St George Girls High School

Year 11

Common Test – 1

2010

Mathematics Extension 1

Marks: 72

Instructions
1. Reading time – 5 minutes
2. Working time – 65 minutes
3. All questions should be attempted.
4. Show all working.
5. **Start each question on a new page.**
6. Marks will be deducted for careless work or poorly presented solutions.
7. On the cover sheet of the answer booklet clearly show:

   a) your name
   b) your mathematics class and teacher

Question 1: (12 Marks) – Start A New Page Marks

a) Factorise

(i) $a^2 - 2a + 1 - b^2$ 2

(ii) $4x^4 - 12x^2 - 16$ 3

b) Simplify:

(i) $\frac{x^2-x}{3x^2+3x^3} \div \frac{x^2-1}{6x}$ 2

(ii) $\frac{a}{a^2+ab} + \frac{b}{b^2+ab}$ 2

(iii) $\frac{(k+1)^3-(k-1)^3}{k+3k^3}$ 2

c) Make x the subject 1

$$y = 1 + \frac{3}{x}$$

Question 2: (12 Marks) – Start A New Page — Marks

a) Solve for y — 2

$$\frac{y-3}{y+1}=\frac{y+1}{y+2}$$

b) Solve simultaneously: — 4

$x+3y+2z=13$

$x+y+z=6$

$x-2y-z=-5$

c) Solve the following problem by forming and solving a suitable quadratic equation:

A rectangular vegetable patch 9 m by 6 m is to be surrounded by a border of flowers of uniform width. The area of the border of flowers should equal the area of the vegetable patch.

Find the width of the border. — 6

Question 3: (12 Marks) – Start A New Page — Marks

a) Solve for x — 2

$$3(2-5x)\leq 36$$

b) Solve by formula, leaving your answer in simplest surd form: — 3

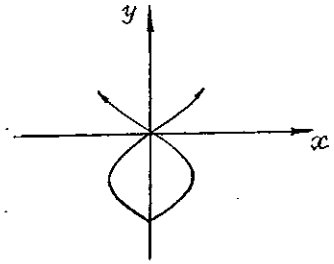
$$2y^2-6y+3=0$$

c) For $\left(a+b\sqrt{5}\right)^2=45-20\sqrt{5}$ — 5

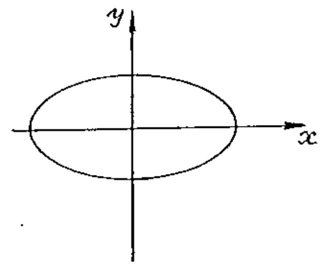
Find the values of a and b, given that they are rational.

d) Which of the following curves represent functions $y=f(x)$? — 2

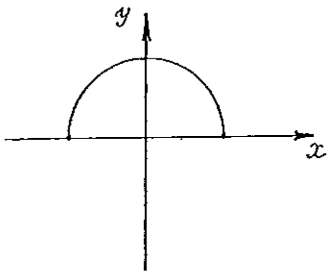
(A)



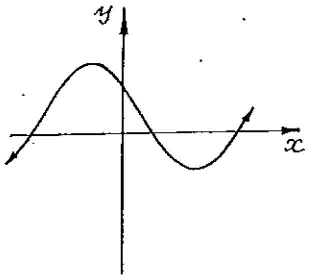
(B)



(C)



(D)



Question 4: (8 Marks) – Start A New Page Marks

a) If $A = \{n: n \text{ is an integer}, -5 \leq n \leq 5\}$

$B = \{1, 3, 5, 7, 9\}$

and the Universal Set $E = \{\text{all integers from } -10 \text{ to } 10\}$

List the elements

(i) $A \cap B$ 1

(ii) $A \cup B$ 1

(iii) $\overline{A \cup B}$ 1

b) If $n(A) = 17,\ n(B) = 20$ 1

and $n(A \cap B) = 9$

Find $n(A \cup B)$

c) (i) What is the natural domain of the function $y = \sqrt{9 - x^2}$ 2

(ii) What is the range of the above function? 1

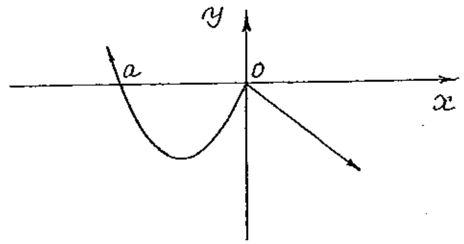
d) (i) Factor the quadratic $3 - 2x - x^2$ 5

(ii) Find the intercepts and vertex of the parabola $y = 3 - 2x - x^2$.

(iii) Sketch the parabola showing all important features.

Question 5: (12 Marks) – Start A New Page Marks

a) Below is the graph of $y = f(x)$



On separate sets of axes sketch the graphs of:

(i) $y = -f(x)$ 1

(ii) $y = f(-x)$ 1

b) Find the centre and the radius of the circle 3

$$x^2 + y^2 - 2x + 6y + 1 = 0$$

c) Given that $f(x) = 2x^2 - 3x + 5$

(i) Evaluate $f(-4)$ 1

(ii) Find $\frac{f(x+a) - f(x)}{a}$ in the simplest form. 3

d) Find the inverse function $y = f^{-1}(x)$ if 3

(i) $f(x) = 4x + 1$

(ii) $f(x) = \frac{5x-2}{x+3}$

Question 6: (12 Marks) – Start A New Page **Marks**

a) What would be a possible restriction on the domain of $y = (x+3)^2$ so that its inverse relation is a function? 2

b) Find the exact value of 3

$$x^2 + \frac{1}{x^2} \quad \text{if} \quad x = \frac{1+2\sqrt{3}}{1-2\sqrt{3}}$$

Express your answer as a simplified fraction with rational denominator.

c) A square number is divisible by 5 if and only if the square root of that number is divisible by 5.

Without proving the above, prove that $\sqrt{5}$ is irrational. 3

d) If $a = 5\sqrt{3} - 7$ and $b = 5 - 2\sqrt{3}$ show that $a > b$. 4

Do not use your calculator to find decimal approximations for any irrational numbers in your solution.

End of Paper

Question 1

a) (i) $1+a^2-2a-b^2$

$= a^2-2a+1-b^2$

$= (a-1)^2-b^2$

$= ((a-1)-b)(a-1+b)$

(ii) $4x^4-12x^2-16$

$= 4(x^4-3x^2-4)$

$= 4(x^2-4)(x^2+1)$

$= 4(x-2)(x+2)(x^2+1)$

(iii) $(k+1)^3 = k^3+3k^2+3k+1$

$(k-1)^3 = k^3-3k^2+3k-1$

$(k+1)^3-(k-1)^3 = 6k^2+2$

$= 2(3k^2+1)$

$\dfrac{(k+1)^3-(k-1)^3}{k+3k^2}$

$= \dfrac{2(3k^2+1)}{k\cdot(1+3k^2)}$

$= \dfrac{2}{k}$

b) (i) $\dfrac{x^2-x}{3x^2+3x^3} \div \dfrac{x^2-1}{6x}$

$= \dfrac{x(x-1)}{3x^2(1+x)} \times \dfrac{6x}{(x-1)(x+1)}$

$= \dfrac{2}{(x+1)^2}$

(ii) $\dfrac{a}{a^2+ab} + \dfrac{b}{b^2+ab}$

$= \dfrac{a}{a(a+b)} + \dfrac{b}{b(a+b)}$

$= \dfrac{1}{a+b} + \dfrac{1}{a+b}$

$= \dfrac{2}{a+b}$

(c) $y = 1+\dfrac{3}{x}$

$y-1 = \dfrac{3}{x}$

$\dfrac{1}{y-1} = \dfrac{x}{3}$

$x = \dfrac{3}{y-1}$

Question 2

(a) $\dfrac{y-3}{y+1} = \dfrac{y+1}{y+2}$

$(y-3)(y+2) = (y+1)^2$

$y^2-y-6 = y^2+2y+1$

$-3y = 7$

$y = -\dfrac{7}{3}$

(b) $x+3y+2z = 13$ ①

$x+y+z = 6$ ②

$x-2y-z = -5$ ③

① − ② $\quad 2y+z = 7$ ④

② − ③ $\quad 3y+2z = 11$ ⑤

④ × 2 $\quad 4y+2z = 14$ ⑥

⑥ − ⑤ $\quad y = 3$

Subst in ④ $\quad 6+z = 7$

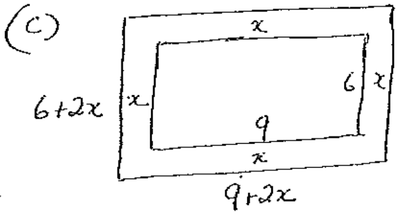
$z = 1$

Subst in ② $\quad x+3+1 = 6$

$x = 2$

$(x,y,z) = (2,3,1)$

(c)



Let x m be width of border

Let A m^2 be area of bord

$A = (2x+9)(2x+6) - 9\times 6$

$= 4x^2+30x+54-54$

$= 4x^2+30x$

If $A = 54$ then

$4x^2+30x-54 = 0$

$2x^2+15x-27 = 0$

$(2x-3)(x+9) = 0$

$x = \dfrac{3}{2}, -9$

Since x is a length $x>0$

$\therefore x = \dfrac{3}{2}$

$\therefore$ Width of border is 1.5 m

Question 3

(a) $3(2-5x) \le 36$

$2-5x \le 12$

$-5x \le 10$

$x \ge -2$

(b) $2y^2-6y+3=0$

$b^2-4ac = (-6)^2-4\times2\times3$

$= 36-24$

$= 12$

$y = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{6 \pm \sqrt{12}}{4}$

$= \dfrac{6 \pm 2\sqrt{3}}{4}$

$= \dfrac{3 \pm \sqrt{3}}{2}$

(c) $(a+b\sqrt{5})^2 = 45-20\sqrt{5}$

$a^2+5b^2+2ab\sqrt{5} = 45-20\sqrt{5}$

$a^2+5b^2 = 45$ — ①

$2ab = -20$

$b = \dfrac{-10}{a}$ — ②

Subst ② in ①

$a^2 + \dfrac{500}{a^2} = 45$

$a^4 + 500 = 45a^2$

$a^4 - 45a^2 + 500 = 0$

$(a^2-20)(a^2-25) = 0$

$a^2 = 20, 25$

$a = \pm\sqrt{20}, \ \pm 5$

Since a is rational

$a = 5, -5$

$b = -2, 2$

$a = 5, b = -2$ or

$a = -5, b = 2$

(d) C and D are graphs of functions

Question 4

(a) $A = \{n : n \text{ is an integer}, -5 \le n \le 5\}$

$B = \{1, 3, 5, 7, 9\}$

$E = \{\text{integers } -10 \text{ to } 10\}$

(i) $A \cap B = \{1, 3, 5\}$

(ii) $A \cup B = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 7, 9\}$

(iii) $\overline{A \cup B} = \{-10, -9, -8, -7, -6, 6, 8, 10\}$

(c) (i) $y = \sqrt{9-x^2}$

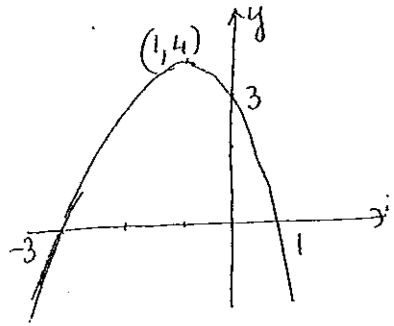
$D: -3 \le x \le 3$

(ii) $R: 0 \le y \le 3$

(d) (i) $y = -x^2-2x+3$

$= 3-2x-x^2$

$= (3+x)(1-x)$



(ii) When $y=0$ $x = -3, 1$

$x = 0$ $y = 3$

Vertex : $x = -\dfrac{3+1}{2} = -1$

$y = -(-1)^2 - 2(-1) + 3$

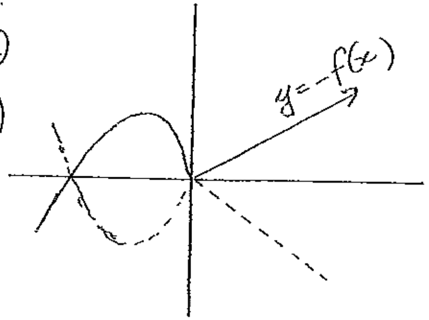
$= 4$

$(-1, 4)$

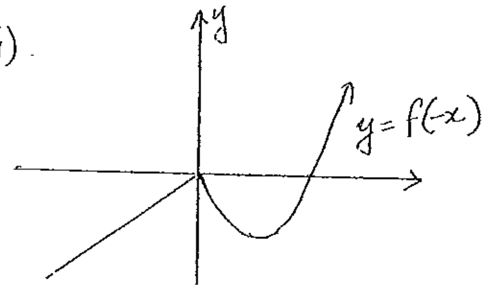
Question 5

(a)

(i)



(ii)



(b) $x^2+y^2-2x+6y+1=0$

$x^2-2x+1+y^2+6y+9=-1+1+9$

$(x-1)^2+(y+3)^2=9$

Centre $(1,-3)$

Radius $=3$

(c) $f(x)=2x^2-3x+5$

(i) $f(-4)=2(-4)^2-3(-4)+5$

$=32+12+5$

$=49$

(ii) $f(x+a)=2(x+a)^2-3(x+a)+5$

$=2(x^2+2ax+a^2)-3x-3a+5$

$=2x^2+4ax+2a^2-3x-3a+5$

$f(x)=\cancel{2x^2-3x+5}$

$\dfrac{f(x+a)-f(x)}{a}$

$=\dfrac{4ax+2a^2-3a}{a}$

$=\dfrac{a(4x+2a-3)}{a}$

$=4x+2a-3$

(d) (i) $f(x)=4x+1$

$y=4x+1$

Inverse: $x=4y+1$

$y=\dfrac{x-1}{4}$

$f^{-1}(x)=\dfrac{x-1}{4}$

(ii) $y=\dfrac{5x-2}{x+3}$

Inverse:

$x=\dfrac{5y-2}{y+3}$

$xy+3x=5y-2$

~~$bxy+5y=$~~

$3x+2=5y-xy$

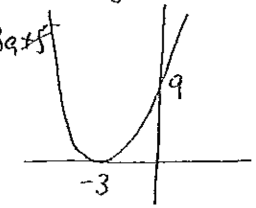
$=y(5-x)$

$y=\dfrac{3x+2}{5-x}$

$f^{-1}(x)=\dfrac{3x+2}{5-x}$

Question 6

(a) $y=(x+3)^2$



$x\le -3$ or $x\ge -3$

OR

[$x\le a$ where $a\le -3$
or
$x\ge a$ where $a\ge -3$]

(b) $x=\dfrac{1+2\sqrt3}{1-2\sqrt3}\times\dfrac{1+2\sqrt3}{1+2\sqrt3}$

$=\dfrac{1+4\sqrt3+4\times3}{1-4\times3}$

$=\dfrac{13+4\sqrt3}{-11}$

$\dfrac{1}{x}=\dfrac{1-2\sqrt3}{1+2\sqrt3}\times\dfrac{1-2\sqrt3}{1-2\sqrt3}$

$=\dfrac{1-4\sqrt3+12}{1-4\times3}$

$=\dfrac{13-4\sqrt3}{-11}$

$x^2+\dfrac{1}{x^2}=\left(\dfrac{13+4\sqrt3}{-11}\right)^2+\left(\dfrac{13-4\sqrt3}{-11}\right)^2$

$=\dfrac{169+2\times13\times4\sqrt3+16\times3}{121}+\dfrac{169-2\times13\times4\sqrt3+16\times3}{121}$

$=\dfrac{2\times(169+48)}{121}$

$=\dfrac{434}{121}$

OR $x^2+\dfrac{1}{x^2}=\left(x+\dfrac1x\right)^2-2\times x\times\dfrac1x$

$=\left(x+\dfrac1x\right)^2-2$

$x+\dfrac1x=\dfrac{1+2\sqrt3}{1-2\sqrt3}+\dfrac{1-2\sqrt3}{1+2\sqrt3}$

$=\dfrac{(1+2\sqrt3)^2+(1-2\sqrt3)^2}{1-4\times3}$

$=\dfrac{1+4\sqrt3+12+1-4\sqrt3+12}{-11}$

$=\dfrac{26}{-11}$

$x^2+\dfrac{1}{x^2}=\left(\dfrac{26}{-11}\right)^2-2$

$=\dfrac{676}{121}-2$

$=\dfrac{434}{121}$

(c) Assume $\sqrt{5}$ is rational

ie $\sqrt{5} = \frac{p}{q}$ where p and q are integers that have no common factors and $q \neq 0$

$\therefore \quad 5 = \frac{p^2}{q^2}$

$p^2 = 5q^2$ — ①

ie 5 is a factor of p^2

Hence 5 is a factor of p.

Let $p = 5k$ where k is an integer

Subst into ①

$(5k)^2 = 5q^2$

$25k^2 = 5q^2$

$q^2 = 5k^2$

$\therefore$ 5 is a factor of q^2 and hence 5 is a factor of q, meaning that p and q have a common factor

This contradicts the original assumption

Hence $\sqrt{5}$ cannot be expressed in form $\frac{p}{q}$, where p & q are integers with no common factors

$\therefore \sqrt{5}$ is irrational

(d) $a - b = (5\sqrt{3} - 7) - (5 - 2\sqrt{3})$

$= 7\sqrt{3} - 12$

$= \sqrt{49 \times 3} - \sqrt{144}$

$= \sqrt{147} - \sqrt{144}$

> 0

$\therefore \quad a > b.$