Year 12 Maths Extension 1
Assessment Task 2
March 2009

ST VINCENT'S COLLEGE

Name____________________

Note all necessary work must be shown

MARKS

1) Sketch the curve: $y = log(x+2)$. (2)

2)

i. Simplify: $\dfrac{\log_2 5}{\log_2 25}$ (1)

ii. Simplify: $e^{2\log_e 2}$ (1)

iii Solve $2^x = 12$ to 3 decimal places (2)

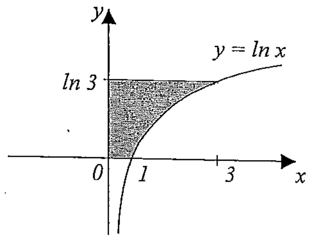
3) Evaluate $\displaystyle\int_0^1 \frac{dx}{2x+1}$. (2)

4) Differentiate with respect to x: $log_e\left[\dfrac{x+4}{x-3}\right]$. (2)

5) Differentiate $y = \log_{10} x$ (1)

6) Find the equation of the tangent to the curve $y = x\ ln\ x$ at the point $(1,\ 0)$. (3)

7) The diagram shows the area bounded by the graph $y = ln\ x$, the co-ordinate axes and the line $y = ln\ 3$.



NOT TO SCALE

i. Find the shaded area. (3)

ii. Hence find the exact value of $\displaystyle\int_1^3 ln\ x dx$. (1)

8) The curve $y = log_e x$ between the lines $x = 1$ and $x = 3$ is rotated about the y axis. Find the volume of the solid formed. (Leave your answer in terms of π.) (4)

9) Evaluate $\displaystyle\int_1^9 \frac{dx}{x+\sqrt{x}}$ using the substitution $x = u^2$. (3)

10) Find the value of $\displaystyle\int_1^6 x\sqrt{x+3}\,dx$, by means of the substitution $u^2 = x + 3$. (3)

11) Consider the function $y = \dfrac{1}{x}e^{-x}$:

a. For what values of x is this function defined? (1)

b. Describe the behaviour of the function as x:

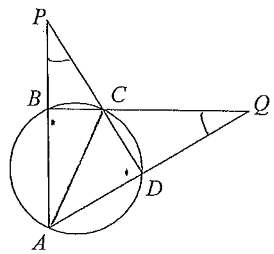
α. approaches zero. (2)

β. increases indefinitely.

c. Find any stationary points and determine their nature. (3)

d. Sketch the curve of this function. (2)

12)



In the diagram above ABP, DCP, BCQ, and ADQ are all straight lines and $\angle APD = \angle BQA$.

a. Show that $\angle ABC = \angle ADC$. (2)

b. Prove that AC is a diameter of the circle. (2)

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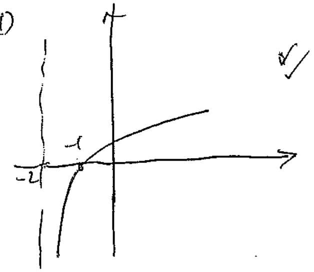
Solutions

(1)



✓

2. (i) 0.5 ✓

(ii) 4 ✓

(iii) $x = \frac{\ln 12}{\ln 2}$ ✓

$= 3.585$ ✓

(3) $\int_0^1 \frac{1}{2x+1}\,dx$

$= \frac{1}{2}\left[\ln(2x+1)\right]_0^1$ ✓

$= \frac{1}{2}\ln 3.$ ✓

(4) $\frac{d}{dx}\left[\ln(x+4) - \ln(x-3)\right]$ ✓

$= \frac{1}{x+4} - \frac{1}{x-3}.$ ✓

(5) $y = \frac{\ln x}{\ln 10}$

$y' = \frac{1}{x \cdot \ln 10}$ ✓

6. $y = x\ln x$

$y' = \ln x + 1$ ✓

at $x=1$, $y' = 1$ ✓

$\therefore y - 0 = 1(x-1)$

$y = x - 1$ ✓

(11) $y = \frac{1}{x e^x}$

(a) defined for all $x \neq 0$. ✓

(b) $x \to 0$ $y \to \infty$. ✓

$x \to \infty$, $y \to 0$. ✓

(c) $y = (xe^x)^{-1}$

$y' = -(xe^x)^{-2} \cdot (e^x + xe^x)$ ✓

$= \frac{-e^x(1+x)}{x^2 e^x e^x}$ ✓

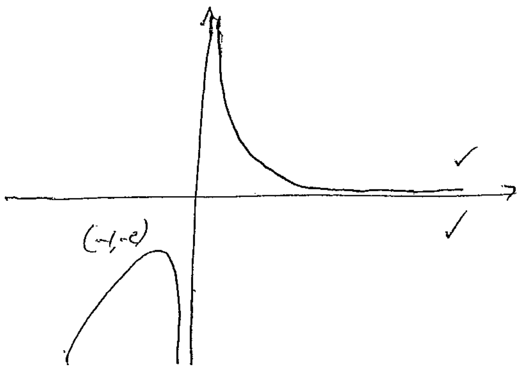
$= \frac{-(1+x)}{x^2 e^x}$

Stat pts at $x = -1$,

x	-2	-1	0
y'	$+$	0	$-$

✓

$\therefore$ Max T.pt at $(-1, -e)$



✓ ✓

(7) (i) $A = \int_0^{\ln 3} x\,dy$

$= \int_0^{\ln 3} e^y\,dy$ ✓

$= \left[e^y\right]_0^{\ln 3}$ ✓ ✓

$= 3 - 1$

$= 2.$

(ii) $\int_1^3 \ln x\,dx = 3\ln 3 - 2$ ✓

(9) $\int_1^9 \frac{dx}{x+\sqrt{x}}$

$x = u^2$

$dx = 2u\,du$ ✓

$x=9$ $u=3$

$x=1$ $u=1$

$= \int_1^3 \frac{2u\,du}{u^2+u}$

$= \int_1^3 \frac{2\,du}{1+u}$ ✓

$= \left[2\ln(1+u)\right]_1^3$

$= 2(\ln 4 - \ln 2)$ ✓

$= 2\ln 2.$

(10) $\int_1^6 x\sqrt{x+3}\,dx$

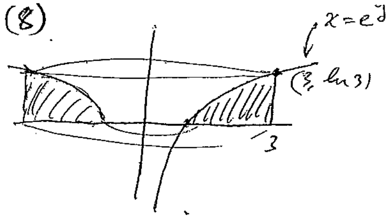
$x = u^2 - 3$

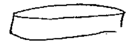
$dx = 2u\,du$

$= \int_2^3 (u^2-3)\sqrt{u^2} \cdot 2u\,du$ ✓ ✓

$= \int_2^3 (2u^4 - 6u^2)\,du$ ✓

(8)



Volume required is the volume of cylinder  subtract the volume of 

$= \pi \times 3^2 \times \ln 3 - \int_0^{\ln 3} \pi e^{2y}\,dy.$

$= 9\pi\ln 3 - \frac{\pi}{2}\left[e^{2y}\right]_0^{\ln 3}$

$= 9\pi\ln 3 - \frac{\pi}{2}\left[6 - 1\right]$

$= 9\pi\ln 3 - \frac{5\pi}{2}.$

(12)