



NAME _____

2007

Year 10

Pathway A

School Certificate Assessment Task 3

Wednesday, 6 June

Mathematics

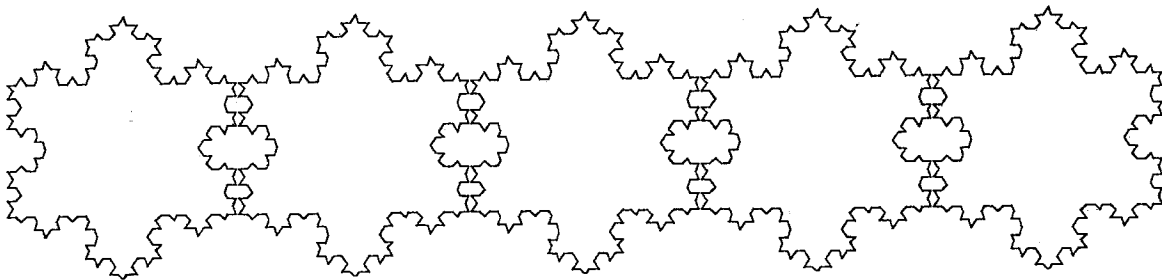
Weighting: 20%

Total marks: 30

Topics examined: Number
Measurement
Patterns and algebra

General Instructions

- Working time – 50 minutes
- Answer each question in the space provided for each answer. If you need more space you may write on the reverse of the page
- All necessary working should be shown in every question
- You may refer to your text book and your own grid book during this test



Total marks (30)

Please attempt all questions 1 - 5 on this paper.

REMEMBER TO SHOW WORKING FOR PART MARKS.

The value of each question or part of a question is shown inside the [].

Calculators ARE allowed.

1. A good way of saving money is to shop when merchandise is on sale i.e. when the original or marked price of an item is discounted. Sometimes two successive discounts are given. For example, a discount may be given because the item is discontinued and a further discount may be given on this reduced price, for paying cash.

Successive discounts of 70% and 30% are given on a computer with a marked price of \$1800.

(a) Does this mean the computer is now free? **Explain.**

[1]

(b) How much does the computer cost after both discounts? Show working.

[1]

(c) The **single** discount equivalent to successive discounts of $x\%$ and $y\%$ is $\left[x + y - \frac{xy}{100} \right] \%$.

Use this fact to find the single discount **equivalent** to successive discounts of 70% and 30%?

[1]

(d) Use your answer to part (c) to check your answer in part (b).

[1]

2.

(a) Nestor finds a rectangular sheet of metal with an area of 6 m^2 . Explain why this sheet is too small to make a **lidless** water tank in the shape of a cube, with a **volume** of at least 6 m^3 . [2]

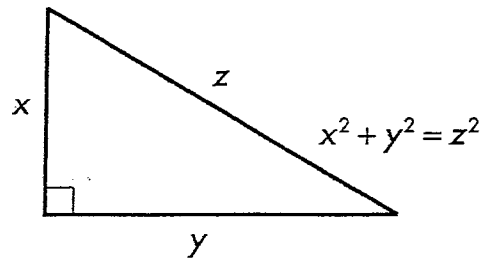
(b) Nestor finds another rectangular sheet of metal. The **breadth** of this sheet is given by $4x + 1$ and its **area** by $28x^2 - x - 2$. Find the **length** of this sheet in terms of x . [1]

(c) Nestor cuts square pieces from the corners of this second sheet and folds up the four flaps to make a lidless water tank. Find an algebraic expression for the **volume** of the tank if the **square** pieces cut from the corners of the sheet have area x^2 . [2]

(d) Calculate the **volume** of the tank formed in part (c) if $x = 1 \text{ m}$. [1]

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3. Pythagoras' theorem gives a relationship between the sides of right-angled triangles as follows.



An electrician needs to reach a spot-light that is high on a wall. He has a 5 m ladder with him.

(a) He places the ladder against the wall, with the bottom end of the ladder 3 m from the base of the wall. At what **height** does the top of the ladder touch the wall? [1]

(b) As he climbs onto the ladder, the foot (bottom end) of the ladder slips x m, away from the wall. The top of the ladder now touches the wall y m lower down. Show that the relationship between the length of the ladder, the distance of the foot of the ladder from the wall, and the height at which the ladder touches the wall is given by the expression $x^2 + y^2 = 8y - 6x$. [2]

(c) If the height at which the ladder touches the wall is written as $a - b$ and the distance of the foot of the ladder from the wall is written as $a + b$, show that the **length** of the ladder can be written as $\sqrt{2(a^2 + b^2)}$. [2]

4. A new dry-cleaning machine has been designed so that at the end of each cleaning cycle, the dry-cleaning liquid will be purified by evaporation and condensation. This will lead to a **loss** of two per cent of the dry-cleaning liquid **every** time the machine is used.

(a) When new, the machine is filled with 1000 mL of dry-cleaning liquid. How much liquid will remain after the machine has been used **once**? Show working. [1]

(b) Complete the table below. [2]

Number of uses (n)	2	3	4	5
Amount of liquid remaining (A)				

(c) Write a **formula** that can be used to find the amount of liquid (A) remaining after n uses. [1]

(d) The designers of the existing dry-cleaning machines claim that the new machine will lose all of the dry-cleaning liquid after 50 uses. Is this claim correct? Show working. [1]

(e) When the quantity of liquid is reduced to $\frac{1}{4}$ of the original amount it is time to replace the liquid. After how many uses will the liquid need to be replaced? [1]

SOLUTIONS

Total marks (30)
Please attempt all questions 1 - 5 on this paper.

REMEMBER TO SHOW WORKING FOR PART MARKS.

The value of each question or part of a question is shown inside the [].

Calculators ARE allowed.

1. A good way of saving money is to shop when merchandise is on sale i.e. when the original or marked price of an item is discounted. Sometimes two successive discounts are given. For example, a discount may be given because the item is discontinued and a further discount may be given on this reduced price, for paying cash.

Successive discounts of 70% and 30% are given on a computer with a marked price of \$1800.

(a) Does this mean the computer is now free? Explain. [1]

No. The successive discount are not accumulating in the way questiond. The second discount is applied after the first one and will result in a reduction of a quantity not the elimination of it. [1]

(b) How much does the computer cost after both discounts? Show working. [1]

After first discount the computer is worth \$1800 x 0.3 = \$540
After the second discount the computer is worth \$540 x 0.7 = \$378

(c) The single discount equivalent to successive discounts of x% and y% is $\left[x + y - \frac{xy}{100} \right]$ %.

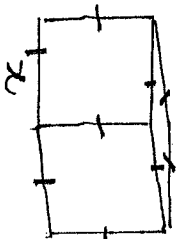
Use this fact to find the single discount equivalent to successive discounts of 70% and 30%? [1]

Single discount = $\left[70 + 30 - \frac{70 \times 30}{100} \right]$ %
= $79 \frac{1}{10}$ %

(d) Use your answer to part (c) to check your answer in part (b). [1]

\$1800 x $\frac{100 - 79}{100}$
= \$1800 x 0.21
= \$378

2. (a) Nestor finds a rectangular sheet of metal with an area of 6 m². Explain why this sheet is too small to make a lidless water tank in the shape of a cube, with a volume of at least 6 m³. [2]

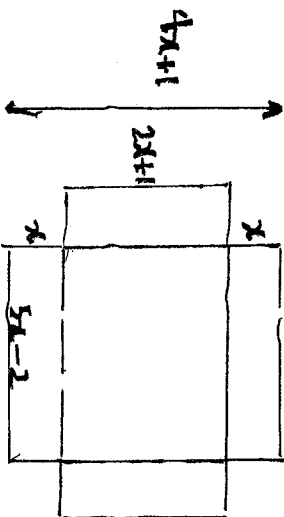


Let side length be x m clearly 6m³ of metal is not enough

(b) Nestor finds another rectangular sheet of metal. The breadth of this sheet is given by 4x + 1 and its area by 28x² - x - 2. Find the length of this sheet in terms of x. [1]

Area = length x breadth
Area = 28x² - x - 2
= (4x+1)(7x-2)
⇒ length = 7x-2

(c) Nestor cuts square pieces from the corners of this second sheet and folds up the four flaps to make a lidless water tank. Find an algebraic expression for the volume of the tank if the square pieces cut from the corners of the sheet have area x². [2]

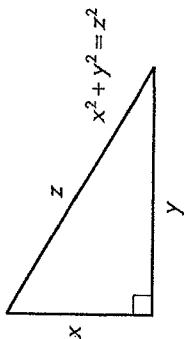


V = x x (2x+1) x (2x-2)
= x [10x² - 4x + 5x - 2]
= x (10x² + x - 2)
= 10x³ + x - 2x

(d) Calculate the volume of the tank formed in part (c) if x = 1 m. [1]

V = 10(1)³ + 1² - 2(1)
= 10 + 1 - 2
= 9 m³

3. Pythagoras' theorem gives a relationship between the sides of right-angled triangles as follows.

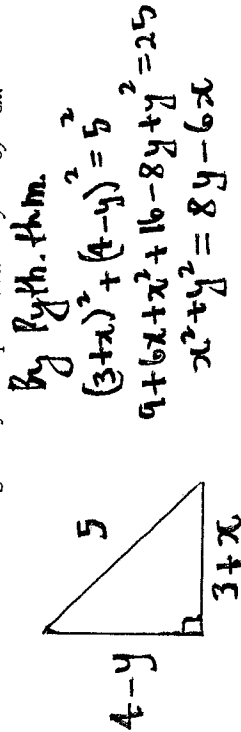


An electrician needs to reach a spot-light that is high on a wall. He has a 5 m ladder with him.

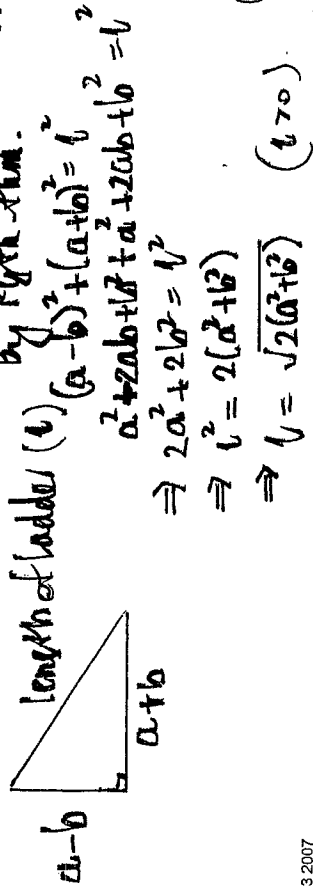
(a) He places the ladder against the wall, with the bottom end of the ladder 3 m from the base of the wall. At what height does the top of the ladder touch the wall? [1]



(b) As he climbs onto the ladder, the foot (bottom end) of the ladder slips x m, away from the wall. The top of the ladder now touches the wall y m lower down. Show that the relationship between the length of the ladder, the distance of the foot of the ladder from the wall, and the height at which the ladder touches the wall is given by the expression $x^2 + y^2 = 8y - 6x$. [2]



(c) If the height at which the ladder touches the wall is written as $a - b$ and the distance of the foot of the ladder from the wall is written as $a + b$, show that the length of the ladder can be written as $\sqrt{2(a^2 + b^2)}$. [2]



4. A new dry-cleaning machine has been designed so that at the end of each cleaning cycle, the dry-cleaning liquid will be purified by evaporation and condensation. This will lead to a loss of two per cent of the dry-cleaning liquid every time the machine is used.

(a) When new, the machine is filled with 1000 mL of dry-cleaning liquid. How much liquid will remain after the machine has been used once? Show working. [1]

After 1 use there will be $1000 \times 0.98 = 980$ mL

(b) Complete the table below. [2]

Number of uses (n)	2	3	4	5
Amount of liquid remaining (A)	980×0.98 $= 960.4$ mL	960.4×0.98 ≈ 941.2 mL	922.4 mL	903.9 mL

actually
941.142

actually
922.36816

actually
903.9207168

(c) Write a formula that can be used to find the amount of liquid (A) remaining after n uses. [1]

After n uses $A = 0.98^n \times 1000$

(d) The designers of the existing dry-cleaning machines claim that the new machine will lose all of the dry-cleaning liquid after 50 uses. Is this claim correct? Show working. [1]

After 50 uses $A = 0.98^{50} \times 1000$
 $= 364.1696801...$
 ≈ 364 mL

The claim is wrong as more than a third of the liquid will still remain

(e) When the quantity of liquid is reduced to $\frac{1}{4}$ of the original amount it is time to replace the liquid. After how many uses will the liquid need to be replaced? [1]

One-quarter of 1000 mL is 250 mL
Using guess and check $0.98^n \times 1000 = 250$
when $n = 68$ $A = 253.147...$ mL
when $n = 69$ $A = 248.084...$ mL