2009
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time –3 hours
- Write using black or blue pen
- Approved calculators may be used
- A table of standard integrals is provided
- All necessary working should be shown in every question
- Please complete Questions 1,2 &3 in one booklet
  Questions 4,5 &6 in one booklet
  Questions 7&8 in one booklet
  Questions 9&10 in one booklet

Total marks – 120

- Attempt Questions 1 – 10
- All questions are of equal value

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question 1 (12 Marks) Start a New Booklet **Marks**

(a) Evaluate $\frac{3.24^2 - 2.1^2}{\sqrt{36+2.1}}$ correct to 3 significant figures. **2**

(b) Express $3.5\dot{3}$ as a fraction in simplest form. **2**

(c) If $\tan\theta = \frac{7}{8}$ and $\cos\theta > 0$, find the exact value of $\sin\theta$ **2**

(d) Solve $|15+4x| \leq 3$ **2**

(e) If $k = m(v^2 - u^2)$ find the value of m,

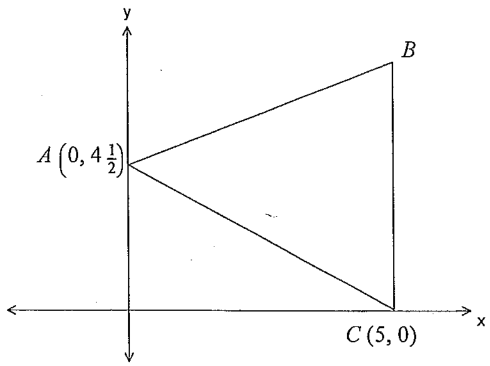
when $k = 724$, $v = 14{\cdot}2$ and $u = 7{\cdot}4$, correct to 3 decimal places. 1

(f) Factorise $x^3 - 8$ 1

(g) Paint at the local hardware store is sold at a profit of 30% on the cost price. If a drum of paint is sold for $67·50, find the cost price. **1**

(h) State the domain of $y = \sqrt{2x-3}$ 1

Question 2 (12 Marks) **Marks**



The lines AB and CB have equations $x - 2y + 9 = 0$ and $4x - y - 20 = 0$ respectively.

(a) Show that the coordinates of the point B are (7,8) **2**

(b) Show that the equation of the line AC is $9x + 10y - 45 = 0$. **2**

(c) Calculate the distance AC in exact form. **2**

(d) Find the equation of the line perpendicular to BC which passes passes through A. **2**

(e) Calculate the shortest distance between the point B and the line AC. Hence find the area of the triangle ABC. **2**

(f) State the inequalities that together define the area bounded by the triangle ABC. **2**

Question 3 (12 Marks) **Marks**

(a) Differentiate with respect to x.

i. $\dfrac{\ln x}{x}$ **2**

ii. $(\sin 2x)\, e^{2x}$ **2**

(b) Find:

i. $\int \dfrac{dx}{e^{3x}}$ **2**

ii. $\int_0^{\pi} \sec^2 \dfrac{x}{4}\, dx$. **2**

(c) If α and β are the roots of the equation $3x^2 - 4x - 7 = 0$
Find:

i. $\alpha + \beta$. **1**

ii. $\alpha\beta$ **1**

iii. $2\alpha^2 + 2\beta^2$. **2**

Question 4 (12 Marks) Start a New Booklet **Marks**

(a) Express $\dfrac{\sqrt{3}}{2\sqrt{7} - \sqrt{3}}$ with a rational denominator. 3

(b) The federal government distributes $500 million in order to stimulate the economy. Each recipient spends 80% of the money that he or she receives. In turn, the secondary recipient spends 80% of the money that they receive, and so on. What was the total spending that results from the original $500 million into the economy? 2

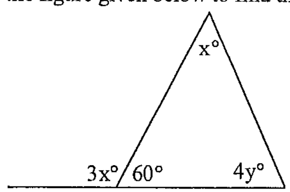
(c) A ship sails 60 nautical miles due west, from port A to a port B. It then proceeds a distance of 50 nautical miles on a bearing of 210° to a port C.

i. Draw a diagram to illustrate the information given. 1

ii. Find the distance from port A to port B.
(correct to the nearest nautical mile). 2

iii. Find the bearing of port C from port A.
(correct to the nearest degree) 1

(d) Use the information in the figure given below to find the values of x and y.

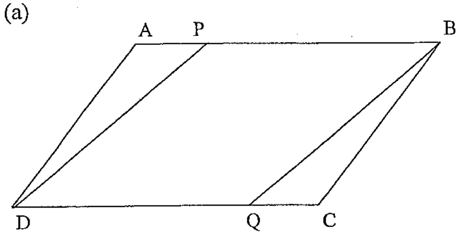


2

(e) What is the value of the total interior angles of a pentagon. 1

Question 5 (12 Marks) **Marks**

(a) 3



Copy this diagram into your booklet.

ABCD is a parallelogram, BP = DQ.

Prove DP = BQ

(b) i. Show $\ln 9 = 2 \ln 3$ 1

ii. Is the series $\ln 3 + \ln 9 + \ln 27 + \ldots$ arithmetic or geometric? Give reasons for your answer. 1

iii. Find the sum of the first 10 terms of the series. 1

(c) Find the equation of the tangent to the curve $y = \log_e x$ at the point where $x = e^3$. 3

(d) Solve $2(x^2 + 1)^2 - 19(x^2 + 1) - 10 = 0$ 3

Question 6 (12 Marks) **Marks**

(a) A curve has a gradient function with equation $\frac{dy}{dx} = 6x^2 - 18x + 12$

i. If the curve passes through the point (1, 2), what is the equation of the curve? 2

ii. Find the coordinates of the stationary points and determine their nature. 2

iii. Find any points of inflexion. 2

iv. Graph the function showing all the main features. 2

(b) Prove that

$$\frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\operatorname{cosec}\theta$$ 3

(c) Evaluate $\lim_{\theta\to 0}\frac{\sin 2\theta}{3\theta}$ 1

Question 7 (12 Marks) **Start a New Booklet** **Marks**

(a) The parabola $y = x^2$ and the line $y = x + 2$ intersect at points A and B respectively.

i. Find the coordinates of the points A and B. **2**

ii. Hence, find the area bounded by the parabola and the line. **2**

(b) The minute hand on a clock face is 12 centimetres long.

i. Through what angle does the hand move (in radians) in 40 minutes? **1**

ii. How far does the tip of the minute hand move in 40 minutes? **1**

iii. What area does the minute hand sweep through in 40 minutes? **1**

(c) Use Simpson's rule to evaluate $\int_1^2 f(x)dx$,

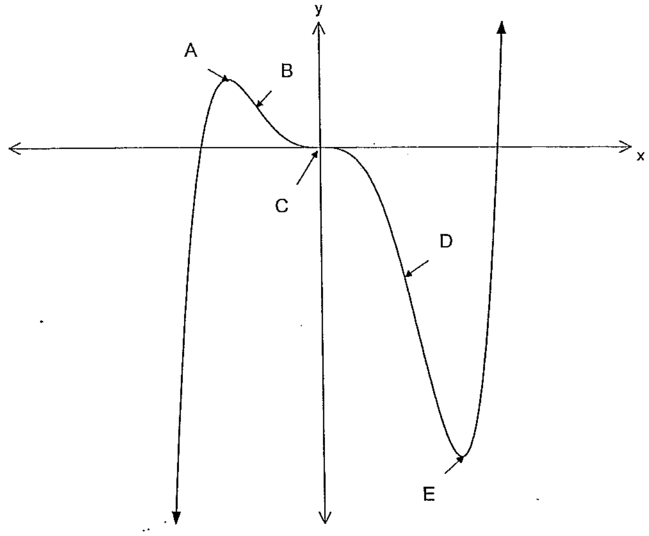
to 1 decimal place, using the 5 function values in the table below. **3**

x	1·00	1·25	1·50	1·75	2·00
$f(x)$	3·43	2·17	0·38	1·87	2·65

(d) Find the period and amplitude for the graph of $3y = \sin(2x)$. **2**

Question 8 (12 Marks) **Marks**

(a) The graph of the curve $y = f(x)$ is drawn below.



i. Name the points of inflexion. **1**

ii. When is the graph decreasing? **1**

iii. Sketch the gradient function.
Clearly label the points A, B, C, D and E on your sketch. **1**

(b) Steve borrows \$15 000 for a new car. He decides to repay the loan plus interest at 6% pa compounded monthly. He repays the loan in monthly installments of \$P.

i. Show that after three months the amount that Steve owes is \$[15226·13 — P(3·015025)]. **2**

ii. After two years of repaying his loan, Steve still owes \$10 000 on the loan. What was the monthly repayment? **3**

(c) i. Express $2x = y^2 - 8y + 4$ in the form $(y - k)^2 = 4a(x - h)$ **1**

ii. Hence or otherwise, sketch the parabola, showing the vertex, focus and the directrix. **3**

Question 9 (12 Marks) Start a New Booklet **Marks**

(a) The area bounded by the curve $y=\sqrt{\dfrac{2x}{3x^2-1}}$ between the lines $x=1$ and $x=3$ is rotated about the x-axis. Find the volume of the solid of revolution formed. (Leave your answer in exact form.) **3**

(b) The rate at which Carbon Dioxide will be produced when conducting an experiment is given by $\dfrac{dV}{dt}=\dfrac{1}{100}\left(30t-t^2\right)$ where V cm^3 is the volume of gas produced after t minutes.

i. At what rate is the gas being produced 15 minutes after the experiment begins? **1**

ii. How much Carbon Dioxide has been produced during this time? **2**

(c) Solve $\cos 2\theta=-\dfrac{1}{\sqrt{2}}$, for $0\le\theta\le 2\pi$ **3**

(d) Find the values of m, if $-4x^2+4(m+1)x-(4m+1)$ is negative definite. **3**

Question 10 (12 Marks) **Marks**

(a) An open cylindrical can is made from a sheet of metal with an area of 300cm^2.

i. Show that the volume of the can is given by $V=150r-\dfrac{1}{2}\pi r^3$. **2**

ii. Find the radius of the cylinder that gives the maximum volume and calculate this volume. **4**

(b) The population of a certain town grows at a rate proportional to the population. If the population grows from 20 000 to 25 000 in two years, find:

i. The population of the town, to the nearest hundred, after a further 8 years. **3**

ii. Calculate the rate of change at this time. **1**

(c) If $\log_a 2+2\log_a x-\log_a 6=\log_a 3$ find the value of x. **2**

End of Examination.

2009 MATHEMATICS – HSC TRIAL SOLUTIONS

1) a) 0.986 (3 sig figs) ✓✓

b) let $x = 0.5333\ldots$

$10x = 5.333\ldots$

$100x = 53.333\ldots$

$90x = 48$

$x = \frac{8}{15}$ $\quad \therefore 3.5\dot{3} = 3\frac{8}{15}$. ✓✓

OR $3.5\dot{3} = 3.5 + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10\,000}$

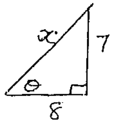
$\left[S_\infty = \frac{\frac{3}{100}}{1-\frac{1}{10}}\right]$

$= 3\frac{1}{2} + \frac{3}{100}\times\frac{10}{9}$

$= 3\frac{1}{2} + \frac{1}{30}$

$= 3\frac{16}{30} = 3\frac{8}{15}$

c) $\tan\theta = \frac{7}{8}$, $\cos\theta > 0$ $\therefore$ 1ST QUAD



$x^2 = 7^2 + 8^2$

$x = \sqrt{113}$

$\therefore \sin\theta = \frac{7}{\sqrt{113}}$ ✓✓

d) $|15+4x| \le 3$

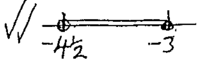
$15+4x \le 3$ (or) $-(15+4x) \le 3$

$4x \le -12$ $\quad 15+4x \ge -3$

$x \le -3$ $\quad 4x \ge -18$

$\quad x \ge -4\frac{1}{2}$

$\therefore -4\frac{1}{2} \le x \le -3$ ✓✓



[Check $f(-4) \le 3$]

e) $k = m(v^2 - u^2)$

$724 = m(14.2^2 - 7.4^2)$

$m = \frac{724}{(14.2^2 - 7.4^2)}$

$= 4.929$ (3 dec. pl. ✓

f) $x^3 - 8 = x^3 - 2^3$

$= (x-2)(x^2+2x+4)$ ✓

g) SP $= 130\%$ of CP

$1.3\times$ CP $= 67.50$

$\therefore$ CP $= \frac{67.5}{1.3}$

$= \$51.92$ ✓

h) Domain: $2x - 3 \ge 0$

$2x \ge 3$

$x \ge \frac{3}{2}$ or $1\frac{1}{2}$ ✓

2) a) AB has eqn. $x - 2y + 9 = 0$ ①

CB has eqn $4x - y - 20 = 0$. ②

① $x - 2y = -9$

(2a) $-8x + 2y = -40$, ② $\times -2$

①+(2a) $-7x = -49$ $\therefore x = 7$

Sub $x = 7$ into ①

$7 - 2y + 9 = 0$ $\therefore y = \frac{16}{2} = 8$. ✓✓

$\therefore$ Coords of $B(7,8)$

b) AC passes thr $(0, 4\frac{1}{2})$, $C(5,0)$

$m_{AC} = \frac{0-4.5}{5-0} = -\frac{9}{10}$

$y - 0 = -\frac{9}{10}(x-5)$

$10y = -9x + 45$ ✓✓

$\therefore 9x + 10y - 45 = 0$ is eqn. of AC

c) $AC = \sqrt{(5-0)^2 + (0-4\frac{1}{2})^2}$

$= \sqrt{25 + \frac{81}{4}} = \sqrt{\frac{181}{4}} = \frac{\sqrt{181}}{2}$ ✓✓

d) $m_{BC} = \frac{8-0}{7-5} = 4$ $\quad \therefore$ Grad of line $\perp$ BC $m_2 = -\frac{1}{4}$

Eqn of line $\perp$ BC thr $A(0, 4\frac{1}{2})$

$y - 4\frac{1}{2} = -\frac{1}{4}(x - 0)$

$4y - 18 = -x$

$\therefore x + 4y - 18 = 0$ is eqn. ✓✓

e) $\perp$ dis from $B(7,8)$ to AC.

$d = \frac{|9(7)+10(8)-45|}{\sqrt{81+100}}$

$= \frac{98}{\sqrt{181}}$

Area of $\triangle ABC = \frac{1}{2}bh$

$= \frac{1}{2}\times\frac{\sqrt{181}}{2}\times\frac{98}{\sqrt{181}}$

$= 24.5$ units^2 ✓✓

f) $x - 2y + 9 \ge 0$

and $9x + 10y - 45 \ge 0$ ✓✓

and $4x - y - 20 \le 0$

3) a) i) $y = \frac{\ln x}{x}$

$\frac{dy}{dx} = \frac{x\cdot\frac{1}{x} - \ln x\cdot 1}{x^2}$

$= \frac{1 - \ln x}{x^2}$ ✓✓

ii) $y = (\sin 2x)\cdot e^{2x}$

$= \sin 2x \cdot 2e^{2x} + e^{2x}\cdot 2\cos 2x$

$= 2e^{2x}(\sin 2x + \cos 2x)$ ✓✓

b) i) $\int \frac{dx}{e^{3x}} = -\frac{1}{3}e^{-3x} + C$ ✓✓

ii) $\int_0^\pi \sec^2\frac{x}{4}\,dx = 4\left[\tan\frac{x}{4}\right]_0^\pi = 4$ ✓✓

c) $3x^2 - 4x - 7 = 0$

i) $\alpha + \beta = -\frac{b}{a} = \frac{4}{3}$ ✓ $\quad$ ii) $\alpha\beta = \frac{c}{a} = -\frac{7}{3} = -2\frac{1}{3}$ ✓

iii) $2\alpha^2 + 2\beta^2$

$= 2(\alpha^2 + \beta^2)$

$= 2[(\alpha+\beta)^2 - 2\alpha\beta]$

$= 2[(\frac{4}{3})^2 - 2(-\frac{7}{3})]$

$= 2[\frac{16}{9} + \frac{42}{9}]$

$= 2\times\frac{58}{9}$

$= 12\frac{8}{9}$ ✓✓

4a) $\frac{\sqrt{3}}{2\sqrt{7}-\sqrt{3}}$

$= \frac{\sqrt{3}}{2\sqrt{7}-\sqrt{3}}\times\frac{2\sqrt{7}+\sqrt{3}}{2\sqrt{7}+\sqrt{3}}$

$= \frac{\sqrt{3}(2\sqrt{7}+\sqrt{3})}{4\times 7 - 3}$

$= \frac{2\sqrt{21}+3}{25}$ ✓✓✓

4 b) $500\,000\,000 + 0.8\times 500\,000\,000 + 0.8\times 0.8\times 500\,000\,000\ldots$

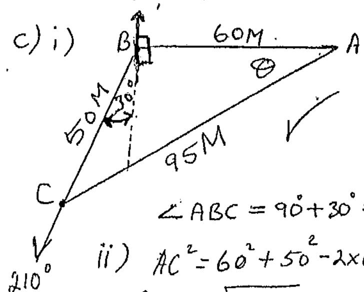
Geom. series with $r = 0.8$, $a = 500\,000\,000$

$S_\infty = \frac{a}{1-r}$

$= \frac{500\,000\,000}{(1-0.8)}$

$= \$2\,500\,000\,000$ ✓✓

c) i)



$\angle ABC = 90° + 30° = 120°$.

ii) $AC^2 = 60^2 + 50^2 - 2\times 60\times 50\cos 120°$

$AC = \sqrt{9100}$

$= 95.39$

$= 95\,M$ ✓✓

iii) $\frac{\sin\theta}{\cancel{50}} = \frac{\sin 120}{95}\times 50$

$\theta = 27°7' = 27°$

$\therefore$ Bearing of C from A

$= 270° - 27°$

$= 243°$. ✓

d) $3x - 60 = 180°$ (Straight $\angle$)

$\therefore x = 40°$

$x + 4y = 120°$ (Ext. $\angle$ of $\triangle$)

$4y = 80°$

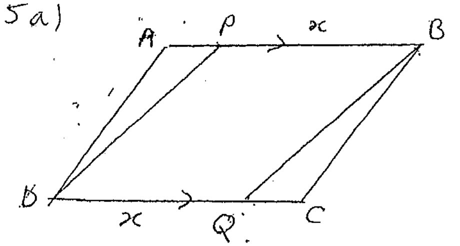
$y = 20°$ ✓✓

e) Interior $\angle$s of a pentagon.

Total $= (5-2)\times 180°$

$= 540°$. ✓

5a)



In $ABCD$

$AB = CD$ (property of a parallelogram.)

If $BP = DQ$ (given)

$\therefore AP = CQ$

In $\triangle$'s APD and CQB

$\angle A = \angle C$ (Property of a parallelogram.)

$AD = CB$ (Opp sides of p'gram)

$AP = QC$ (Shown above)

$\therefore \triangle APD \equiv \triangle CQB$ (SAS)

$\therefore DP = BQ$ (corresp. sides of congruent $\triangle$'s). ✓✓✓

b) i) Show $\log 9 = 2\log 3$

$RHS = \log 3^2$

$= \log 9$

$= LHS.$ ✓

ii) $\log 3 + \log 9 + \log 27 + \ldots$

If A.S $T_3 - T_2 = T_2 - T_1$.

$T_2 - T_1 = \log 9 - \log 3$

$= \log(\frac{9}{3})$

$= \log 3.$

$T_3 - T_2 = \log 27 - \log 9$

$= \log(\frac{27}{9})$

$= \log 3 = d$

$\therefore T_2 - T_1 = T_3 - T_2$ ✓

which shows that it's an Arithmetic Series

5b iii) $S_{10} = \frac{n}{2}(2a + (n-1)d)$

$= \frac{10}{2}(2\log 3 + 9\log 3)$

$= 5 \times 11 \log 3$

$= 55 \log 3$ ✓

5c) $y = \log_e x$

$\frac{dy}{dx} = \frac{1}{x} \quad \therefore m = \frac{1}{e^3}$ (at $x = e^3$)

$m = e^{-3}$

When $x = e^3$, $y = \log_e e^3$

$= 3\log_e e$

$= 3$

Eqn of tangent:

$y - 3 = e^{-3}(x - e^3)$

$y = e^{-3}x - e^0 + 3$

$y = e^{-3}x + 2$ ✓✓✓

(OR) $x - e^3y + 2e^3 = 0$

d) Let $y = x^2 + 1$

$2y^2 - 19y - 10 = 0$

$(2y+1)(y-10) = 0$

$y = -\frac{1}{2}$ or $y = 10$

But $y = x^2 + 1$

$\therefore x^2 + 1 = -\frac{1}{2}$ OR $x^2 + 1 = 10$

$x^2 = -\frac{3}{2}$ $\qquad x^2 = 9$

$x = \pm 3.$

No soln. ✓✓✓

6 a) $\frac{dy}{dx} = 6x^2 - 18x + 12$

i) $y = \frac{6x^3}{3} - \frac{18x^2}{2} + 12x + c$

$= 2x^3 - 9x^2 + 12x + C$

Passes thr $(1, 2)$

$2 = 2 - 9 + 12 + c$

$c = -3$

$\therefore y = 2x^3 - 9x^2 + 12x - 3$ ✓✓

is the eqn. of the curve.

ii) Let $\frac{dy}{dx} = 0$ for stat pts.

$6(x^2 - 3x + 2) = 0$

$(x-2)(x-1) = 0.$

$x = 1$ or $x = 2.$

When $x = 1$, $y = 2 - 9 + 12 - 3 = 2$

when $x = 2$, $y = 16 - 36 + 24 - 3 = 1$

$\frac{d^2y}{dx^2} = 12x - 18$

$f''(1) < 0 \therefore$ Max T.pt at $(1, 2)$ ✓

$f''(2) > 0 \therefore$ Min T.pt at $(2, 1)$ ✓

iii) For Pts of inflexion let

$\frac{d^2y}{dx^2} = 0$

$12x - 18 = 0$

$x = \frac{18}{12} = 1\frac{1}{2}.$

x	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$
y''	<0	0	>0

Change of concavity

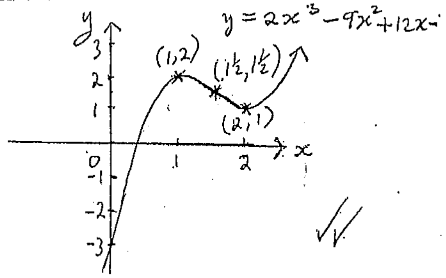
$\therefore$ pt of inflexion at $(1\frac{1}{2}, 1\frac{1}{2})$ ✓✓

[When $x = 1\frac{1}{2}$

$y = 2(1\frac{1}{2})^3 - 9(1\frac{1}{2})^2 + 12 \times 1\frac{1}{2} - 3$

$= 1\frac{1}{2}$]

iv)



✓✓

b) $\frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\operatorname{cosec}\theta$

$LHS = \frac{\sin\theta(1+\cos\theta) + \sin\theta(1-\cos\theta)}{1-\cos^2\theta}$

$= \frac{\sin\theta(1 + \cos\theta + 1 - \cos\theta)}{\sin^2\theta}$

$= \frac{2\sin\theta}{\sin^2\theta}$

$= \frac{2}{\sin\theta}$

$= 2\operatorname{cosec}\theta = RHS.$ ✓✓✓

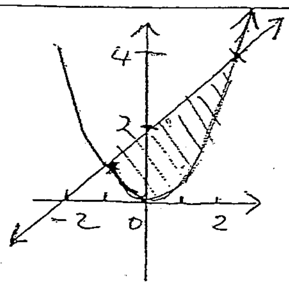
c) $\lim_{\theta \to 0} \frac{\sin 2\theta}{3\theta} = \frac{2}{3}\lim_{\theta \to 0} \frac{\sin 2\theta}{2\theta}$

$= \frac{2}{3} \times 1,$

$= \frac{2}{3}$ ✓

7a) (i) $y = x^2$ ①

$y = x + 2$ ②



Sub ① into ②

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\therefore x = 2, \; y = 4$

or $x = -1, \; y = 1$

$\therefore A(-1, 1), \; B(2, 4)$ ✓✓

ii) $A = \int_{-1}^{2} (x+2)\,dx - \int_{-1}^{2} x^2\,dx$

$= \int_{-1}^{2} (x + 2 - x^2)\,dx$

$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{-1}^{2}$

$= \left[2 + 4 - \frac{8}{3}\right] - \left[\frac{1}{2} - 2 + \frac{1}{3}\right]$

$= 3\frac{1}{3} - (-1\frac{1}{6})$

$= 4\frac{1}{2}$ units^2 ✓✓

b) i) $\frac{40}{60} \times 360° = 240° \times \frac{\pi}{180}$

$= \frac{4\pi}{3}$ ✓

ii) $l = r\theta$

$= 12 \times \frac{4\pi}{3}$

$= 16\pi$ cm ✓

iii) $A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 12^2 \times \frac{4\pi}{3}$

$= 96\pi$ cm^2 ✓

c) $\int_{1}^{2} f(x)\,dx$

$= \frac{1.5-1}{6}(3.43 + 4(2.17) + 0.38) + \frac{2-1.5}{6}(0.38 + 4(1.87) + 2.65)$

$= \frac{1}{12}[3.43 + 2.65 + 4(2.17 + 1.87) + 2(0.38)]$

$= \frac{1}{12} \times 23 = 1.9$ ✓✓✓

d) $y = \frac{1}{3}\sin(2x)$, amplitude $= \frac{1}{3}$ ✓

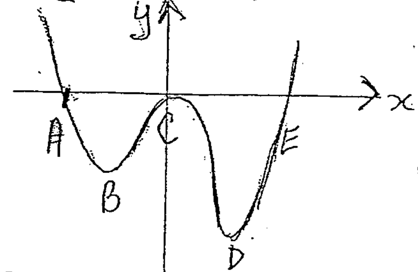
period $= \frac{2\pi}{2} = \pi$ ✓

8a) i) Pts of inflexion: B, C, D ✓

ii) Curve decreasing if $f'(x) < 0$

$\therefore$ From A to C, then C to E: curve decreasing ✓

iii)



✓

b) Let P = monthly Repayments.

$I = 6\%$ p.a Amount $= 15000$

$= 0.5\%$/month

$\doteqdot 0.005$

After 1 month, $A_1 = 15000(1.005^1) - P$

After 2 months, $A_2 = A_1(1.005^1) - P$

$= 15000(1.005^2) - P(1.005 + 1)$

After 3 months

$A_3 = A_2(1.005) - P$

$= [15000(1.005^2) - P(1.005+1)](1.005) - P$

$= 15000(1.005^3) - P(1.005^2 + 1.005 + 1)$

$= 15226.13 - P(3.015025)$ ✓✓

ii) Following this pattern, given $A_{24} = 10000$

$10000 = 15000(1.005^{24}) - P(1.005^{23} + 1.005^{22} + \ldots + 1)$

$P(1.005^{23} + 1.005^{22} + \ldots + 1) = 15000(1.005^{24}) - 10000$

↑ Geom. Series. $S_{24} = \frac{1(1.005^{24} - 1)}{1.005 - 1}$

$\doteqdot 25.431955$

$P = [15000(1.005^{24}) - 10000] \div \frac{1.005^{24} - 1}{0.005}$

$= 6907.3966 \times \frac{0.005}{1.005^{24} - 1}$

$= \$271.60$ ✓✓✓

c) $y^2 - 8y + (-4)^2 = 2x - 4 + 16$

$(y-4)^2 = 2(x+6)$ ✓

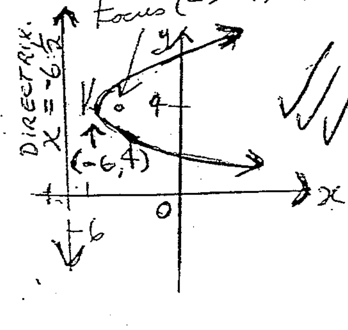
Parabola has

Vertex $(-6, 4)$

$4a = 2 \therefore a = \frac{1}{2}$

Focus $(-5\frac{1}{2}, 4)$

Directrix $x = -6\frac{1}{2}$.



✓✓✓

9) a) $y = \sqrt{\dfrac{2x}{3x^2-1}}$

$V = \pi \int_1^3 y^2 \, dx$

$= \pi \int_1^3 \dfrac{2x}{3x^2-1} \, dx$

$= \dfrac{\pi}{3} \int_1^3 \dfrac{6x}{3x^2-1} \, dx$

$= \dfrac{\pi}{3} \left[\ln(3x^2-1) \right]_1^3$

$= \dfrac{\pi}{3} \left[\ln 26 - \ln 2 \right]$

$= \dfrac{\pi}{3} \ln 13$. ✓✓✓

b) $\dfrac{dV}{dt} = \dfrac{1}{100}(30t - t^2)$

(i) When $t = 15$

$\dfrac{dV}{dt} = \dfrac{1}{100}(30(15) - 15^2)$ ✓

$= 2.25 \text{ cm}^3/\text{min}$

ii) $V = \int_0^{15} \dfrac{1}{100}(30t - t^2) dt$

$= \dfrac{1}{100}\left[\dfrac{30t^2}{2} - \dfrac{t^3}{3} \right]_0^{15}$

$= \dfrac{1}{100}\left[15^3 - \dfrac{15^3}{3} - 0 \right]$

$= 22.5 \text{ cm}^3$. ✓✓

c) $\cos 2\theta = -\dfrac{1}{\sqrt{2}}$, $0 \le \theta \le 2\pi$, $0 \le 2\theta \le 4\pi$

$2\theta = 45°$

$= \dfrac{\pi}{4}$ (in 2nd, 3rd Quad)

$2\theta = \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{3\pi}{4} + 2\pi, \dfrac{5\pi}{4} + 2\pi$

$\therefore \theta = \dfrac{3\pi}{8}, \dfrac{5\pi}{8}, \dfrac{11\pi}{8}, \dfrac{13\pi}{8}$.

✓✓✓

9d) Negative definite

$\Delta < 0$, $a < 0$.

$a = -4 \therefore a < 0$.

$\Delta = 16(m+1)^2 + 4(-4)(4m+1)$

$= 16(m+1)^2 - 16(4m+1)$

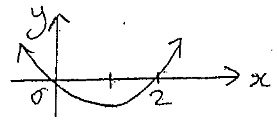
$= 16m^2 + 32m + 16 - 64m - 16$

$= 16m^2 - 32m$

$= 16m(m-2)$

Let $16m(m-2) = 0$

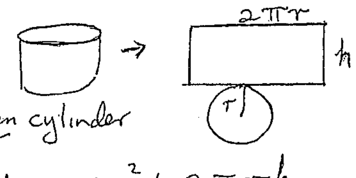
$m = 0$, $m = 2$.



$\Delta < 0$ when

$0 < m < 2$. ✓✓✓

10) i)



$A = \pi r^2 + 2\pi r h$

$2\pi r h + \pi r^2 = 300$

$2\pi r h = 300 - \pi r^2$

$h = \dfrac{300}{2\pi r} - \dfrac{\pi r^2}{2\pi r}$

$= \dfrac{150}{\pi r} - \dfrac{r}{2}$

$V = \pi r^2 h$

$= \pi r^2 \left(\dfrac{150}{\pi r} - \dfrac{r}{2} \right)$

$\therefore V = 150r - \dfrac{\pi r^3}{2}$ ✓✓

ii) $\dfrac{dV}{dr} = 150 - \dfrac{3\pi r^2}{2}$

Let $V' = 0$ for stat. pts.

$\dfrac{3\pi r^2}{2} = 150$

$3\pi r^2 = 300$

$r^2 = \dfrac{100}{\pi}$

$r = \pm\sqrt{\dfrac{100}{\pi}} = \dfrac{10}{\sqrt{\pi}}$ $(r > 0)$

$\dfrac{d^2V}{dr^2} = -3\pi r$

when $r = \dfrac{10}{\sqrt{\pi}}$, $\dfrac{d^2V}{dr^2} < 0$

$\therefore$ max volume at $r = \dfrac{10}{\sqrt{\pi}}$

At $r = \dfrac{10}{\sqrt{\pi}}$,

$V_{Max} = 150 \times \dfrac{10}{\sqrt{\pi}} - \dfrac{\pi}{2}\left(\dfrac{10}{\sqrt{\pi}}\right)^3$

$= \dfrac{1500}{\sqrt{\pi}} - \dfrac{500}{\sqrt{\pi}}$

$= \dfrac{1000}{\sqrt{\pi}}$ ✓✓✓

b). $\dfrac{dP}{dt} = kP$

$\therefore P = P_0 e^{kt}$

$P_0 = 20\,000$,

When $t = 2$, $P = 25000$

$25000 = 20\,000 e^{2k}$

$\ln \dfrac{25\,000}{20\,000} = 2k . \ln e$, $(\ln e = 1)$

$2k = \ln \dfrac{5}{4}$

$\therefore k \doteqdot 0.11157$

$\therefore P = 20\,000 e^{0.11157t}$

i) If $t = 10$, $P = 20\,000 e^{0.11157 \times 10}$

$= 61\,034$ people. ✓✓✓

ii) $\dfrac{dP}{dt} = kP$

$= 0.11157 \times 61035$

Rate of change $= 6809$ people p.a.

(OR)

$\dfrac{dP}{dt} = 20000(0.11157) e^{0.11157}$

$= 2231.4 \, e^{0.11157t}$

When $t = 10$

$\dfrac{dP}{dt} = 2231.4 \, e^{0.11157 \times 10}$

$= 6809$ people/year ✓

c) $\log_a 2 + 2\log_a x - \log_a 6 = \log_a 3$

$2 \log_a x = \log_a 3 + \log_a 6 - \log_a 2$

$= \log_a \left(\dfrac{3 \times 6}{2} \right)$

$= \log_a 9$

$2\log_a x = 2\log_a 3$

$\therefore x = 3$. ✓✓