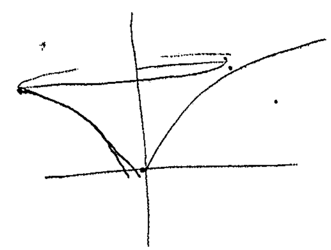
$\int_0^2 y^2 \cdot dy = \left[\frac{y^3}{3}\right]_0^2$ ✓

$= \left(\frac{(2)^3}{3} - 0\right)$

$= \frac{8}{3}$ units^2 ✓✓

H1
H8

ii)



$\pi\int_0^2 x^2 \cdot dy$

$x = y^2$

$x^2 = (y^2)^2$

$= y^4$ ✓

$\pi\int_0^2 y^4 \cdot dy = \pi\left[\frac{y^5}{5}\right]_0^2 = \pi\left(\frac{32}{5} - 0\right)$

$= 20.106$ units^3 $= \frac{32\pi}{5}$ units^3 ✓✓

H1
H8
H9