St Patrick's College Strathfield

MATHEMATICS & EXTENSION I MATHEMATICS

PRELIMINARY COURSE EXAMINATION

September 2007

TIME ALLOWED: 2 hours plus 5 minutes reading time

TOTAL POSSIBLE SCORE: 84 marks

INSTRUCTIONS:

- There are seven questions on this paper.
- Attempt all questions. All questions are of equal value
- Start each question in a separate booklet
- All working must be shown.
- The marks allocated for each question are clearly shown.
- Marks may not be awarded for careless or badly arranged work.
- Write your Student Number *only* on the front of each booklet.

Question 1 (12 marks) *Use a separate booklet* **Marks**

(a) Factorise $x^2-7x+12$ **1**

(b) Find correct to 3 significant figures places $\dfrac{5.31^2}{\sqrt{6.84-2.91}}$ **2**

(c) Solve for x and y where **3**

$$3x-2y=7 \quad \textit{and}$$
$$4x+3y=-2$$

(d) Solve the inequality below and sketch its solution on a number line: **2**

$$7-4x \le 35$$

(e) Rationalise the denominator of $\dfrac{2}{\sqrt{3}-1}$. Simplify your answer. **2**

(f) Solve $\dfrac{2}{5}-\dfrac{x-2}{3}=15$. **2**

Question 2 (12 marks) *Use a separate booklet* **Marks**

(a) If $\sqrt{45}+\sqrt{20}=\sqrt{a}$ find the value of a. **2**

(b) Factorise: x^3+27 **1**

(c) Simplify fully : $\dfrac{3}{x+1}+\dfrac{x-2}{x^2+6x+5}$ **2**

(d) A retailer increased the price for a pair of shoes by 6%. The new price of the shoes is \$132.50. What was the *old price*? **1**

(e) Show that the points $A(-4,5)$, $B(2,-3)$ and $C(-1,1)$ are collinear. **3**

(f) Shade the region of the number plane where the following three inequalities hold *simultaneously*:

$$y \le \sqrt{9-x^2}$$
$$x \ge -2$$
$$y \ge 0$$

3

Question 3 (12 marks) *Use a separate booklet.* **Marks**

(a) Write the exact value of: *tan* 210^0 **1**

(b) Write an expression in terms of x for the *exact* area of an equilateral triangle with a side length of x cm. **2**

(c) Solve for x where: $0^0 \le x \le 360^0$: $\cos x^\circ = \dfrac{\sqrt{3}}{2}$ **2**

(d) Solve for x: $|x-4|<2$ **2**

(e) State the natural domain *and* range of the function $y=\sqrt{x-2}$. **2**

(f) Solve the equation: $x-\dfrac{1}{3x}=2$.
Give your answer(s) correct to 3 decimal places. **3**

Question 4 (12 marks) *Use a separate booklet.* **Marks**

(a) A function $y=f(x)$ is given by the rule

$$f(x)=2x \text{ if } x \ge 2$$
$$=4 \text{ if } x<2$$

(i) Draw this graph from $x=-4$ to $x=4$ **2**

(ii) Evaluate $2f(5)+f(2)$ **1**

(b) Make neat sketches of the following equations on *separate* sets of axes.
Mark clearly, where appropriate, any points where the graphs cut the coordinate axes.

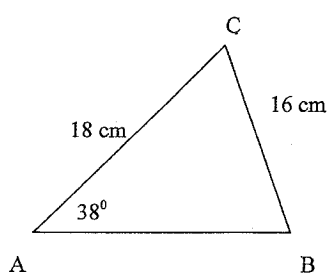
(i) $xy=2$ **1**

(ii) $y=25-x^2$ **2**

(iii) $x^2+y^2=25$ **2**

(iv) $y=2^x+1$ **2**

(c) In a triangle ABC, drawn below, $\angle CAB=38^0$, $BC=16\,cm$ and $AC=18\,cm$.



Find the size of $\angle ABC$ to the *nearest minute*. **2**

Question 5 (12 marks) *Use a separate booklet.* **Marks**

$A(2,-4)$, $B(6,-8)$ and $C(2,6)$ are the vertices of a triangle ABC.
M is the midpoint of BC.

(a) On a coordinate plane, plot the points A, B, C and M.
Write their coordinates besides them. **2**

(b) Through M, draw a line *parallel* to AC and intersecting AB at N. **1**

(c) Write down the equation of MN. **1**

(d) Show that the line MN bisects the interval AB. **2**

(e) Show that the equation of the line through A and M is:
$3x-2y-14=0$ **2**

(f) Find the perpendicular distance from B to the line AM. **2**

(g) Use your answer from (f) to find the area of triangle ABM. **2**

Question 6 (12 marks) *Use a separate booklet.* **Marks**

(a) Differentiate with respect to x

(i) $2x^3+3x+5$. **1**

(ii) $(x+5)(x^3-2)$ **2**

(iii) $\sqrt{2x-3}$ **2**

(iv) $\dfrac{2x+1}{x-2}$ **2**

(v) $\dfrac{1}{\sqrt{x}}$ **1**

(b) Consider the graph of the function: $y=(x^2+2)^3$

(i) Find the gradient of the tangent at the point $(1,27)$. **2**

(ii) Hence find the equation of the tangent at the point $(1,27)$. **2**
Express your answer in general form.

Question 7 (12 marks) *Use a separate booklet.* **Marks**

(a) The quadratic equation $2x^2+4x+1=0$ has roots α and β.
Without solving the equation, find the values of:

(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(b) Find all values of k for which the quadratic equation: $x^2+kx+(k+3)=0$ has:

(i) *real* and *distinct* roots. **3**

(ii) one root equal to 5 **2**

(c) The function $f(x)$ is given by: $f(x)=x^3+3x^2-9x+1$

(i) Find the coordinates of any stationary points on the graph of $y=f(x)$ and determine their nature. **3**

(ii) Draw a neat sketch of $y=f(x)$ *in the domain:* $-2\le x\le 4$. **2**

END OF EXAMINATION

Year 11 - Preliminary Exam

Question 1)

a) $x^2-7x+12=(x-4)(x-3)$

b) $=14.22305099$
$=14.2$ to 3 sf

c) $3x-2y=7$ —(1)
$4x+3y=-2$ —(2)

(1) × 3
(2) × 2
$9x-6y=21$ —(3)
$8x+6y=-4$ —(4)

(3) + (4)
$17x=17$
$\boxed{x=1}$

$3(1)-2y=7$
$-2y=4$
$\boxed{y=-2}$

d) $7-4x\le 35$
$7-35\le 4x$
$-28\le 4x$
$-7\le x$

e) $\frac{2}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$
$=\frac{2\sqrt{3}+2}{3-1}$
$=\frac{2\sqrt{3}+2}{2}$
$=\sqrt{3}+1$

f) $\frac{2}{5}-\frac{x-2}{3}=15$
$\frac{6-5x+10}{15}=15$
$-5x+16=225$
$-5x=209$
$x=-41.8$

Question 2)

a) $\sqrt{45}+\sqrt{20}=\sqrt{a}$
$3\sqrt{5}+2\sqrt{5}$
$=5\sqrt{5}$
$=\sqrt{125}\Rightarrow a=125$

b) x^3+27
$=x^3+3^3$
$=(x+3)(x^2-3x+9)$

c) $\frac{3}{x+1}+\frac{x-2}{x^2+6x+5}$
$=\frac{3}{(x+1)}+\frac{x-2}{(x+1)(x+5)}$
$=\frac{3(x+5)}{(x+5)(x+1)}+\frac{x-2}{(x+1)(x+5)}$
$=\frac{4x+13}{(x+5)(x+1)}$

P4 applies algebraic techniques

P3 performs routine arithmetic manipulations.

P4 applies algebraic techniques

P3 performs routine algebraic and arithmetic manipulations to surds and rational expressions
P4 uses appropriate algebraic techniques

P3 routine manipulations of surds

P4 applies appropriate algebraic techniques

P3 performs routine algebraic manipulation of rational expressions

c) $x + \frac{6}{100}x = 132.5$

$x(1 + \frac{6}{100}) = 132.5$

$1.06x = 132.5$

$x = 132.5 \div 1.06$

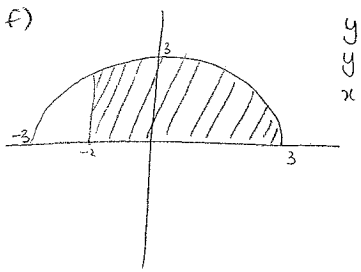
$x = \$125$

P1 obtain realistic solutions to problems

e) $m_{AB} = \frac{-3-5}{2+4} = \frac{-8}{6} = \frac{-4}{3}$ $\quad m_{AC} = \frac{1-5}{-1+4} = \frac{-4}{3}$ $\quad m_{BC} = \frac{1+3}{-1-2} = \frac{4}{-3}$ $\quad \therefore m_{AB} = m_{BC} = m_{AC}$ $\Rightarrow$ collinear

P4 applies appropriate geometric techniques

f)



$y \leqslant \sqrt{9-x^2}$

$y \geqslant 0$

$x \geqslant -2$

P5 understands the relationship between a function and its graph

Question 3)

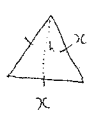
a) $\tan 210^\circ = \tan(180+30)$

$= \tan 30$

$= \frac{1}{\sqrt{3}}$

P4 applies appropriate trigonometric techniques

b)



$A = \frac{1}{2}bh$

$x^2 = h^2 + (\frac{1}{2}x)^2$

$x^2 = h^2 + \frac{1}{4}x^2$

$\frac{3}{4}x^2 = h^2$

$\Rightarrow h = \frac{\sqrt{3}}{2}x$

$\therefore A = \frac{1}{2}x(\frac{\sqrt{3}}{2}x)$

$= \frac{\sqrt{3}}{4}x^2$

P4 applies appropriate geometric or trigonometric techniques

c) $\cos x = \frac{\sqrt{3}}{2}$, $0 \leqslant x \leqslant 360^\circ$

$x = \cos^{-1}(\frac{\sqrt{3}}{2})$

$x = 30^\circ$ or 330°

P4 applies appropriate trigonometric techniques

d) $|x-4| < 2$

$x - 4 < -2$ or $x - 4 < 2$

$x < 2$ or $x < 6$

P4 applies appropriate algebraic techniques

e) $y = \sqrt{x-2}$

$x - 2 \geqslant 0$

$x \geqslant 2$ domain

$y \geqslant 0$ range

P5 understands the relationship between a function and its graph

f) $x - \frac{1}{3x} = 2$

$3x^2 - 1 = 6x$

$3x^2 - 6x - 1 = 0$

$x = \frac{6 \pm \sqrt{36 - 4(3)(-1)}}{6}$

$x = \frac{6 \pm \sqrt{36+12}}{6}$

$= \frac{6 \pm \sqrt{48}}{6}$

$= \frac{6 \pm 2\sqrt{12}}{6}$

$= \frac{6 \pm 4\sqrt{3}}{6}$

$= 1 \pm \frac{2}{3}\sqrt{3}$

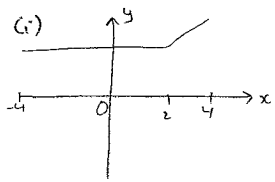
P3 performs routine manipulation of rational expressions

P4 applies appropriate algebraic techniques

Question 4)

a) $f(x) = \begin{cases} 2x, & x \geqslant 2 \\ 4, & x < 2 \end{cases}$

(i)



P5 understands the relationship between a function and its graph

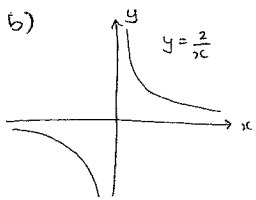
(ii) $2f(5) + f(2)$

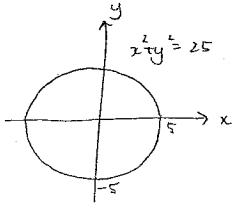
$= 2(10) + 4$

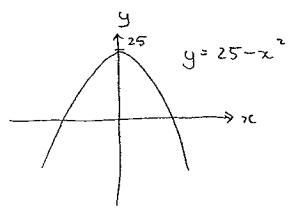
$= 24$

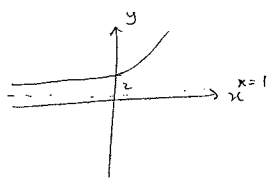
P3 performs routine arithmetic manipulation

b)









P5 understands the relationship between a function and its graph

c) $\frac{18}{\sin B} = \frac{16}{\sin 38}$

$\sin B = \frac{18 \sin 38}{16}$

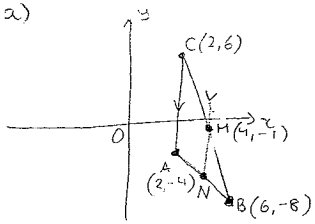
$B = \sin^{-1}(\frac{18 \sin 38^\circ}{16})$

$B = 43^\circ 50'$

P4 applies appropriate trigonometric techniques

Question 5)

a)



b)

c) $MN = 4$

d) For MN to bisect AB, the x-coordinate of N should be the same as the x coordinate of the midpoint of AB

$M_{AB} = \left(\frac{2+6}{2}, \frac{-4-8}{2}\right)$

$= (4, -6)$

∴ MN bisects AB as the x coordinates are both 4.

e) $M(4,-1)$ $\quad m_{AM} = \frac{-1+4}{4-2} = \frac{3}{2}$ $\quad \therefore$

$y+1 = \frac{3}{2}(x-4)$

$y+1 = \frac{3}{2}(x-4)$

$2y+2 = 3x-12$

$3x-2y-14 = 0$

f) $d = \frac{|3(6) + (-2)(-8) - 14|}{\sqrt{9+4}}$

$= \frac{|18+16-14|}{\sqrt{13}}$

$= \frac{20}{\sqrt{13}}$

g) $AM = \sqrt{(2-4)^2 + (-4+1)^2}$

$= \sqrt{4+9}$

$= \sqrt{13}$

$A = \frac{1}{2} \frac{20}{\sqrt{13}} \sqrt{13}$ $\quad (A = \frac{1}{2}bh)$

$= 10 \text{ units}^2$

Question 6)

a)

(i) $2x^3 + 3x + 5$

$\frac{dy}{dx} = 6x + 3$

(ii) $(x+5)(x^3-2)$

$u = x+5 \quad v = x^3-2$

$u' = 1 \quad v' = 3x^2$

$\frac{dy}{dx} = x^3 - 2 + 3x^3 + 15x^2$

$= 4x^3 + 15x^2 - 2$

(iii) $\sqrt{2x-3}$

$\frac{dy}{dx} = \frac{1}{2}(2x-3)^{-\frac{1}{2}} \cdot 2$

$= \frac{1}{\sqrt{2x-3}}$

(iv) $\frac{2x+1}{x-2}$

$u = 2x+1 \quad v = x-2$

$u' = 2 \quad v' = 1$

$\frac{dy}{dx} = \frac{2x-4-2x-1}{(x-2)^2}$

$= \frac{-5}{(x-2)^2}$

(v) $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

$\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{3}{2}}$

$= \frac{-1}{2\sqrt{x^3}}$

b) $y = (x^2+2)^3$

$\frac{dy}{dx} = 3(x^2+2)^2 \cdot 2x$

$= 6x(x^2+2)^2$

(i) $\left.\frac{dy}{dx}\right|_{x=1} = 6(1+2)^2$

$= 6(3)^2$

$= 6 \times 9$

$= 54$

(ii) $y - 27 = 54(x-1)$

$y = 54x - 54 + 27$

$y = 54x - 27$

$54x - y - 27 = 0$

P4 applies appropriate geometric techniques and algebraic techniques

P8 performs routine manipulations of rational expressions

P7 determines the derivative of a function through routine applications of the rules of differentiation

P9 understands the language and notation of calculus

P6 relates the derivative of a function to the slope of its graph

Question 7)

a) $2x^2 + 4x + 1 = 0$

(i) $\alpha + \beta = \frac{-4}{2}$

$= -2$

(ii) $\alpha\beta = \frac{1}{2}$

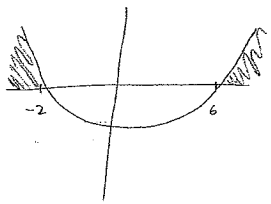
b) $x^2 + kx + (k+3) = 0$

(i) $b^2 - 4ac > 0$

$k^2 - 4(k+3) > 0$

$k^2 - 4k - 12 > 0$

$(k+2)(k-6) > 0$



$k < -2$ and $k > 6$

(ii) $(5)^2 + 5k + k + 3 = 0$

$25 + 5k + k + 3 = 0$

$28 + 6k = 0$

$6k = -28$

$k = -\frac{14}{3}$

c) $f(x) = x^3 + 3x^2 - 9x + 1$

(i) $f'(x) = 3x^2 + 6x - 9$

let $f'(x) = 0$

$0 = 3x^2 + 6x - 9$

$0 = x^2 + 2x - 3$

$0 = (x+3)(x-1)$

$\Rightarrow x = 1$ or -3

$A(1, -4)$ and $B(-3, 28)$

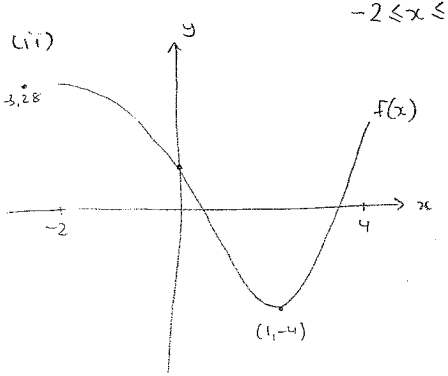
x	-4	-3	-2	0	1	2
$\frac{dy}{dx}$	+	0	−	−	0	+

$\therefore$ A is minimum

B is maximum

$-2 \leq x \leq 4$

(ii)



P4 applies appropriate algebraic techniques

P3 applies appropriate routine arithmetic and algebraic manipulation to simple rational expressions

P5 understands the relationship between a function and its graph

P6 relates the derivative of a function to the slope of its graph.

P7 determines the derivative of a function through applications of the rules of differentiation

P8 understands and uses the language and notation of calculus

P3 performs routine algebraic manipulation of rational expressions