Name: _____ Class: _____

St George Girls High School

Year 11

Common Test – 1

2015

Mathematics Extension 1

General Instructions

- Time: 65 minutes (including reading time)
- Write using blue or black pen
- All questions should be attempted
- Show all working
- Marks will be deducted for careless work or poorly presented solutions.

Total marks - 65

Section I
Total marks (5)
Attempt Questions 1 – 5
Use the answer sheet provided

Section II
Total marks (60)
Attempt Questions 6 – 11
Start each question in a new booklet.
On the cover sheet of the answer booklet clearly show your name and your mathematics class and teacher

Section I

5 Marks
Attempt Questions 1 – 5

Use the multiple choice answer sheet provided for Questions 1 – 5.

1. $\left(x+\frac{1}{x}\right)^2 =$

   (A) $x^2+\frac{1}{x^2}=$

   (B) $x^2+1+\frac{1}{x^2}$

   (C) $x^2+2+\frac{1}{x^2}$

   (D) $\frac{x^2+1}{x^2}$

2. $1-2x \geq x+5$ then

   (A) $x \leq -2$

   (B) $x \leq -\frac{4}{3}$

   (C) $x \geq -2$

   (D) $x \geq -\frac{4}{3}$

3. If $x=\frac{4-3p}{5p-1}$ then $p=$

   (A) $\frac{4-x}{5x-3}$

   (B) $\frac{4+x}{5x-3}$

   (C) $\frac{4-x}{5x+3}$

   (D) $\frac{4+x}{5x+3}$

Section I (cont'd)

4. If $x = \log_2 8$ then

(A) $x^2 = 8$

(B) $x^8 = 2$

(C) $2^x = 8$

(D) ~~$2^x = 8$~~ $8^x = 2$

5. When solving $3x^2 - 7x + 6 = 0$ by completing the square mistakes were made. On which line did the **first** error occur?

(A) $3x^2 - 7x = -6$

(B) $x^2 - \frac{7}{3}x = -6$

(C) $x^2 + \frac{7}{3}x + \frac{49}{9} = -6 + \frac{49}{9}$

(D) $\left(x + \frac{7}{3}\right)^2 = -\frac{5}{9}$

(E) $x + \frac{7}{3} = \pm\frac{\sqrt{5}}{9}$

(F) $x = -\frac{7}{3} \pm \frac{\sqrt{5}}{9}$

Section II

60 marks
Attempt Questions 6 – 11
Start each question in a new booklet.

Question 6 (10 Marks) – Start A New Booklet Marks

a) Simplify:

(i) $\frac{2p+2p+2p}{2p \times 2p \times 2p}$ 1

(ii) $\frac{x^2-4}{2x-4}$ 1

b) If $P = 4x + 2y - 3$ and $Q = 2x - 3y + 5$ then write in simplest form $2P - 3Q$ 2

c) Draw a neat sketch of:

(i) $y = \sqrt{4 - x^2}$ 1

(ii) $y = \sqrt{4 - x}$ 1

d) Factorise $a^3 - (a + b)^3$ 2

e) The value of a certain fraction becomes $\frac{1}{5}$ if one is added to the numerator. If one is subtracted from the numerator of the original fraction its value becomes $\frac{1}{7}$. By forming suitable equations find the value of the original fraction. 2

Question 7 (10 Marks) – Start A New Booklet Marks

a) Which term of the sequence $T_n = 7n + 3$ is 458? 1

b) G.S.T. of 10% is added to the cost price of an item. The new price is now \$374. What was the amount of G.S.T. added? 1

c) Given $(2-3x)^3 = (2-3x)(2-3x)^2$ expand $(2-3x)^3$ and write your answer in simplest form. 2

d) Solve for x: $a^{2x+1} = \frac{1}{a^3}$. 1

e) Find $\sum_{n=1}^{4} 2(1.1)^n$ 2

f) Express as a single fraction with a rational denominator 3

$$\frac{5-\sqrt{3}}{2\sqrt{2}+1} - \frac{5\sqrt{2}}{\sqrt{5}+\sqrt{3}}$$

Question 8 (10 Marks) – Start A New Booklet Marks

a) If $4+\sqrt{x} = 4(1+\sqrt{3})$ then find the value of x. 1

b) Solve $3^x = 50$ correct to 3 decimal places. 2

c) What is the domain of the function $f(x) = \sqrt{x+2} + \sqrt{x-5}$ 1

d) State the range of $y = 1 + 2^x$ 1

e) Solve $\frac{|x-2|}{2-x} = x$ 2

f) For the arithmetic sequence $\log_a 250$, $\log_a 50$, $\log_a 10$

(i) Find the common difference. 1

(ii) Show that $T_n = \log_a \frac{1250}{5^n}$ 2

Question 9 (10 Marks) – Start A New Booklet — Marks

a) Simplify $\frac{x^2}{x^2+3x+2} - \frac{2x}{x+2}$ — 3

b) If $f(x) = x^2 + 5x - 3$ find the value of $\frac{f(x+h)-f(x)}{h}$ — 2

c) Draw a neat sketch of $y - 1 = \frac{1}{x+2}$ marking all relevant features. — 2

d) Solve $|x + 2| + |3x + 1| < 5$. — 3

Question 10 (10 Marks) – Start A New Booklet — Marks

a) Simplify $\frac{5^{n+1}-25}{5^n-5}$ — 2

b) Factorise $12y^4 + 20xy^2 - 8x^2$ — 2

c) Find the solutions of $y = \frac{11}{2} + \frac{3}{y}$ — 3

d) Solve $\frac{x+4}{x} \leq x + 1$ — 3

Question 11 (10 Marks) – Start A New Booklet Marks

a) Solve $3x^2 = 8(2x - 1)$ writing your answer in simplest surd form. 2

b) Given 2

$$f(x) = \begin{cases} 4 - x^2 & \text{if } x \geq 0 \\ x + 4 & \text{if } x < 0 \end{cases}$$

Sketch $y = f(x)$.

c) Find the first term and the common difference of an arithmetic sequence if $T_3 + T_4 = 32$ and $T_8 + T_9 = 72$. 3

d) Solve simultaneously 3

$$3x + 2y = 4$$
$$x^2 + xy - y^2 = 1$$

End of Paper

SOLUTIONS

Year 11 Mathematics Ext 1 Common Test 1 2015

Section I

1. $\left(x^2 + 2.x.\frac{1}{x} + \frac{1}{x^2}\right)$ (C)

$= x^2 + 2 + \frac{1}{x^2}$

2 $1 - 2x \geq x + 5$

$-3x \geq 4$ (B)

$x \leq -\frac{4}{3}$

3 $x = \frac{4 - 3p}{5p - 1}$

$5px - x = 4 - 3p$

$5px + 3p = 4 + x$ (D)

$p(5x + 3) = 4 + x$

$p = \frac{4 + x}{5x + 3}$

4. $x = \log_2 8$ (C)

$2^x = 8$

5. $3x^2 - 7x = -6$

$x^2 - \frac{7}{3}x = -2$ (B)

Question 6

(a) (i) $\frac{2p + 2p + 2p}{2p \times 2p \times 2p} = \frac{6p}{8p^3} = \frac{3}{4p^2}$

(ii) $\frac{x^2 - 4}{2x - 4} = \frac{(x-2)(x+2)}{2(x-2)} = \frac{x+2}{2}$

(b) $P = 4x + 2y - 3 \qquad Q = 2x - 3y + 5$

$2P - 3Q = 8x + 4y - 6 - 3(2x - 3y + 5)$

$= 8x + 4y - 6 - 6x + 9y - 15$

$= 2x + 13y - 21$

$\therefore y = \sqrt{4 - x^2}$

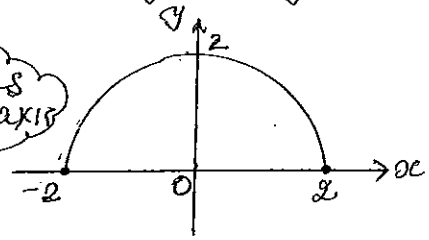
Domain : $4 - x^2 \geq 0$

$x^2 \leq 4$

$-2 \leq x \leq 2$

Range: $0 \leq y \leq 2$

When sketching circles the scale on x and y-axis must be the same!



(ii) $y = \sqrt{4 - x}$

Domain :

$4 - x \geq 0$

$-x \geq -4$

$x \leq 4$

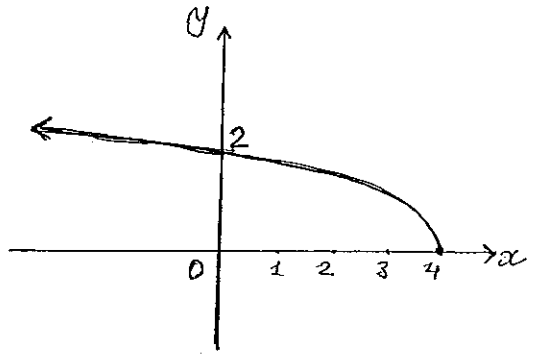
Range :

When $x = 4$ $y = 0$

As $x \to -\infty$ $y \to \infty$

y-intercept :

$x = 0$ $y = 2$



(d) $a^3 - (a+b)^3$

$= (a-(a+b))(a^2 + a(a+b) + (a+b)^2)$

$= -b(a^2 + a^2 + ab + a^2 + 2ab + b^2)$

$= -b(3a^2 + 3ab + b^2)$

(e) Let the original numerator be x and the original denominator be y. Then:

$\frac{x+1}{y} = \frac{1}{5}$ and $\frac{x-1}{y} = \frac{1}{7}$

$y = 5(x+1)$ and $y = 7(x-1)$

$y = 5x + 5$ ~① $y = 7x - 7$ ~②

M1: Substituting ① into ②:

$5x + 5 = 7x - 7$

$2x = 12$

$x = 6$

Substituting $x = 6$ into ①:

$y = 35$

M2: ② − ①:

$0 = 2x - 12$

$2x = 12$

$x = 6$

Sub $x = 6$ into 1:

$y = 35$.

∴ Original fraction is $\frac{6}{35}$

M3: The Best! Finding $\frac{x}{y}$ without finding x or y:

$\frac{x+1}{y} = \frac{1}{5}$ ~① $\frac{x-1}{y} = \frac{1}{7}$ ~②;

① + ②: $\frac{x+1}{y} + \frac{x-1}{y} = \frac{1}{5} + \frac{1}{7}$

$\frac{2x}{y} = \frac{12}{35} \Rightarrow \frac{x}{y} = \frac{6}{35}$ //

Question 7

a) $T_n = 7n + 3$

If $T_n = 458$ then

$458 = 7n + 3$

$455 = 7n$

$n = \frac{455}{7}$

$n = 65$

b) $374 \div 1.1 = 340$

∴ GST $= 374 - 340$

$= \$34$

Alternative methods

- $340 \times 0.1 = \$34$

or: $374 \div 11 = \$34$

c) $(2-3x)^3(2-3x)^2$

$= (2-3x)(4 - 12x + 9x^2)$

$= 8 - 24x + 18x^2 - 12x + 36x^2 - 27x^3$

$= -27x^3 + 52x^2 - 36x + 8$

d) $a^{2x+1} = a^{-3}$

$2x + 1 = -3$

$2x = -4$

$x = -2$

e) $\sum_{n=1}^{4} 2(1.1)^n = 2(1.1^1 + 1.1^2 + 1.1^3 + 1.1^4)$

$= 2 \times 5.1051$

$= 10.2102$

7f) $\frac{5-\sqrt{3}}{2\sqrt{2}+1} \times \frac{2\sqrt{2}-1}{2\sqrt{2}-1} - \frac{5\sqrt{2}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$= \frac{10\sqrt{2}-5-2\sqrt{6}+\sqrt{3}}{8-1} - \frac{(5\sqrt{10}-5\sqrt{6})}{5-3}$

$= \frac{10\sqrt{2}-5-2\sqrt{6}+\sqrt{3}}{7} - \frac{(5\sqrt{10}-5\sqrt{6})}{2}$

$= \frac{20\sqrt{2}-10-4\sqrt{6}+2\sqrt{3}-35\sqrt{10}+35\sqrt{6}}{14}$

$= \frac{20\sqrt{2}+2\sqrt{3}-35\sqrt{10}+31\sqrt{6}-10}{14}$

Question 8

a) $4+\sqrt{x} = 4+4\sqrt{3}$

$= 4+\sqrt{16\times3}$

$= 4+\sqrt{48}$

$\therefore x = 48$

b) $3^x = 50$

$\log_3 50 = x$

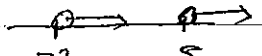
$x = \frac{\log_{10}50}{\log_{10}3}$ change of base

$= 3.5608767...$

$= 3.561$ (to 3 dp)

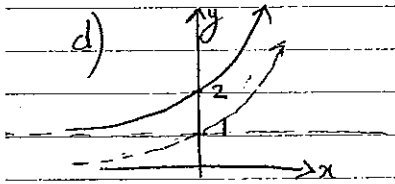
c) $x+2 \geq 0$ and $x-5 \geq 0$

$x \geq -2$ $x \geq 5$



$\therefore x \geq 5$

d)



Range is $y > 1$

8e) $\frac{|x-2|}{2-x} = x \quad , \quad x \neq 2$

Method 1

$x-2<0$	$x-2 \geq 0$
$x<2$	$x \geq 2$
$\frac{-(x-2)}{2-x} = x$	$\frac{x-2}{2-x} = x$
$\frac{2-x}{2-x} = x$	$\frac{-(2-x)}{2-x} = x$
$\therefore x = 1$	$x = -1$
Test $x=1$	Test $x=-1$
LHS $= \frac{1}{1}$	LHS $= 1$
$= 1$	RHS $= -1$
RHS $= 1$	Since LHS $\neq$ RHS
$\therefore x = 1$	no solution

$\therefore x = 1$

Method 2

Square both sides

$\frac{x^2-4x+4}{4-4x+x^2} = x^2$

$1 = x^2$

$x = \pm 1$

Test $x=1$ or $x=-1$ (no solution)

$\therefore x = 1$

8f) i) $\log_a 250$, $\log_a 50$, $\log_a 10$

$d = T_2 - T_1$ and $d = T_3 - T_2$

$= \log_a 50 - \log_a 250$ $= \log_a 10 - \log_a 50$

$= \log_a \frac{250}{50}$ $= \log_a \frac{10}{50}$

$= \log_a \frac{1}{5}$ $d = \log_a \frac{1}{5}$

$\therefore$ common difference is $\log_a \frac{1}{5}$

ii) $a = \log_a 25$

$T_n = a + (n-1)d$

$= \log_a 250 + (n-1)\left[\log_a \frac{1}{5}\right]$

$= \log_a 250 + n \log_a \frac{1}{5} - \log_a \frac{1}{5}$

$= \frac{\log_a 250}{\log_a \frac{1}{5}} + n \log_a \left(\frac{1}{5}\right)^n$

$= \log_a 1250 + \log_a 1 - \log_a 5^n$

$= \log_a 1250 + \log_a 5^n$

$= \log_a \frac{1250}{5^n}$

Question 9

a) $\dfrac{x^2}{(x+2)(x+1)} - \dfrac{2x}{x+2}$

$= \dfrac{x^2 - 2x(x+1)}{(x+2)(x+1)}$

$= \dfrac{x^2 - 2x^2 - 2x}{(x+2)(x+1)}$

$= \dfrac{-x^2 - 2x}{(x+2)(x+1)}$

$= \dfrac{-x(x+2)}{(x+2)(x+1)}$

$= \dfrac{-x}{x+1}$

b) $f(x) = x^2 + 5x - 3$

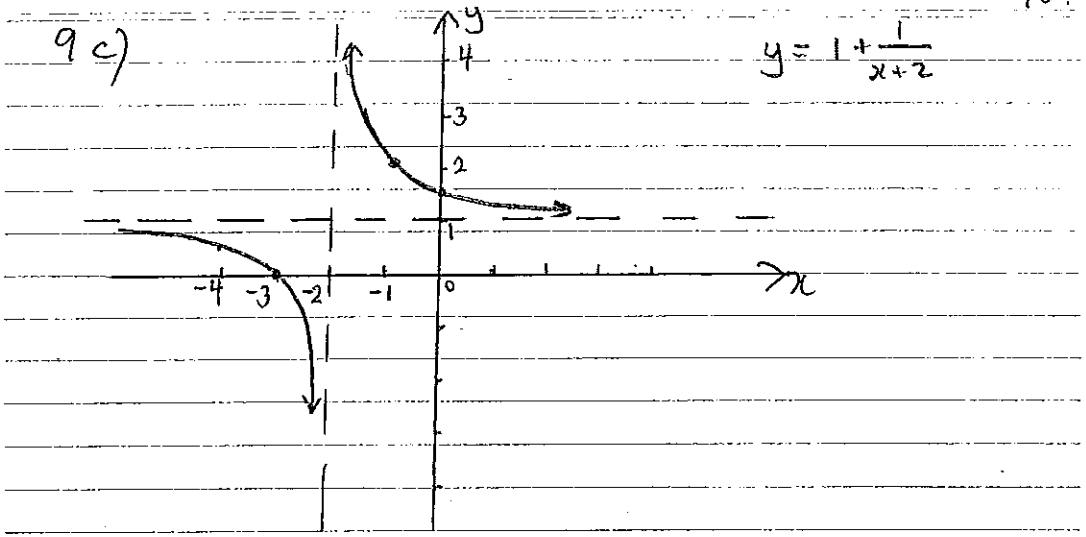
$\dfrac{f(x+h) - f(x)}{h} = \dfrac{(x+h)^2 + 5(x+h) - 3 - (x^2 + 5x - 3)}{h}$

$= \dfrac{x^2 + 2xh + h^2 + 5x + 5h - 3 - x^2 - 5x + 3}{h}$

$= \dfrac{h(2x + h + 5)}{h}$

$= 2x + h + 5$

9 c)

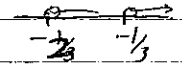


$y = 1 + \dfrac{1}{x+2}$

9 d) $x+2>0$ $\quad 3x+1>0$

$x>-2$ $\quad 3x>-1$

$x>-\frac{1}{3}$ $\quad \therefore x>-\frac{1}{3}$



$x+2+3x+1<5$

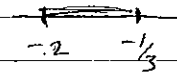
$4x+3<5$ ✓

$4x<2$

$x<\frac{1}{2}$ $\quad \therefore -\frac{1}{3}<x<\frac{1}{2}$

$x+2>0$ or $3x+1<0$

$x>-2$ $\quad x<-\frac{1}{3}$



$x+2-3x-1<5$ $\quad \therefore x>-2$ ✓

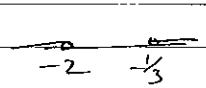
$-2x+1<5$

$-2x<4$

$x>-2$

$x+2<0$ or $3x+1>0$

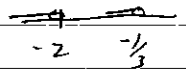
$x<-2$ $\quad x>-\frac{1}{3}$



no solution

$x+2<0$ $\quad 3x+1<0$

$x<-2$ $\quad x<-\frac{1}{3}$



$-(x+2)-(3x+1)<5$

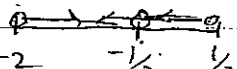
$-x-2-3x-3<5$

$-4x-5<5$

$-4x<0$

$x>0$ no solution

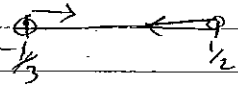
$\therefore$ Solution $-2<x<\frac{1}{2}$.



9 d) Alternative solution

For $|x+2|+|3x+1|<5$

$x<-2$	$-2<x<-\frac{1}{3}$	$x>-\frac{1}{3}$
$-(x+2)-(3x+1)<5$	$x+2-(3x+1)<5$	$x+2+3x+1<5$
$-x-2-3x-1<5$	$x+2-3x-1<5$	$4x+3<5$
$-4x-3<5$	$-2x+1<5$	$4x<2$
$-4x<8$	$-2x<4$	$x<\frac{1}{2}$
$x>-2$	$x>-2$	
At $x=-2$		
$\|0\|+\|-5\|<5$	$\therefore -2<x<-\frac{1}{3}$	
$5\not<5$		
$\therefore$ No solution		



$\therefore$ Solution

$-2<x<\frac{1}{2}$

Question 10

a) $\frac{5^{n+1}-5^2}{5^n-5} = \frac{5^n \cdot 5 - 5^2}{5^n-5}$

$= \frac{5^n 5 - 5\times 5}{5^n-5}$

$= \frac{5(5^n-5)}{5^n-5}$

$= 5$ (2)

b) $12y^4 + 20xy^2 - 8x^2$

$= 4(3y^4 + 5xy^2 - 2x^2)$

$= 4(3y^4 + 6xy^2 - xy^2 - 2x^2)$

$= 4[3y^2(y^2+2x) - x(y^2+2x)]$

$= 4(3y^2 - x)(y^2+2x)$ (2)

c) $y = \frac{11}{2} + \frac{3}{y}$

$2y^2 = 11y + 6$

$2y^2 - 11y - 6 = 0$

$2y^2 - 12y + y - 6 = 0$

$2y(y-6) + 1(y-6) = 0$

$(2y+1)(y-6) = 0$

$\therefore y = -\frac{1}{2}$ or $y = 6$ (3)

10d)

$x(x+4) \leq x^2(x+1)$

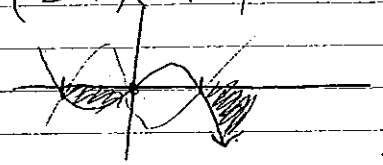
$x(x+4) - x^2(x+1) \leq 0$

$x[x+4 - x(x+1)] \leq 0$

$x[x+4 - x^2 - x] \leq 0$

$x(4 - x^2) \leq 0$

$x(2-x)(2+x) \leq 0$



Solution

$-2 \leq x < 0$ as $x \neq 0$

or

$x \geq 2$

(3)

Question 11

a) $3x^2 - 16x + 8 = 0$

$$x = \frac{16 \pm \sqrt{16^2 - 4 \times 3 \times 8}}{2 \times 3}$$

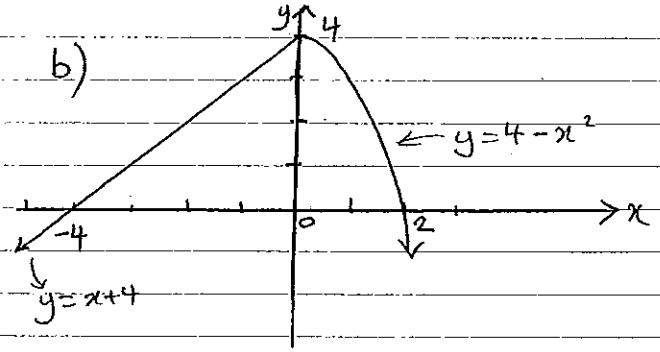
$$= \frac{16 \pm \sqrt{256 - 96}}{6}$$

$$= \frac{16 \pm \sqrt{160}}{6}$$

$$= \frac{16 \pm 4\sqrt{10}}{6}$$

$$= \frac{8 \pm 2\sqrt{10}}{3}$$

b)



c) $T_3 + T_4 = 32$

$a + 2d + a + 3d = 32$

$2a + 5d = 32$ ---①

$T_8 + T_9 = 72$

$a + 7d + a + 8d = 72$

$2a + 15d = 72$ ---②

② - ①

$10d = 40$

$d = 4$ sub in ①

$2a + 20 = 32$

$2a = 12$ $\therefore a = 6$ and $d = 4$

11 d) $3x + 2y = 4$ ---①

$x^2 + xy + -y^2 = 1$ ---②

From ① $y = \frac{4 - 3x}{2}$ ---③

sub in ②

$$x^2 + x\left(\frac{4-3x}{2}\right) - \left(\frac{4-3x}{2}\right)^2 = 1$$

$$4x^2 + 2x(4-3x) - (16 - 24x + 9x^2) = 4$$

$$4x^2 + 8x - 6x^2 - 16 + 24x - 9x^2 = 4$$

$$-11x^2 + 32x - 20 = 0$$

$$11x^2 - 32x + 20 = 0$$

$$11x^2 - 22x - 10x + 20 = 0$$

$$11x(x-2) - 10(x-2) = 0$$

$$(x-2)(11x-10) = 0$$

$x = 2$ or $x = \frac{10}{11}$

when $x = 2$, sub in ③

$y = \frac{4-6}{2}$

$y = -1$

when $x = \frac{10}{11}$, sub in ③

$$y = \frac{4 - 3\left(\frac{10}{11}\right)}{2}$$

$$= \frac{4 - \frac{30}{11}}{2}$$

$y = \frac{7}{11}$

$\therefore x = 2, y = -1$

or

$x = \frac{10}{11}, y = \frac{7}{11}$