Name: ..... Maths Class: .....

SYDNEY TECHNICAL HIGH SCHOOL

Year 11
Mathematics Extension 1

Preliminary Course

Assessment 1

May, 2015

Time allowed: 70 minutes

General Instructions:

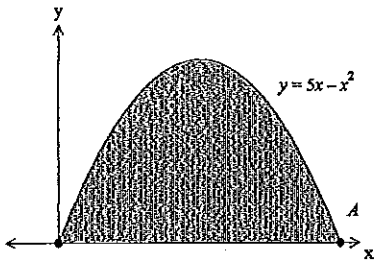
- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided

Section 1 Multiple Choice
Questions 1-5
5 Marks

Section II Questions 6-11
51 Marks

Section 1 – Multiple Choice – Answer on the sheet provided.

1 The diagram shows the graph of the function $y = 5x - x^2$.



Which pair of inequalities specify the shaded region?

(A) $y \le 5x - x^2$ and $y \le 0$.

(B) $y \le 5x - x^2$ and $y \ge 0$.

(C) $y \ge 5x - x^2$ and $y \le 0$.

(D) $y \ge 5x - x^2$ and $y \ge 0$.

2 What is the solution to the equation $|2x-5| = x+2$?

(A) $x = 1$

(B) $x = 7$

(C) $x = 1$ or $x = 7$

(D) $x = 1$ or $x = -7$

3 If $3\cos\theta + 2 = 0$ and $\tan\theta > 0$, what is the exact value of $\sin(\theta + 180°)$?

(A) $-\frac{\sqrt{5}}{3}$

(B) $-\frac{\sqrt{5}}{2}$

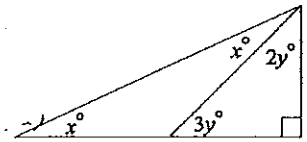
(C) $\frac{\sqrt{5}}{2}$

(D) $\frac{\sqrt{5}}{3}$

4 A woman is standing on level ground 70 metres from the base of a vertical cliff. If the angle of elevation to the top of the cliff is 40°, what is the height of the cliff, correct to the nearest metre?

(A) 58 metres
(B) 59 metres
(C) 60 metres
(D) 61 metres

5



What is the value of x?

(A) 18°
(B) 27°
(C) 36°
(D) 45°

End of section 1

SECTION II

(Start each new question on a new page)

QUESTION 6: (8 Marks)

Marks

(a) Fully factorise, $x^4 - xy^3$ — **2**

(b) Write down the exact value of $sin^2225^o + cosec150^o$ — **2**

(c) Solve for x: $27^x \times (\frac{1}{3})^{x-1} = 81$ — **2**

(d) State the Domain and Range of $y = \frac{2x+1}{x-2}$ — **2**

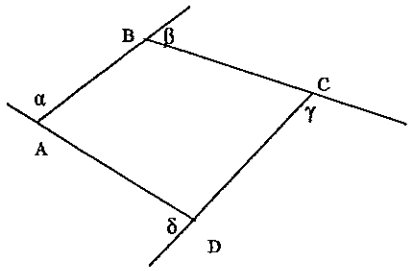
QUESTION 7: (8 Marks) *Start a new page*

Marks

(a) If $\tan\theta = p$ and $\sec\theta < 0$, find an expression for $\sin\theta$ — 2

(b) ABCD is a quadrilateral with external angles α, β, γ and δ. — 2

Explain why $\sin(\alpha+\beta+\gamma+\delta) = 0$

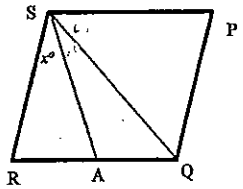


(c) PQRS is a rhombus. SA bisects ∠RSQ

∠RSA = x^o — 2

Prove: (i) ∠RSP = $4x^o$ — 2

(ii) ∠SAR = $3x^o$



QUESTION 8: (8 Marks) *Start a new page*

Marks

(a) Solve for θ, if $\sin 2\theta = \cos\theta$ and $0^o < \theta < 90^o$ — 1

(b) If $f(x) = \frac{1}{x}$ write $\frac{f(x+h)-f(x)}{h}$ as a simplified fraction. — 3

(c) If $f(x) = 2x - 3$, find a simplified expression for $f(f(-x))$ — 2

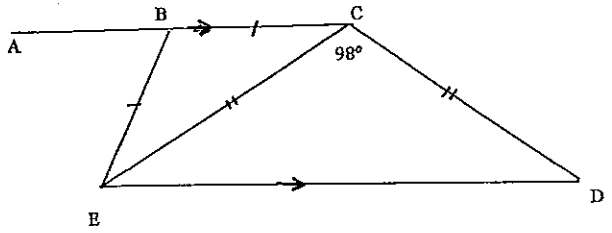
(d) Sketch the function $y = \frac{1}{\sqrt{4-x}}$ showing all necessary information. — 2

QUESTION 9: (8 Marks) *Start a new page*

Marks

(a) Solve $\sec\theta = -2$ for $-180^o \le \theta \le 180^o$ — 2

(b) Consider the quadrilateral BCDE where BC is parallel to ED and CB is produced to A, ∠ECD = 98^o, BC = BE and EC = CD



Copy the diagram showing all given information and find the size of angle ABE, giving reasons. — 3

(c) Solve the inequality $\frac{x-2}{x+3} > -2$ — 3

QUESTION 10: (10 Marks) *Start a new page*

Marks

(a) (i) Sketch the region $y \leq 6 - |2x|$ on a number plane **3**

(ii) Solve $6 - |2x| = |x|$ **2**

(iii) Find the area of the region held simultaneously by **2**
$y \leq 6 - |2x|$ and $y \geq |x|$

(b) Solve for θ, $2sin^2\theta = sin\theta cos\theta$, $0^o \leq \theta \leq 360^o$, correct to the nearest minute. **3**

QUESTION 11: (9 Marks) *Start a new page*

Marks

(a) Show that $sec\theta + tan\theta = \frac{cos\theta}{1-sin\theta}$ **3**

(b) (i) Sketch the function $f(x) = \frac{1}{x^2+1}$ **2**

(ii) On a separate number plane, sketch the function $y = -f(x) - 1$ **2**

(c) Solve $|x+2| + |x-2| = 6 - 4x$ **2**

End of Assessment task

IEHK
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2015

Extension 1 ~ Term 2

Solutions.

Section 1

1. B
2. C
3. D.
4. B
5. B

Section 2

Question 6

a) $x^4 - xy^3 = x(x-y)(x^2+xy+y^2)$

b) $(\sin 225°)^2 + \frac{1}{\sin 150°} = (-1/\sqrt{2})^2 + (1/\frac{1}{2})$

$= 1/2 + 2$

$= 2\frac{1}{2}$

c) $27^x \times (\frac{1}{3})^{x-1} = 81$

$3^{3x} \times 3^{-x+1} = 3^4$

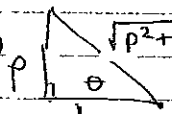
$3^{2x+1} = 3^4$

$2x = 3$

$x = 3/2$

d) $y = \frac{2x+1}{x-2}$ D: $x \in \mathbb{R}, x \neq 2$ R: $y \in \mathbb{R}, y \neq 2$

Question 7

a) 

$\sec\theta < 0$ ie 3rd quad

$\therefore \cos\theta < 0$

$\sin\theta = -p/\sqrt{p^2+1}$

b) exterior angles of a polygon equal 360°

$\therefore \alpha + \beta + \gamma + \delta = 360°$

ie $\sin(\alpha + \beta + \gamma + \delta) = \sin 360°$

$= 0$

c) i. $\angle RSA = x$ given

$\angle ASQ = \angle RSA$ (given SA bisects $\angle RSQ$)

$= x$

$\angle QSP = \angle RSQ$ (diagonal of a Rhombus bisects interior angles)

$= 2x$

$\therefore \angle RSP = \angle RSA + \angle ASQ + \angle QSP$ (sum of adjacent angles)

$= 4x°$

$\angle PSA = \angle PSQ + \angle QSA$ (adjacent angles)

$= 2x + x$

$\angle SAR = \angle PSA$ (alternate angles, SP // RQ opposite sides of Rhombus equal).

$= 3x°$

Question 8

a) $\sin 2\theta = \cos\theta$ as $\sin A = \cos(90 - A)$

then $2\theta + \theta = 90°$

$\theta = 30°$

b) $\frac{\frac{1}{x+h} - \frac{1}{x}}{h} \times \frac{(x+h)x}{(x+h)x}$

$= \frac{x - (x+h)}{h(x+h)x}$

$= \frac{-h}{h(x+h)x}$

$= \frac{-1}{x(x+h)}$

c) $f(x) = 2x - 3$ then $f(-x) = -2x - 3$

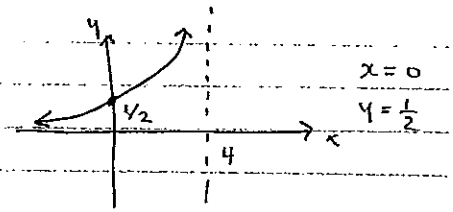
and $f[f(-x)] =$

$= 2[-2x - 3] - 3$

$= -4x - 6 - 3$

$= -4x - 9.$

$y = \frac{1}{\sqrt{4-x}}$ D: $x < 4$ R: $y > 0$

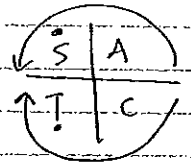


$x = 0$, $y = \frac{1}{2}$

Question 9

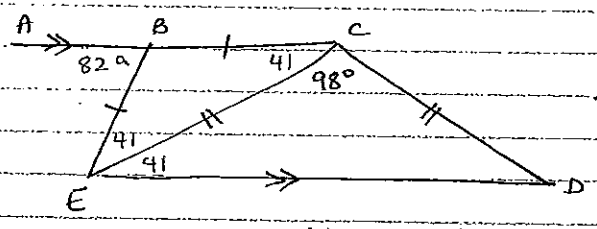
a) $\sec\theta = -2$ $\quad -180° \le \theta \le 180°$

$\cos\theta = -\frac{1}{2}$



$\therefore \theta = \pm 120°$

b)



$\angle CED = \angle CDE$ (base angles isosceles $\triangle CED$)

$\angle CED = 41°$ (angle sum $\triangle CED$)

$\angle BCE = \angle CED$ (alternate angles $AC \parallel ED$)

$= 41°$

$\angle BCE = \angle BEC$ (base angles of isosceles $\triangle BCE$, $BC = BE$)

$= 41°$

$\angle ABE = \angle BCE + \angle BEC$ (exterior angle $\triangle BCE$)

$= 82°$

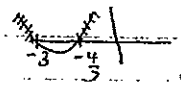
c) $\frac{x-2}{x+3} > -2 \qquad x \neq -3$

$(x-2)(x+3) > -2(x+3)^2$

$(x-2)(x+3) + 2(x+3)^2 > 0$

$(x+3)[(x-2) + 2(x+3)] > 0$

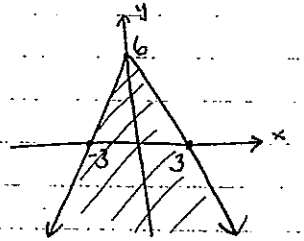
$(x+3)(3x+4) > 0$



$\therefore x < -3, \; x > -4/3$

Question 10

a. i. $y = 6 - |2x|$



ii. $y = |x|$ and $6 - |2x|$

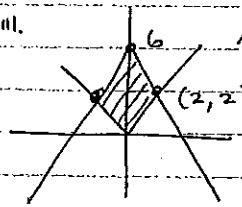
$6 - |2x| = |x|$

let $6 - 2x = x$ $\qquad 6 + 2x = -x$

$6 = 3x$ $\qquad 6 = -3x$

$x = 2$ $\qquad x = -2$

iii.



Area $= 2 \times \frac{1}{2} \times 2 \times 6$

$= 12 \text{ u}^2$

b. $2\sin^2\theta = \sin\theta\cos\theta \qquad 0° \le \theta \le 360°$

$2\sin^2\theta - \sin\theta\cos\theta = 0$

$\sin\theta(2\sin\theta - \cos\theta) = 0$

$\sin\theta = 0$ $\qquad$ ~~or~~ $\qquad 2\sin\theta = \cos\theta$

$\tan\theta = \frac{1}{2}$

$\theta = 0°, 180°, 360°$ $\qquad \theta = \tan^{-1}(\frac{1}{2}), \; 180 + \tan^{-1}(\frac{1}{2})$

$= 26°34', \; 206°34'$

Question 11

a. Show that $\sec\theta + \tan\theta = \frac{\cos\theta}{1-\sin\theta}$

$$\text{LHS} = \sec\theta + \tan\theta$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$

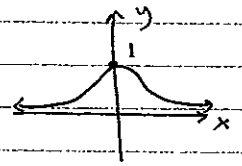
$$= \frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)}$$

$$= \frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$$

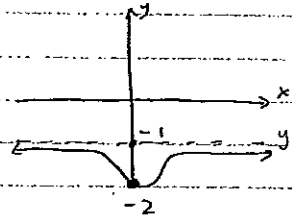
$$= \frac{\cos\theta}{1-\sin\theta}$$

$$= \text{RHS}$$

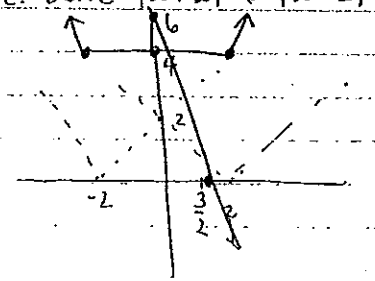
b. i $f(x) = \frac{1}{x^2+1}$



ii.



c. Solve $|x+2| + |x-2| = 6 - 4x$



∴ from the graph

Soln $6 - 4x = 4$

$4x = 2$

$x = 1/2$