Student _____

BRIGIDINE COLLEGE RANDWICK

PRELIMINARY
EXTENSION 1
MATHEMATICS

HALF-YEARLY

2014

(Time - 90 minutes)

Directions to candidates

* *Put your name at the top of this paper and on each of the 5 sections that are to be collected.*
* *All questions are to be answered on separate pages and will be collected in separate bundles at the end of this exam.*
* *All necessary working should be shown in every question.*
* *Full marks may not be awarded for careless or badly arranged work.*

Brigidine College
2014 Half Yearly Extension 1 Exam

Question		Marks
1	How many ways can six children be arranged around a circular table? (A) 120 (B) 600 (C) 720 (D) 5040	1
2	Find the solution to the inequality: $\frac{15}{2x-6} \leq 5$. (A) $x \leq 3, x \geq 4.5$ (B) $x < 3, x \geq 4.5$ (C) $3 < x \leq 4.5$ (D) $x \leq 3, x > 4.5$	1
3	Find the coordinates of the point which divides the interval joining A(2, 1) and B(2, 8) internally in the ratio 3: 4. (A) (1, 7) (B) (2, 4) (C) (2, 7) (D) (4, 2)	1
4	From a group of 4 men and 6 women a committee of 3 is to be chosen. What is the probability that the committee contains at least one woman? (A) $\frac{29}{30}$ (B) $\frac{13}{30}$ (C) $\frac{3}{10}$ (D) $\frac{1}{6}$	1

Question	Marks

5 Evaluate $\lim_{x \to 2} \frac{x^3-8}{x-2}$ 1

(A) 0

(B) 4

(C) 12

(D) Undefined

6 How many four-digit numbers can be formed with the digits 1, 2, 3, 4 and 5 if no digit is repeated? 1

(A) 20

(B) 120

(C) 625

(D) 3125

7 What is the value of $\frac{a}{b}$ if the lines $ax+2y=6$ and $4y=bx-9$ are parallel?

(A) $\frac{1}{2}$

(B) $-\frac{1}{2}$

(C) -2

(D) 2

8 Three markers are placed out to sea. Marker B is 4 km north of marker A. However to sail from A to B a boat must first sail from A to C on a bearing 025° and then turn and sail from C to B on a bearing of 335°. What is the distance from A to C? 1

(A) 2.2 km

(B) 4.0 km

(C) 6.3 km

(D) 28.1 km

Question **Marks**

9 If $3\cos\theta+2=0$ and $\tan\theta>0$, what is the exact value of $\sin(\theta+180)$? 1

(A) $-\frac{\sqrt{5}}{3}$

(B) $-\frac{\sqrt{5}}{2}$

(C) $\frac{\sqrt{5}}{2}$

(D) $\frac{\sqrt{5}}{3}$

10 What is $\frac{1+\sqrt{3}}{5-2\sqrt{3}}$ as a fraction with a rational denominator? 1

(A) $\frac{-5-\sqrt{3}}{7}$

(B) $\frac{11-7\sqrt{3}}{7}$

(C) $\frac{-5+\sqrt{3}}{13}$

(D) $\frac{11+7\sqrt{3}}{13}$

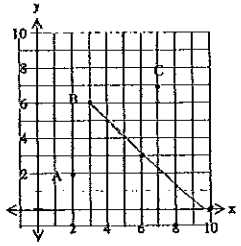
End of Multiple Choice

Question 11 (15 Marks) Start a New Page **Marks**

(a) Simplify $\dfrac{x^2+2x}{x^3-27}\times\dfrac{x^2+4x-21}{x^2+9x+14}$ **2**

(b) The points A, B and C have coordinates (2, 2), (3, 6) and (7, 7) respectively.

Point D is a point on the number plane so that ABCD a rhombus.



(i) Show that the coordinates of point D are (6, 3) **1**

(ii) Find the exact length of the diagonal AC. **1**

(iii) Find the equation of the diagonal BD. **2**

(iv) Explain the relationship between the gradients of AC and BD. **1**

(v) Find the point of intersection of the diagonals AC and BD. **2**

(vi) Find the area of ABCD. **2**

(vii) If point E exists such that ABCE is a kite with twice the area of the rhombus ABCD, find the coordinates of E. **2**

(c) Find the equation of the circle whose centre is at the origin and which has the line $2x-y+5=0$ a tangent to it. **2**

Question 12 (15 Marks) Start a New Page **Marks**

(a) Solve $\left(3+\dfrac{1}{x}\right)^2+4\left(3+\dfrac{1}{x}\right)-21=0$ **2**

(b) Find the coordinates of the point P that divides the interval AB joining A(-4, -3) and B(1, 5) externally in the ratio 3 : 2. **2**

(c) Find the exact value of the following :

(i) $\sin 135°$ **1**

(ii) $\cot 240°$ **2**

(d) What is the solution to the equation $2\cos 2\theta-1=0$ in the domain $0\le\theta\le 360°$ **2**

(e) Find the number of ways in which 3 boys and 3 girls can be arranged in a straight line so that the 3 boys are not all next to each other. **2**

(f) Find all possible values of x such that $|2x-3|=5-x$ **2**

(g) State the natural domain for the curve $y=\sqrt{4-x}$, and sketch this curve, showing intercepts. **2**

Question 13 (15 Marks) Start a New Page **Marks**

(a) Solve simultaneously: 3
$y = x^2 + 5x + 1$ and $y = 7x + 9$

(b) Solve the following quadratic inequality: 3

$3x^2 - 5x - 12 \geq 0$

(c) Consider the curve $y = \dfrac{x^2 - 4}{x^2 - 1}$

(i) Show it is an even function. 1

(ii) Find the limit as $x \to \infty$. 2

(iii) Determine the domain. 1

(iv) Sketch the curve showing all the important features. 2

(d) Consider the function $f(x) = (x-2)^2, x \geq 2$.

(i) Find the equation of the inverse function $f^{-1}(x)$ 2

(ii) On the same diagram sketch the graphs of 2
$y = f(x), y = f^{-1}(x)$, and $y = x$.

Question 14 (12 Marks) Start a New Page **Marks**

(a) Find the equation of the line through $(-2,3)$ that passes through the intersection of the lines $3x + y - 1 = 0$ and $2x - y + 6 = 0$. 3

(b) Find the integer values for A, B and C such that 3
$A(x-1)^2 + B(x-1) + C \equiv 2x^2 + 3x + 4$

(c) The Pacific Star cruise ship travels 215 km on a bearing of 085° from Sydney. It then travels 112km on a bearing of 135°.

(i) How far is the ship from Sydney? 2

(ii) What is the final bearing of the ship from Sydney? (give answer correct to the nearest degree) 2

(d) Find the number of ways the letters of the word RHOMBUS can be arranged in a straight line so that the vowels are together. 2

End of Exam

SOLUTIONS

13

QUESTION 11

a) $\frac{x^2+2x}{x^3-27} \times \frac{x^2+4x-21}{x^2+9x+14}$

$= \frac{x(x+2)}{(x-3)(x^2+3x+9)} \times \frac{(x+7)(x-3)}{(x+7)(x+2)}$ ✓✓

$= \frac{x}{x^2+3x+9}$

MULTIPLE CHOICE ANSWERS/10 (1 each)				
1. A	2. B	3. B	4. A	5. C
6. B	7. B	8. A	9. A	10. D

b) (i) BC ∥ AD

$m_{BC} = \frac{7-6}{7-3}$

$= \frac{1}{4}$

$\therefore m_{AD} = \frac{1}{4}$

$\frac{1}{4} = \frac{y_2-2}{x_2-2}$

$y_2 - 2 = 1$

$y_2 = 3$ ✓

$x_2 - 2 = 4$

$x_2 = 6$

$\therefore D = (6,3)$

(ii) $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(7-2)^2+(7-2)^2}$

$= \sqrt{5^2+5^2}$

$= \sqrt{25+25}$

$= \sqrt{50}$ ✓

$= 5\sqrt{2}$

(iii) ~~Diagonals of Rhomus are perpendicular~~

Gradient BD $= \frac{3-6}{6-3}$ ✓

$= \frac{-3}{3}$

Equation: $y = mx + b$

B(3,6)

$6 = -1(3) + b$

$6 = -3 + b$ ✓

$b = 9$

$\therefore$ Equation $\Rightarrow y = -x + 9$

$x + y - 9 = 0$

(iv) The diagonals of a ~~Rhomes~~ Rhombus are perpendicular

Therefore the ~~Midpoint~~ gradient of the diagonals AC and BD ✓

~~and~~ are perpendicular. and times to equal -1

Therefore $m_{AC} \times m_{BD} = -1$ Show calc.

(v) $M_{AC} = \left(\frac{7+2}{2}, \frac{7+2}{2}\right)$

$= (4.5, 4.5)$

$M_{BD} = \left(\frac{3+6}{2}, \frac{6+3}{2}\right)$

$= (4.5, 4.5)$

$\therefore$ Point of Intersection $= \left(\frac{9}{2}, \frac{9}{2}\right)$ ✓✓

$= \left(4\frac{1}{2}, 4\frac{1}{2}\right)$

(vi) Area of ~~Re~~ Rhombus $= \frac{1}{2}xy$

(From q. ii) $\therefore d_{AC} = 5\sqrt{2}$

$d_{BD} = \sqrt{(6-3)^2+(3-6)^2}$

$= \sqrt{3^2+(-3)^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$ ✓

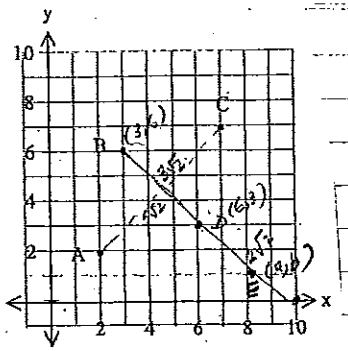
$= 3\sqrt{2}$ ✓

$A = \frac{1}{2} \times 5\sqrt{2} \times 3\sqrt{2}$

$= \frac{1}{2} \times 30$

$= 15$ units^2

vii.



Let the coordinates of E be (a, b)

Note: that ratio of $BD : DE = 3\sqrt{2} : 2\sqrt{2}$

$= 3 : 2 = k : l$

(Using Area of $\triangle BCE = 15$ units^2)

$\therefore$ using ratio formula

$$x = \frac{kx_2 + lx_1}{k+l} \qquad y = \frac{ky_2 + ly_1}{k+l}$$

$$6 = \frac{3a + 3\times 2}{3+2} \qquad 3 = \frac{3b+12}{5}$$

$$\therefore 30 = 3a + 6 \qquad 15 = 3b + 12$$

$$\therefore 3a = 24 \qquad 3 = 3b$$

$$a = 8 \qquad b = 1$$

$\therefore E(8, 1)$

Perpendicular ~~Form~~ Distance Formula : $\frac{|ax + by + c|}{\sqrt{a^2+b^2}}$

Circle = $x^2 + y^2 = r^2$

$$r = \frac{|2(0) + 1(0) + 5|}{\sqrt{2^2 + (-1)^2}}$$

$$= \frac{|0 - 0 + 5|}{\sqrt{4+1}}$$

$$= \frac{|5|}{\sqrt{5}}$$

$$= \frac{5}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{5\sqrt{5}}{5}$$

$$= \sqrt{5}$$

$\therefore$ Equation of circle : $x^2 + y^2 = 5$

Question 12

a) $\left(3+\frac{1}{x}\right)^2 + 4\left(3+\frac{1}{x}\right) - 21 = 0$

$u^2 + 4u - 21 = 0$

$(u+7)(u-3) = 0$

$u = -7, 3$ ✓ ✗

b) $-3 : 2$

$\frac{2(-4)+3(1)}{2-3}$

$= \frac{-8-3}{-1}$

$= \frac{-11}{-1}$

$= 11$

$\frac{2(-3)-3(5)}{2-3}$

$\frac{-6-15}{-1}$

$= \frac{-21}{-1}$

$= 21$

$P(11, 21)$ ✓✓

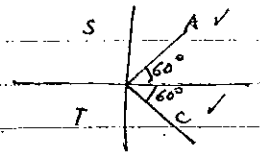
c) i. $\frac{1}{\sqrt{2}}$ ✓

ii. $\frac{1}{\tan 240}$ ✓ $\frac{1}{\tan 60}$ Answer $= \frac{1}{\sqrt{3}}$

d) $2\cos 2\theta = 1$

$\cos 2\theta = \frac{1}{2}$

$\cos 2\theta = \frac{1}{2}$ where $0 \leq \theta \leq 360°$



$2\theta = 60, 300°, 420°, 660° \quad 0 \leq 2\theta \leq 720°$

$\theta = 30°, 150°, 210°, 330°$

e) 3 × 5 × 5 × 4 × 3 × 2 × 1

Total no. of ways of arranging 3 boys + 3 girls = 6!

No. of ways all 3 boys are together is

$(B_1 B_2 B_3)\ G_1\ G_2\ G_3$

$4! \times 3!$

∴ No of ways that the 3 boys are together $= 6! - 4!\,3! = 720 - 144 = 576$ ways

f) $|2x-3| = 5-x$

$2x-3 = 5-x$

$2x-3 = 5-x$

$3x = 8$

$x = \frac{8}{3}$

$= 2\frac{2}{3}$

$2x-3 = -(5-x)$

$2x-3 = -5+x$

$x = -2$

$5 = (-2)$

$5+2$

$= 7$

Answer $x = \frac{8}{3}, -2$

$5 - \frac{8}{3}$

$\frac{15-8}{3}$

$= \frac{7}{3}$

$2\left(\frac{8}{3}\right) - 3$

$\frac{16}{3} - \frac{9}{3}$

$= \frac{7}{3}$

$|2(-2)-3|$

$|-4-3|$

$= |-7|$

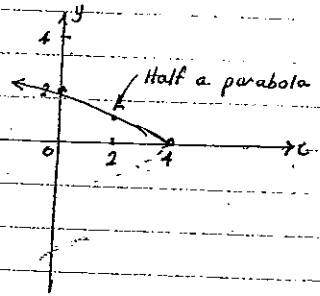
$= 7$

(12) (g) $y = \sqrt{4-x}$

D: $4-x \geq 0$

$x \leq 4$

R: $y \geq 0$.



QUESTION 13

a) $y = x^2 + 5x + 1$

$y = 7x + 9$

$7x + 9 = x^2 + 5x + 1$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4$

$x = -2$

for $x = 4$

$y = 7(4) + 9$

$= 28 + 9$

$\therefore x = 4, y = 37$

for $x = -2$

$y = 7(-2) + 9$

$= -14 + 9$

$= \cancel{23}\ -5$

$\therefore x = -2, y = \cancel{23}\ -5$

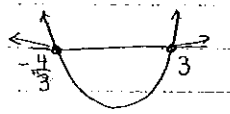
Answer: $x = 4, y = 37$

~~$x = -2, y = 23 - 5$~~ $x = -2, y = 5$

b) $3x^2 - 5x - 12 \geq 0$

$(3x+4)(x-3) \geq 0$

$x = -\frac{4}{3}$ $x = 3$



$\geq$

$-\frac{4}{3} \leq x$ $x \geq 3$

$\therefore$ Answer: $x \geq -\frac{4}{3}$

$x \geq 3$

c) (i.) $f(x) = \dfrac{x^2-4}{x^2-1}$

$f(-x) = \dfrac{(-x)^2-4}{(-x)^2-1}$ ✓

$= \dfrac{x^2-4}{x^2-1}$

$= f(x)$

$\therefore$ Even

(ii.) $\lim_{x\to\infty} \dfrac{x^2-4}{x^2-1}$ $\lim_{x\to\pm\infty} \dfrac{1-\frac{4}{x^2}}{1-\frac{1}{x^2}}$ ✗

$\lim_{x\to\infty} \dfrac{\frac{x^2}{x^2}-4}{\frac{x^2}{x^2}-1}$ $= \dfrac{1}{1}$

$= 1$

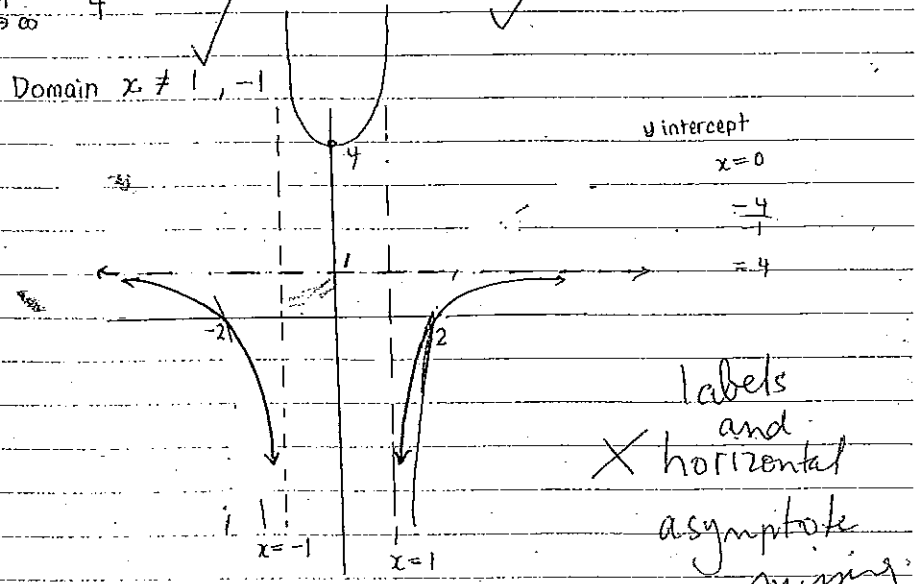
$\lim_{x\to\infty} \dfrac{0-4}{0-1}$ ✗

$\lim_{x\to\infty} \dfrac{-4}{-1}$

$\lim_{x\to\infty} 4$ ✓

(iii) Domain $x \neq 1, -1$ ✓

(iv)



✗ labels and horizontal asymptote missing

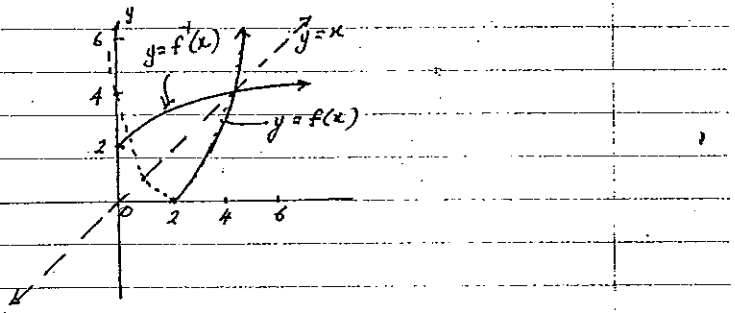
Q 13 (d) (i) $x = (y-2)^2$

$\therefore \pm\sqrt{x} = y-2$

$y = 2 \pm \sqrt{x}$

$\therefore f'(x) = 2+\sqrt{x}$ since $y \geq 2$

(ii)



QUESTION 14 (6)

a) $(3x+y-1)+k(2x-y+6)=0$

$(3(-2)+3-1)+k(2(-2)-3+6)=0$

$(-6+3-1)+k(-4-3+6)=0$

$(-4)+k(-1)=0$

$-4-k=0$

$-k=4$

$k=-4$

$(3x+y-1)-4(2x-y+6)=0$

$3x+y-1-8x+4y-24=0$

$-5x+5y-25=0$

$-5(x-y+5)=0$

$x-y+5=0$

(b) $A(x^2-2x+1)+Bx-B+C \equiv 2x^2+3x+4$

$Ax^2+(B-2A)x+(A-B+C) \equiv 2x^2+3x+4$

$\therefore A=2$

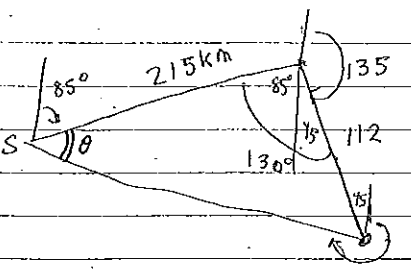
$B-2(2)=3$

$\therefore B=7$

$2-7+C=4$

$\therefore C=9$

c)



i) $c^2=a^2+b^2-2ab\cos C$

$c^2=215^2+112^2-2\times215\times112\times\cos130°$

$C=299.5$ km (1dp)

ii) $\frac{299.5}{\sin130}=\frac{215}{\sin A}$ $\quad \frac{299.5}{\sin130°}=\frac{112}{\sin\theta}$

$\sin A=\frac{215\sin130}{299.5}$ $\quad \therefore \sin\theta=\frac{112\sin130°}{299.5}$

$A=33°$ $\quad \theta=16°48'$

$\therefore$ Bearing is $85°+17°$

$=102°T$ (to the nearest deg)

d) RHOMBUS

$7!=5040$